# ASTROCHALLENGE 2016 DATA RESPONSE QUESTIONS

## SENIOR ROUND

### INSTRUCTIONS

- This paper consists of 12 printed pages, excluding this cover page and appendices.
- Do **NOT** turn over this page until instructed to do so.
- You have 2 hours to finish all questions in this paper.
- At the end of the paper, submit this booklet together with your answer script.
- Your answer script should clearly indicate your school (and team number) on **EVERY** page, as well as the individuals in the said team on the first page.
- It is your team's responsibility to ensure that all pages of your answer script have been submitted.

**DRQ 1: Astronomical Reasoning [20 marks]**

Question I

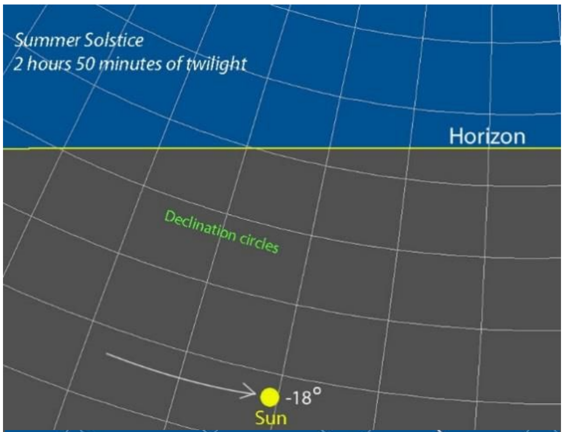
Rank these locations by increasing amount of twilight during the June solstice. Justify your answers. [4 marks]

- An observer at 45°N
- An observer at the equator
- An observer at 45°S

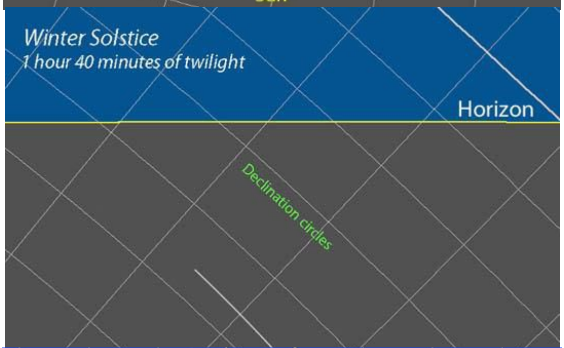
You are given that twilight occurs only if the sun is between 0° to 18° below the horizon.

*Hint: a formal mathematical proof is overkill, simple sketches with explanations will suffice.*

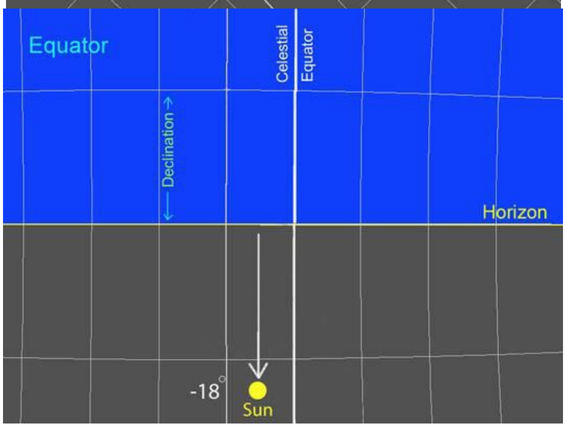
Equator followed by 45S followed by 45N (i.e. 45N has the longest twilight timing.)




45N experiencing summer. At locations away from the equator, declination lines intersect the horizon at a shallower angle. To reach the required –18°, the Earth must rotate more to "move" the Sun far enough below the horizon for night to begin. That means a longer twilight.




In contrast, during the winter, the declination line for the Sun curves downward below the horizon at a steeper angle, causing the altitude of the Sun to plunge faster than during the summer.




Amount of twilight is smallest at the equator as the Sun plunges nearly vertically with respect to the horizon.

Diagrams from [Sky and Telescope]

Question II

With suitable calculations, estimate the thickness of our Galactic Bulge based on the following diagram. State all of your assumptions **explicitly**. [3 marks]

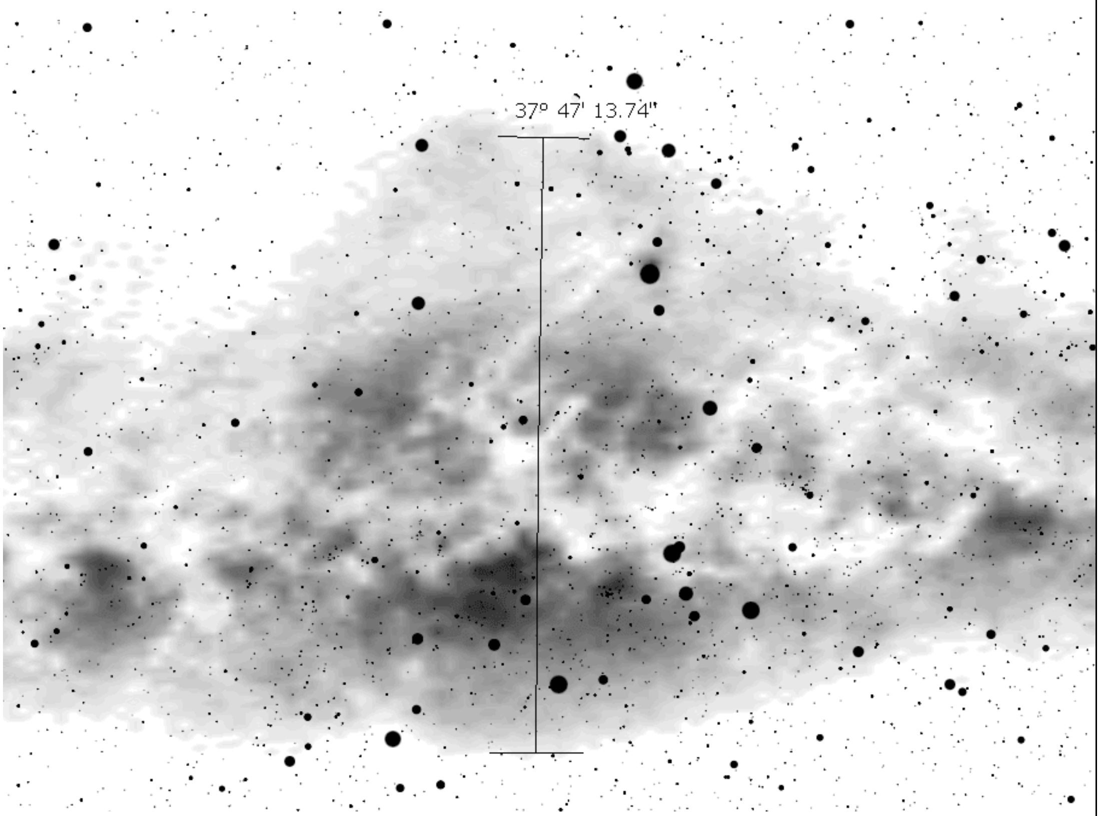



*Figure 1.1* A simulated widefield view of the Milky Way core.
Its colours have been *inverted* for clarity.

The actual distance between the galactic center to Earth/Solar System as 25900 ly with Sgr A* [this distance to be estimated by teams]

Model the galactic bulge as a sphere with uniform radius, $r_g$ and 37°47'≈37.8°

Using trigometric relationship, we have $\tan\frac{37.8°}{2} \approx \frac{r_g}{25900ly} \Rightarrow r_g \approx 8870ly$

Therefore the galactic bulge is approximately 2(8870) = 17740 ly thick

Question III

(a) I have a telescope with an aperture of X mm, set up such that it has a magnification of Y times. Given that a dark adapted pupil has a diameter of 7mm and has a limiting magnitude of 6, what is the limiting magnitude of a star through this setup? [3 marks]

(b) I then turn to a planetary nebula with a surface brightness of $S_{neb}$ and an area of A square arcseconds. The whole planetary nebula also has an apparent integrated magnitude $m_{neb}$. Express $m_{neb}$ in terms of A and $S_{neb}$. [2 marks]

(c) Through the scope, the apparent integrated magnitude of the planetary will increase by the same factor in part (a). Hence, find the new surface brightness S in terms of $S_{neb}$, X and Y. [3 marks]

*Hint: If something is magnified by Y times, its apparent diameter increases by a factor of Y.*

(a) Since brightness is proportional to the collecting aperture, the brightness gain is the ratio of the squares of the diameter, $(\frac{D_1}{D_{eye}})^2$. Recalling the relationship between brightness and magnitude, the equivalent **gain** in magnitude is then given as $5\log\frac{D_1}{D_{eye}}$. Hence, the limiting magnitude of the setup is $6 + 5\log\frac{X}{7}$. This means that the apparent magnitude of any object viewed through this scope changes by $-5\log\frac{X}{7}$. (i.e. it becomes brighter by this factor)

(b) The relationship between the apparent integrated magnitude and surface brightness, is given by $S_{nebula} = m_{nebula} + 2.5\log A$. To derive this, one has to understand that the surface brightness is the magnitude extended over a visual area. In other words, we have a nebula with A square arcseconds, with each square arcsecond having a surface brightness S. We thus simply sum up the brightness of each square arcsecond (brightness is additive), to obtain the integrated brightness of the nebula. After applying the relationship between brightness and magnitudes, you can find that the surface brightness is given by the addition of the integrated apparent magnitude and the area factor $2.5\log A$

(c) As seen in part a, the change in the apparent magnitude is given by $-5\log\frac{X}{7}$. Hence, $m'_{nebula} = m_{nebula} - 5\log\frac{X}{7}$

Using the relation derived in (b), $S'_{nebula} = m'_{nebula} + 2.5\log(Y^2 \cdot A)$

Why? The nebula will now appear to cover $Y^2$ times more area than it did with naked eye.

Which when simplified gives, $S'_{nebula} = m_{nebula} + 2.5\log\left(\frac{7^2 \cdot Y^2 \cdot A}{X^2}\right)$

NB: A more detailed explanation (with more math) can be found at http://www.rocketmime.com/astronomy/Telescope/MagnitudeGain.html

Question IV

Recall the Jean's Length formula:

$$R_J = \sqrt{\frac{15k_bT}{4\pi G\langle m\rangle\langle\rho\rangle}}$$

Where:

- $k_b$ represents the Boltzmann constant
- $T$ is the temperature of the gas cloud
- $G$ represents the gravitational constant
- $\langle m \rangle$ represents the mass per particle (a constant)
- $\langle \rho \rangle$ represents the density of the cloud

Now, suppose we have a molecular cloud with a uniform density. It is suddenly forced to compress such that its diameter decreases by a factor of n. During this process, the temperature of the cloud remains constant.

a) Find $R_J'$ (the new value of $R_J$) in terms of $R_J$ and n. [2 marks]
b) Hence, explain why stars tend to form in clusters, even in the absence of other star forming mechanisms (e.g. supernovae shock waves and stellar winds). [3 marks]

Using above assumptions, we simplify the relationship of Jean's length to just in terms of density of cloud $R_J \propto \frac{1}{\sqrt{\rho}}$

Recall that n represents the compression factor. This means that when n > 1, the cloud contracts (and vice-versa). During this contraction, the density of the cloud increases by a factor of $n^3$.

Hence $R_J$ decreases by a factor of $n^{3/2}$ *and* $R_J' = n^{-1.5}R_J$ [2 marks]

Since this is a contraction, $R_J$ decreases at a faster rate than $R$ (The Jean's length is now smaller than the radius of the cloud). This implies that portions of the cloud are now favoured to undergo their own contraction (especially if there are slight inhomogeneities). Thus, the collapsing molecular cloud tends to fragment into many protostellar cores during the process of collapse, causing the end product to be a cluster rather than a single star [3 marks]

## DRQ 2: Mass-Luminosity Relation [20 marks]

The relationship between the mass of a star and its Luminosity is an important feature which allows use to calibrate our mass measurements. However, this relationship is an empirical relationship. In order to establish a theoretical basis for this relationship, we start off with a massive star filled with an ideal gas of the same temperature throughout.

Question I

(a) Given that the Ideal-Gas Law is

$$PV = NkT$$

and that from integrating the hydrostatic equilibrium equation:

$$P \sim \frac{GM^2}{R^4}$$

find temperature *T* in terms of mass *M*, average mass $\bar{m}$, density, $\rho$. [2 marks]

*Note that number density,

$$n = \frac{N}{V} = \frac{\rho}{\bar{m}}$$

(b) Hence or otherwise, express luminosity *L* in terms of only mass *M* and other constants. [3 marks]

See attached document

We will now attempt to empirically affirm the results found previously. On **Appendix A**, attached at the back of this paper, you will find the data for some of the stars found in the Great Nebula in Orion, M42. In general, we believe that:

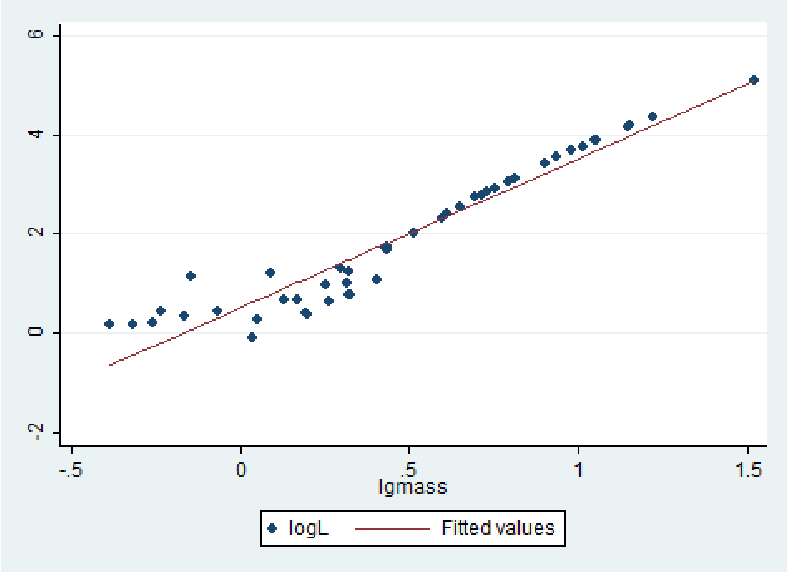
$$L \propto M^\alpha$$

where α depends on the mass of the star on the Main Sequence Branch. This implies that:

$$\frac{L}{L_{sun}} = (\frac{M}{M_{sun}})^\alpha \rightarrow \log\frac{L}{L_{sun}} = \alpha \log\frac{M}{M_{sun}}$$

Question II

(a) Given the data in Appendix A, plot a line of best fit on the graph paper provided and estimate the constant α. [8 marks]




```
Linear regression                               Number of obs =        47
                                                F(  1,     45) =    393.50
                                                Prob > F      =    0.0000
                                                R-squared     =    0.9159
                                                Root MSE      =    .41746
```

| logl | Coef. | Robust Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| lgmass | 2.994455 | .1509544 | 19.84 | 0.000 | 2.690417 | 3.298492 |
| _cons | .524042 | .123227 | 4.25 | 0.000 | .2758501 | .7722339 |

Thus α is approximately 3 (accept ± 0.2)

Question III

If the general mass-luminosity relationship is true, then there should be a relationship between the apparent bolometric magnitude m, and mass M.

(a) Express m as a function of M, α, distance d and other suitable constants. [2 marks]

We know Msun to be 4.7554 (see your formula book!). Working in absolute bolometric magnitude ($M_b$) first, we get

$$\frac{L}{L_{sun}} = 10^{\frac{M_b - 4.7554}{2.5}} \rightarrow \log\frac{L}{L_{sun}} = \frac{M_b - 4.7554}{2.5}$$

Equate this with what we got in the helptext of Q2 to get 1 mark

$$\alpha \log\frac{M}{M_{sun}} = \frac{M_b - 4.7554}{2.5}$$

$$M_b = -2.5\alpha \log\frac{M}{M_{sun}} + 4.7554$$

Next, from the distance modulus, we know that

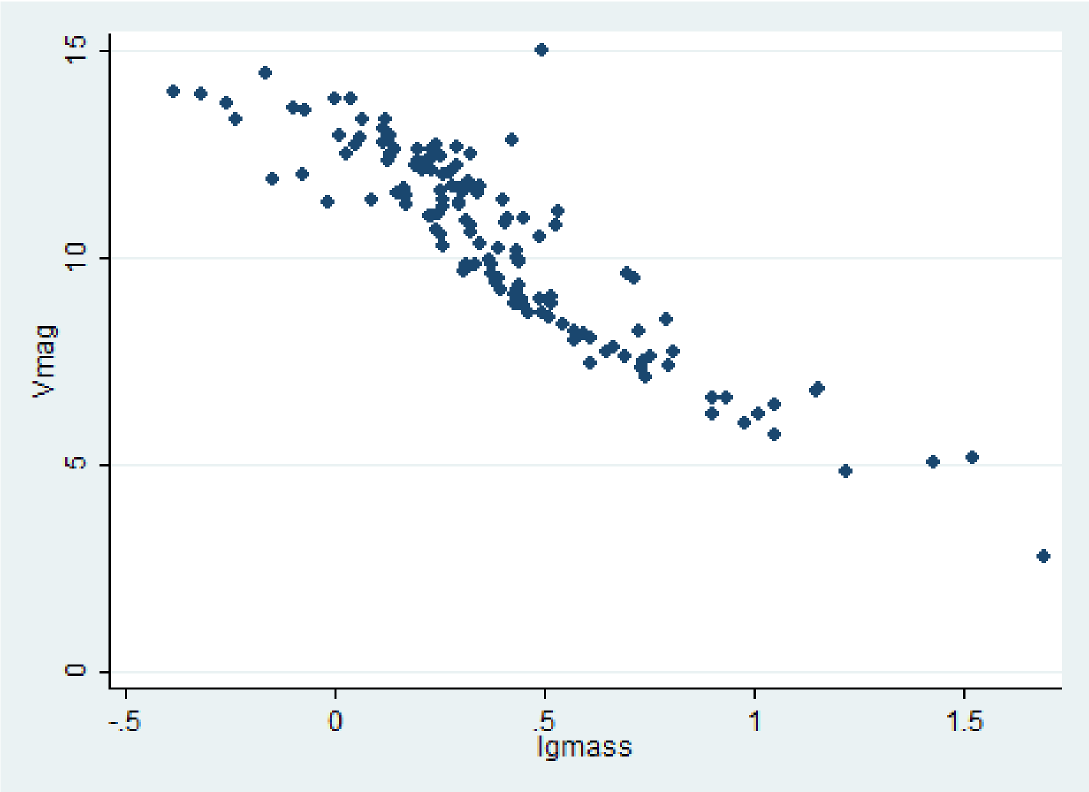
$$m - M_b = 5\log\frac{d}{10pc} \rightarrow M_b = m - 5\log\frac{d}{10pc}$$

Substitution then yields

$$m - 5\log\frac{d}{10pc} = -2.5\alpha\log\frac{M}{M_{sun}} + 4.7554$$

$$m = -2.5\alpha\log\frac{M}{M_{sun}} + 4.7554 + 5\log\frac{d}{10pc}$$

Using the full dataset of stars in M42, a student has plotted the following graph:




Through ordinary least squares, the student claims that the following linear relationship is the best fit to the data:

$$m_v = 12.8 - 6.2\log M$$

Where $m_v$ represents the apparent visual magnitude of the star, as measured through a standard V filter in the UBV photometric system. Log M here refers to the mass of the star, in solar units.

Question IV

(a) Using your answer in III(a), determine the implied value of α and d [2 marks]
Plugging in, you should get α=2.48 and d=406 pc
Amazingly, your value of d is consistent with current estimates of the distance to M42, in spite of the error explained in 4d.

(b) You should find that your two estimates of α are inconsistent. Which estimate of α is likely to be more accurate? [1 mark]
Your value in IIa is likely to be a better estimate

(c) Why is the other estimated value of α so wildly wrong? [2 marks]

Recall that we aren't using bolometric magnitudes here, we are using apparent visual magnitudes. This means that we need to worry about the inherent color of the star. In particular, higher mass main sequence stars tend to be hotter [1 mark]. This means that compared to low mass red dwarfs, they emit a greater proportion of their emitted radiation in the visible (rather than infrared), leading to a steeper gradient [1 mark].

**DRQ 3: The Hertzsprung-Russell Diagram [20 marks]**

**REFER TO ATTACHED DOCUMENT FOR THE WRITTEN SOLUTIONS TO THIS QUESTION.**

When we plot a classical H-R diagram of a cluster, we see a clear "turn-off" where stars are moving into the red giant phase. However, in modern observational versions of the H-R diagram, the temperature (or spectral type) scale on the HR diagram is replaced by the colour index, and the luminosity is replaced by the apparent magnitude to produce a colour-magnitude diagram.

How do we measure "colour"? We do so by taking 2 images of the star through 2 standard filters, giving what is known a color index. Today, most often, the B-V colour index is employed, referring to the difference in apparent magnitudes through a B (blue) and V (visual) filter. Under this system, a blue star will have negative colour index (as its brighter through the B filter than the V filter); while a yellow or red star will have positive colour index. Hence, the color index gives us a gauge of the star's temperature.

For the purpose of our discussion, we will be focusing on the g'-r' index for the SLOAN narrowband filters which allows (peak) wavelength **477 nm** (green) and **6250nm** (red), with bandwidth of **140nm** individually (Fukugita, et al., 1996).

Question I

(a) Other than color, luminosity and temperature, suggest another piece of information a color-magnitude diagram can yield. [1 mark]

(b) Explain how can we obtain this piece of information. [2 marks]

Question II

We will now attempt to prepare the tools required for the calculation of the g' – r' value.

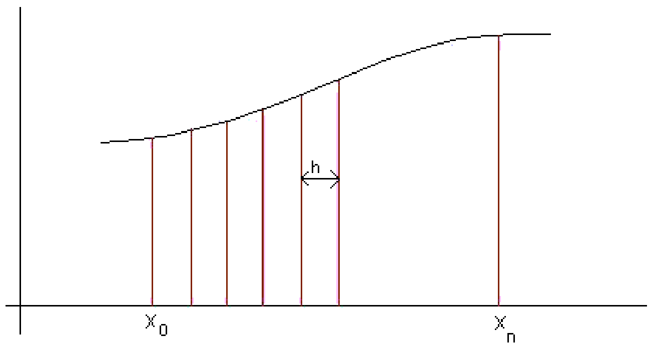
(a) Given, $c = f\lambda$, find $\frac{d\lambda}{df}$. Then, prove that the following relationship is true:

$$I = \int_a^b \frac{2\pi \mathrm{hc}^2}{\lambda^5\left(e^{\frac{\mathrm{hc}}{\lambda \mathrm{kT}}} - 1\right)}\, d\lambda = \int_c^d \frac{2\pi \mathrm{h}}{c^2}\left(\frac{f^3}{e^{\frac{\mathrm{hf}}{\mathrm{kT}}} - 1}\right)\, df$$

where $I$ is the total intensity (aka luminosity per unit surface area) integrated over the entire emission spectrum of the star. [5 marks]

(b) Knowing the bandwidth and the 'peak wavelength' observed through the filter allows us to calculate the observed intensities. Suggest appropriate integration limits to the definite integral above and find the supposed intensities of a star of temperature $T$ for the two different narrowband filters, using the Trapezium Rule or otherwise.

(continued on the next page)




Split the area under a curve into a number of trapeziums with some fixed area.
If we want to find the area under a curve between the points $x_0$ and $x_n$, we divide this interval up into smaller intervals, each of which has length *h* (see diagram above).

$$\int_{x_0}^{x_n} f(x)\,dx \approx \frac{1}{2}h((y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1}))$$

where $y_0 = f(x_0)$ and $y_1 = f(x_1)$ etc.

(i) Suggest an appropriate value for n and hence width *h* for the integration. [1 mark]
(ii) Suggest appropriate integration limits (*a* and *b*) for the equation found in II(a). [1 mark]
(iii) State the assumption required for your answer in part (ii). [1 mark]
(iv) Hence, calculate the respective intensities for the g' and r' filters ($I_{g'}$ and $I_{r'}$), given that *T* = 6000K by summing the trapeziums. [7 marks]

Alternatively, you are more than welcome to do the integration directly to find the exact values. [10 marks]
Note: this is only for the **brave souls** with **too much time and brain power** to spare.

- It might be helpful first to note that $e^{\frac{\mathrm{hc}}{\lambda \mathrm{kT}}} \gg 1$ for $\lambda << 1$
- After the approximation, given that

$$\int \frac{hc}{kT\lambda^2} e^{-\frac{hc}{kT\lambda}}\ \mathrm{d}\lambda = e^{-\frac{hc}{kT\lambda}}$$

- apply integration by parts 4 times

$$\int u\mathrm{dv} = uv - \int v\mathrm{du}$$

An increase in (apparent/absolute) magnitude corresponds to a decrease in brightness by a factor of 2.5. As previously mentioned, the magnitudes can be used to compare the apparent brightness of the two stars or against a reference. For two stars of magnitudes $m_1$ and $m_2$ and apparent brightness $b_1$ and $b_2$, respectively, we have $\frac{b_1}{b_2} = 2.5^{m_2 - m_1}$. Applying this concept for one star instead of 2 but for 2 different intensity spectrum we may find $g' - r$ expressed as the difference between the intensities.

(i) State the equation of $g' - r$ expressed in terms of the intensities. [1 mark]

(ii) Hence calculate the g' – r' value for the given star at 6000K. [1 mark]

**DRQ 4: The Dark Side of Jupiter [20 marks]**

**REFER TO ATTACHED DOCUMENT FOR THE WRITTEN SOLUTIONS TO THIS QUESTION.**

After watching The Transformers: Dark Side of the Moon, Jyh Harng asked during an observation session if it's possible to see the dark side of Jupiter as he was bored of looking at the bands of Jupiter and the Galilean moons through the telescope.

You will now investigate if it is indeed possible to see the dark side of Jupiter from Earth at any time of Jupiter's revolution about the Sun.

Question I

(a) Sketch a diagram showing the Earth, Jupiter and Sun on the same plane, indicating the angle subtended between the Sun and Earth relative to Jupiter, as $\theta$. [3 marks]

(b) When will the angle $\theta$ be largest? [1 mark]

(c) Calculate this largest value of angle $\theta$. You may assume that the orbits of Earth and Jupiter are circular. [2 mark]

(d) Hence or otherwise, calculate the illuminated phase of Jupiter in percentage of the surface area visible from Earth using the information found in (b) and (c). [2 marks]

Hint: Instead of calculating the illuminated surface area of Jupiter, work with longitudes. Under this system, if Jupiter is fully illuminated, we can see all 180° of its visible surface. The current illuminated phase of Jupiter is then a simple ratio, with the denominator being 180°.

Jyh Harng was also eager to look out for the Great Red Spot (GRS) and wondered when the next time he might be able to see it on the surface of Jupiter would be so that he could image Jupiter with the GRS on it.

The table shows transit times of the Great Red Spot on Jupiter, for a certain year:

| Date | Time |
|---|---|
| 7th June | 04:23:00 |
| 7th June | 14:19:00 |
| 8th June | 00:15:00 |
| 8th June | 10:10:00 |
| 8th June | 20:06:00 |
| 9th June | 06:02:00 |
| 9th June | 15:58:00 |
| 10th June | 01:53:00 |
| 10th June | 11:49:00 |

Question II

(a) Using the information provided above calculate the average period of rotation, T of the Great Red Spot. [2 marks]

(b) Jupiter currently has an azimuth of 267° and an altitude of 24°. Hence, when would be the next time Jyh Harng can see the Great Red Spot just appearing east of Jupiter? [4 marks]

Being a physics major, Jyh Harng is now interested to calculate the tangential velocity of the Great Red Spot given the information above.

(c) It is known that the Great Red Spot is 22° south of the equator on Jupiter, find the approximate tangential velocity, $v_{\parallel}$ of the Great Red Spot. [1 mark]

Joel, a friend of Jyh Harng who dreams of being an astronaut wonders how the Dark Side of Jupiter looks like. Joel, a physics major as well, decides to calculate the Hohmann transfer orbit required to arrive at Jupiter from Earth by neglecting the gravitational potential energy of Jupiter and Earth.

The Hohmann transfer orbit is an elliptical orbit used to transfer between two circular orbits of different radii on the same plane. To perform the Hohmann transfer, 2 engine impulses (instantaneous velocity changes, $\Delta v$) are required. The first one to move the spacecraft onto the transfer (intermediate) orbit and the second one to move it off into the orbit of the other planet/object.

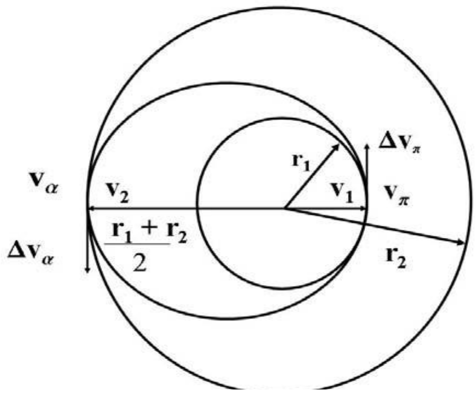



*Figure 4.1* The Hohmann transfer orbit

Question III

(a) It is known that the velocity of an object in circular orbit at some distance $r$ around the sun is $\sqrt{\frac{GM_{\text{sun}}}{r_1}}$, while the velocity required to move into the elliptical orbit at $r = r_1$ from $r_1$ circular orbit is given as $v^2 = GM_{\text{sun}}(\frac{2}{r_1} - \frac{2}{r_1+r_2})$. Given this information, calculate $\Delta v_1$ for the **first impulse** and $\Delta v_2$ for the **second impulse**. [3 marks]

(b) Given that $v_{\text{exh}} = 2989.1$ m/s find the total change in mass for the 2 impulses.
[2 marks]

**DRQ 5: Lost in a Forest Far Far Away [20 + 2 bonus marks]**

Not long ago, there was a group of young astronomers camping in a forest. While they were setting up a telescope, dark clouds began to roll in. Worried, they ran for shelter, and found a road leading to an observatory on top of a faraway hill. As thunder roared in the distance, they found an entrance into the observatory. The hallways were dimly lit without a single person in sight. As the group progressed, they found a monument scribbled with the following formula:

$$\frac{1}{\lambda_{vac}} = R_H\left(\frac{1}{{N_1}^2} - \frac{1}{{N_2}^2}\right)$$

Equation 1: Rydberg formula for Hydrogen

Where $\lambda_{vac}$ is the wavelength of electromagnetic radiation emitted in vacuum (in meters, m),

$R_H$ is the Rydberg constant for Hydrogen, approximately 1.097 x $10^7$ $m^{-1}$,

$N_1$ is the principal quantum number of the orbitals occupied after

$N_2$ is the principal quantum number of the orbitals occupied before

As lightning flashed across the sky, the team decided to enter a door labelled 'Lyman', hoping to find someone. Alas, all they could find was another equation scribbled on the desk with a note: "Should you have absolutely no idea what is going on, read up the appendix for a crash course on the Hydrogen atom. "

Could they solve this mystery?

Question I

(a) Show that Lyman series where $N_1$ = 1, the Rydberg equation can be expressed as:

$$\lambda_{vac} = \left[R_H\left(1 - \frac{1}{{N_2}^2}\right)\right]^{-1}$$

[0.5 mark]

(Its just making $\lambda_{vac}$ the subject after substitution. Giveaway for senior category.)

(b) Calculate the peak emission wavelength (in nm) of the Lyman alpha spectral line (Lyα), where $N_2$ = 2. [0.5 mark]

$$\lambda_{vac} = \left[R_H\left(1 - \frac{1}{2^2}\right)\right]^{-1} \times 10^{-9} = \left[R_H(0.75)\right]^{-1} \times 10^{-9} = 121.544 \text{ nm}$$

As the team looked out of the window, they are greeted by the sight of a dim, misty and rainy forest. On the table next to the equation is an extremely messy graph. One student with some knowledge about cosmology recognised it as a spectrum of an extremely distant quasar. There's no mistake – the telescope and spectroscopy equipment were measuring this high redshift quasar. The Lyα line, the most prominent emission line of Hydrogen, is measured, but interestingly it is followed by many Lyα absorption lines, as shown in the next page.

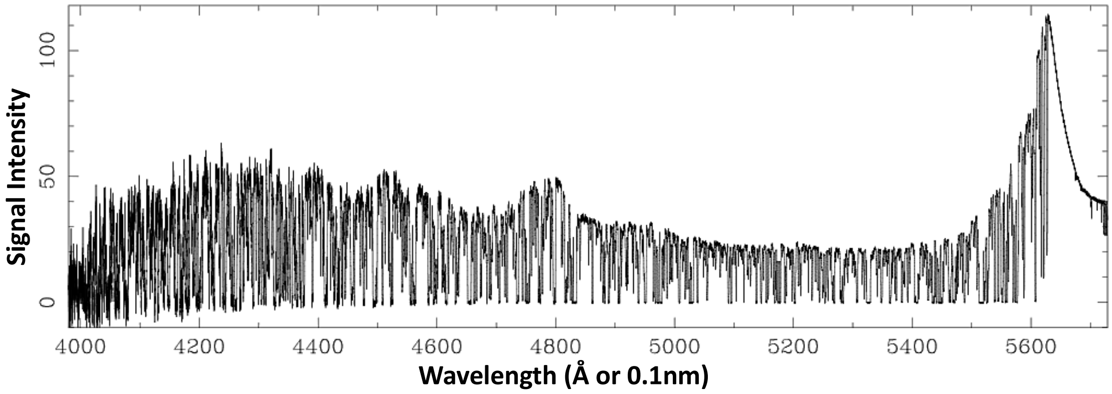



*Figure 5.1* Lyman alpha forest characteristic of the spectrum of a distant, high redshift quasar. The peak Lyα emission wavelength is at 563 nm, followed by the messy 'forest' of shorter Lyα absorption wavelengths. Adapted from Rauch, 1998.

<u>Question II</u>

(a) Briefly explain how Cosmological Redshift occurs (Hint: this phenomena resulted in the difference between your Lyα value from (a) (ii) and the above graph). [1.5 marks]

As a result of the expansion of the universe; Space between Earth and distant objects (e.g. this quasar) is increasing; It is moving away from us at a high velocity due to expansion; Light which eventually reaches us ends up of a lower frequency/ higher wavelength;

(b) Calculate the redshift of the quasar [0.5 mark].

z = (653 – 112.544)/ 112.544 = 4.802175… = 4.802

(c) Show that the velocity of the quasar as a fraction of the speed of light (v/c) based on relativistic redshift (formula in formula booklet) can be expressed in the following form:

$$\frac{v}{c} = \frac{(z+1)^2-1}{(z+1)^2+1}$$

[3 marks]

<u>Suggested Answer:</u>

$$z = \sqrt{\frac{c+v}{c-v}} - 1; \quad \frac{c+v}{c-v} = (z+1)^2; \quad \frac{1+\frac{v}{c}}{1-\frac{v}{c}} = (z+1)^2; \quad \frac{1+\frac{v}{c}}{1-\frac{v}{c}} - \frac{1-\frac{v}{c}}{1-\frac{v}{c}} = (z+1)^2 - 1;$$

$$\frac{2\frac{v}{c}}{1-\frac{v}{c}} = (z+1)^2 - 1; \quad \frac{2\frac{v}{c}}{1-\frac{v}{c}} \div \left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}} + \frac{1-\frac{v}{c}}{1-\frac{v}{c}}\right) = [(z+1)^2 - 1] \div \left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}} + \frac{1-\frac{v}{c}}{1-\frac{v}{c}}\right);$$

$$\frac{2\frac{v}{c}}{1-\frac{v}{c}} \div \frac{2}{1-\frac{v}{c}} = [(z+1)^2 - 1] \div [(z+1)^2 + 1] = \frac{(z+1)^2-1}{(z+1)^2+1}; \quad \frac{v}{c} = \frac{(z+1)^2-1}{(z+1)^2+1}$$

(d) Using the redshift from (b) and the equation from (c), calculate the velocity of the quasar in terms of c (speed of light), and write down any comments/ interesting observations about this value. [1 mark]

$$\frac{v}{c} = \frac{(z+1)^2-1}{(z+1)^2+1} = 0.942305\ldots$$

i.e. The quasar is moving away from us close to the speed of light (c = 1)!

(e) Calculate the distance by which the quasar is away from us, taking $H_0$ to be 67.8km/s/Mpc. [1 mark]

v = $H_0$d; convert all to S.I. units, d = v/$H_0$ = $1.28568\times10^{26}$ m = $4.16660512\times10^{9}$ Parsecs = etc.

(f) Assuming only empty space between Earth and this quasar, would you expect to see a Lyman Alpha forest? With this in mind, suggest what causes the Lyman alpha forest, as well as discuss the significance of this phenomenon. You may illustrate these concepts with the aid of another graph or diagram. (Hint: Think about the misty forest on a cosmological scale, and read the appendix if you're lost) [4 marks]

No; there would have been no means for Hydrogen to be absorbed by empty space to generate this absorption spectrum. This is because space isn't truly empty; many neutral hydrogen clouds and ionised hydrogen clouds exists even between us and a distant object; these clouds are capable of absorbing Lyα lines, causing it to disappear for a portion of the spectrum. Clouds at varying distances absorb the Lyα emission at varying wavelengths, generating this 'forest'. Thus, when observing the quasar, we are indirectly observing all the clouds in between. The absorption lines are indicative of the redshift and thus distance of each Hydrogen cloud, allowing us to use it as a probe of the intergalactic medium to determine the frequency and density of clouds containing neutral hydrogen. Bonus mark: If candidates go on to talk about the Gunn–Peterson trough for quasars that are far away enough and reionisation of the Universe, 1 extra mark.

The team then entered a door named 'Balmer'. Oddly, when they looked out of the window, all they could see was a few distant mountains. Likewise, a note lies on the table, this time with a slightly different equation, and a large optical telescope is in the middle of the room next to a spectrometer.

For the Balmer series, where $N_1$ = 2, the Rydberg equation can be expressed as:

$$\lambda_{vac} = [R_H(0.25 - \frac{1}{{N_2}^2})]^{-1}$$

Question III

(a) Calculate the peak emission wavelength of the first four Balmer spectral lines, then show that all of them exists within the range of the visible spectrum (about 400 – 700 nm). [1 mark]

$$\lambda_{vac} = [R_H(0.25 - \frac{1}{\{3,4,5,6\}^2})]^{-1} = \ 656.335, 486.174, 434.084, 410.210 \ \ nm$$

(b) In addition, a ultraviolet spectrometer report a spectral line reading of 301nm. The object is an old hydrogen lamp at rest. Could that line have been from the Balmer series? (Hint: proof by contradiction by first finding the Balmer limit: The minimum theoretical wavelength of the Balmer series.) [1.5 marks]

$$\lambda_{vac} = \lim_{n\to\infty}\left(R_H(0.25 - \frac{1}{n^2})\right)^{-1} = R_H(0.25)^{-1} = 364.631 \ \ nm \ \ (Balmer \ \ limit)$$

Ans: no; the wavelength is shorter than the Balmer limit (i.e. even an ∞ → 2 transition won't suffice)

Similarly, a graph of two quasars is found on the table, but there are no supporting data on their identity. Nevertheless, since the peaks correspond to the Balmer series, they are probably not very far away compared to the high red shift quasar. Can they solve this mystery as well?

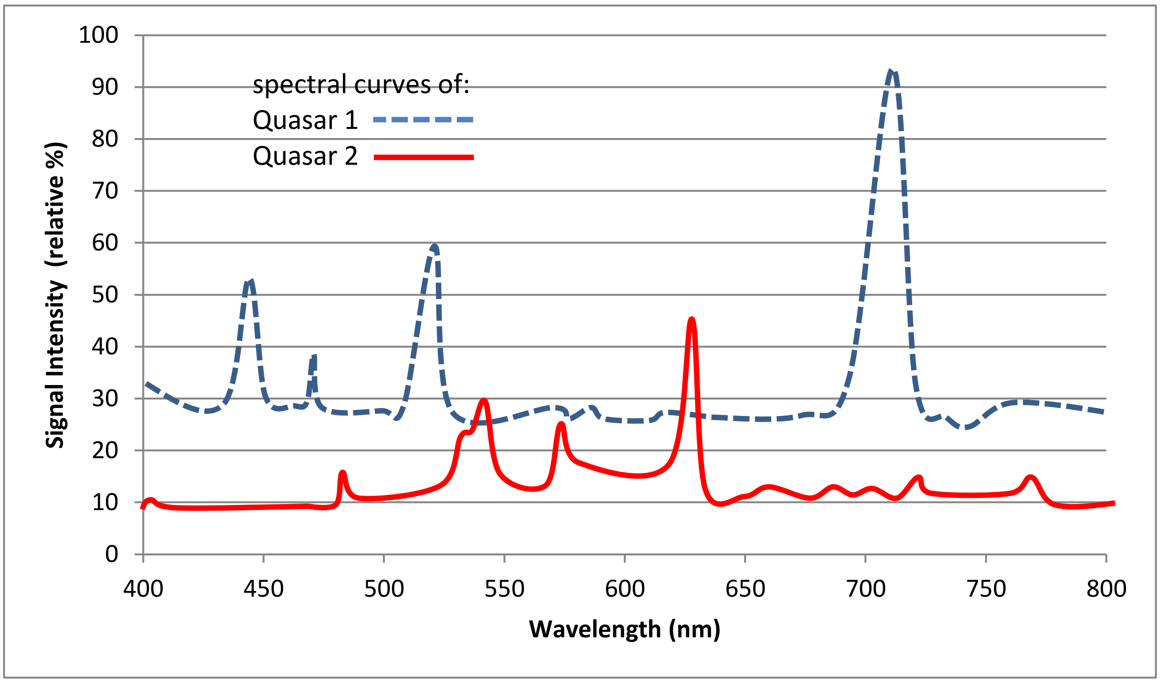



*Figure 5.2* Balmer emission spectra for two hypothetical, relatively lower redshift quasars.

Question IV

(a) Calculate the redshift of both quasars using the Balmer series by estimating the values of the peaks from the graph, and hence determine which quasar is receding away from us at a faster velocity. [3 marks]

Theoretical values for:

Q1 {713.437, 528.472, 471.85, 445.898} Q2 {854.549, 632.999, 565.178, 534.093}

Actual redshift values might differ depending on candidates' interpretation of the peaks, and peaks used (2 marks, one for each redshift). As long as candidates recognise that the triplet peak is the Balmer series, and is able to determine Quasar 2 as the more distant quasar, they get 1 mark. Quasar 2 has a lower signal intensity as well, which indicates it to be less well observed, but they get only 0.5 marks.

(b) Briefly explain the significance of using the Blamer Series over the Lyman Alpha series for low redshift quasars. [0.5 mark]

They are within the visible spectrum and are observable by a good light microscope. OR there are multiple Balmer lines to verify the identity of all the other spectral lines.

(c) Why are only three peaks present in Quasar 2? [0.5 mark]

The Balmer alpha line/ 4$^{th}$ peak (~850 nm) is redshifted outside the range of the graph and the visible spectrum due to its velocity.

(d) On their way back, the team passed by data pertaining to a third object, this time a star. This star is moving away from Earth at 200km/s. However, it is only about 100 Parsecs away from us. Prove that its movement is not a result of the expansion of the universe, and briefly suggest an alternative explanation for its high speed. [1.5 marks]

$V = H_0 d$; $V/H_0 = d$; 200/67.8 = 2.95 MEGA parsecs =/= 100 Pc.

i.e. it is not possible for it to be moving away from us due to the expansion of the universe, but likely because it is a runaway star (e.g. flung out by a supernova of a binary companion)

As the team gathered back in the lobby, they realised they have been on opposite ends of the observatory. In retrospect, they saw a misty forest in the Lyman room, but saw only mountains in the Balmer room. It struck them how coincidental it was that the graphs of each respective spectrum in the rooms ended up in a similar fashion. This leads to a final mystery, as heavy rain droplets begin to pour…

Bonus Question

Why would you observe a Lyman alpha forest, but not a Balmer alpha forest? [2 marks]

Neutral Hydrogen exists at Ground state; electron at n = 1 orbital, and thus have a high tendency to absorb Lyα for a 1 → 2 absorption; 2 → 3 absorption is rare and not likely to occur afterwards; Hydrogen clouds are not energised and do not possess electrons at n = 2 orbitals most of the time; effects might be insignificant or undetectable; etc.

If you're reading this after (hopefully) surviving this ordeal, the Astrochallenge 2016 team will like you to vote for what you thought is the most frightening thing about this question. Please indicate it EITHER next to your School (and team number), OR at the end of the answers to this question. Do not spend too much time describing this section in graphic detail unless your team is confident of finishing the rest of this paper.

The options are:

A. A lot more theory/ explanation questions than expected.

B. Bad weather.

C. Calculation questions were still tough.

D. Difficulty in understanding content.

E. Something else entirely – do indicate briefly.