AstroChallenge 2023
Senior MCQ Round

SOLUTIONS

Monday 29th May 2023

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. This paper consists of **33** printed pages, including this cover page.
2. Do **NOT** turn over this page until instructed to do so.
3. You have **2 hours** to attempt **ALL** questions in this paper. If you think there is more than one correct answer, choose the *most* correct answer.
4. At the end of the paper, submit this booklet together with your answer script.
5. Your answer script should clearly indicate your name, school, and team.
6. It is *your* responsibility to ensure that your answer script has been submitted.

1. Which of the following are **not** one of the ways astronomers measure the Hubble's constant?

   (A) **Using the merger of two black holes or neutron stars and measuring the subsequent gravitational waves to get their relative speeds to Earth**

   (B) **Using the flat rotational curve model and absolute magnitude of Active Galactic Nuclei (AGN) to get their relative speeds to Earth**

   (C) Using spectroscopic analysis of a distant object of known distance to determine its relative redshift to Earth

   (D) Use bubbles in the CMBR and the flat universe model to determine the distance of certain Supernovae to determine its relative speed to Earth

   (E) All of the above methods are valid methods of measuring Hubble's constant

   Solution:

   Correction: A is also correct as the merger of these stellar objects can only imply the distances through the simultaneous detection of light (which velocity can be affected from optical effects) to and gravitational wave (which is largely unaffected). This method is not sufficient to measure to measure the recessional velocity required to determine the value H_0

   The Flat Rotational Curve Model method is used to measure the absolute magnitude of a galaxy and gives no information on the recessional velocity of the galaxy. To determine the value of H_0, we need both the galaxy's distance and their recessional velocity.

2. In both stars and stellar remnants (such as neutron stars), there is an outward force supporting them such that hydrostatic equilibrium is achieved. Which of these is not one of the main supporting forces in stars or stellar remnants?

   (A) **Proton degeneracy pressure**

   (B) Electron degeneracy pressure

   (C) Neutron degeneracy pressure

   (D) Thermal pressure

   (E) Radiation pressure

   Solution:

   Most main-sequence stars and supergiants produce thermal radiation pressure from the various fusion reactions happening within the stellar core. Electron degeneracy and neutron degeneracy pressure are the main outwards force in white dwarves and neutron stars, respectively.

   Proton degeneracy rarely happens in stellar remnants as sufficiently massive stars have the gravitational pressure to combine the existing protons and electrons into neutrons.

3. Which of the following will likely happen in the event of a collision between two spiral galaxies?

   (A) Many stars will collide and collapse into black holes

   (B) The two galaxies' mass will be condensed into a supermassive black hole

   (C) A dramatic increase in stellar formation will be visible

   (D) The galaxies will together form a lenticular galaxy

   (E) None of the above, since intergalactic collisions are improbable given intergalactic distances

   Solution:

   Star collisions and black hole formation will be unlikely since most space within galaxies are empty space. However, there would be an increase in stellar formation as interstellar medium collide with one another, collapse, and subsequently create new cradles for star formation.

4. Carbon is a highly versatile element for life due to its high valency of 4, allowing it to form a wide range of organic molecules. A carbon atom can bond to 4 different atoms or groups of atoms in a tetrahedral structure, which can lead to chirality. What is the relevance of this in Astrobiology?

   (A) There is a preference for a certain chirality of biomolecules, so organisms placed in exotic environments with molecules of the opposite chirality may not survive.

   (B) This opens new possibilities of finding extraterrestrial life using molecules of different chirality.

   (C) Due to the unique configurations of certain molecules, the mismatch in the conformations of biomolecules and the active sites of enzymes may lead to dire consequences.

   (D) Only A and B are correct.

   (E) A, B, and C are all correct.

   Solution:

   It is true that molecules of opposite chirality often exist in an equilibrium, and there are enzymes that facilitate such transitions as well. However, it is also known that molecules of a wrong configuration can cause poisoning due to enzyme inhibition, etc.

5. Scientists have discovered a new star. It has an apparent magnitude of 0.16, a stellar parallax of 0.42”/year, and a temperature of 4758 K. Calculate the luminosity of the star in terms of solar luminosities.

(A) 0.07

(B) 0.58

(C) **3.92**

(D) 5.33

(E) 11.39

Solution:

The distance of the star is $d = \frac{1}{0.42}pc \approx 2.381pc$, from which we can calculate its absolute magnitude.

$$m - M = 5\log\left(\frac{2.381}{10}\right)$$

$$0.16 - M = -3.1162$$

$$M = +3.2762$$

Expressing the star's luminosity in solar luminosity, we get:

$$M = 4.7554 - 2.5\log\left(\frac{L}{L_{Sun}}\right)$$

$$\log\left(\frac{L}{L_{Sun}}\right) = \frac{4.7554 - 3.2762}{2.5}$$

$$\frac{L}{L_{Sun}} = 10^{0.59168} \approx 3.91$$

6. Black holes are regions of space where matter collapsed in on itself, creating a singularity. This collapse is so strong that not even light can escape the black hole's gravity. As a result, black holes are often described as being "invisible" because they do not emit any light or other radiation that we can detect.

   What methods do scientists use to measure the mass of "invisible" black holes?

   i Direct modeling of the motions of resolved stars that are in orbit near the black hole

   ii Finding them in binary systems and measuring the motion of the companion object

   iii Measurement of emitted gravitational waves in black hole mergers

   iv Measuring the temperature difference between the black hole's core and event horizon

   (A) i and ii

   (B) ii and iii

   (C) i and iv

   (D) i, ii, and iii

   (E) All of the above

   Solution:

   We analyze the elongated orbits of visible stars to observe some black hole, including the one in our galaxy's center. If two black holes orbit one another, the motions of each will depend on their masses which may also produce gravtational waves. Measurement of the black hole singularity temperature would be impossible.

7. Let the apparent surface area of an exoplanet be 1.0% of that of its host star. When the exoplanet transits in front of the star, determine the maximum change in apparent magnitude of the star.

   (A) 0.011

   (B) 0.015

   (C) 0.018

   (D) 0.021

   (E) 0.025

   Solution:

   When the exoplanet is completely in front of the star, the apparent luminosity of the star will be 99% as the exoplanet covers 1% of its host star's luminuous area.

   $$\Delta m = -2.5 \log(0.99)$$

   $$\Delta m \approx 0.011$$

8. Scientists use a variety of methods to identify planets outside of the solar systems, with some orbiting other stars and others wandering in interstellar space as rogue planets. Depending on their size, orbit, and circumstances, methods used may also change accordingly. Which of the following are methods used to detect exoplanets (including rogue planets)?

   i Radial Velocity Method

   ii Transit Method

   iii Gravitational Microlensing Method

   iv Direct Imaging Method

   (A) i and iii

   (B) ii, iii, and iv

   (C) ii only

   (D) i and iv

   (E) All of the above

   Solution:

   1: If the exoplanet's orbital plane coincides with our line of sight, their orbital motions will have a periodic radial velocity - from which we can extract information about the orbits through Doppler shift.

   2: Similarly, if the exoplanet is sufficiently large relative to its host star and its orbital plane coincides with our sight, we can observe its transit from the dip in the star's apparent luminosity.

   3: Gravitational microlensing may occur when a sufficiently large exoplanet passes through our sight of a distant star and converges its light, creating a brief uptick in its apparent luminosity.

   4: Albeit much more difficult, an exoplanet can be observed directly given the host star is dim and distant enough from the exoplanet.

9. The central region of a planetary nebula has an apparent magnitude of 13.8 and a surface area of 3.16 square arcseconds. Therefore, its average surface brightness in magnitude per square arcseconds is

   (A) 4.4

   (B) 6.87

   (C) 10.4

   (D) 15.1

   (E) Insufficient information to answer this question

   Solution:

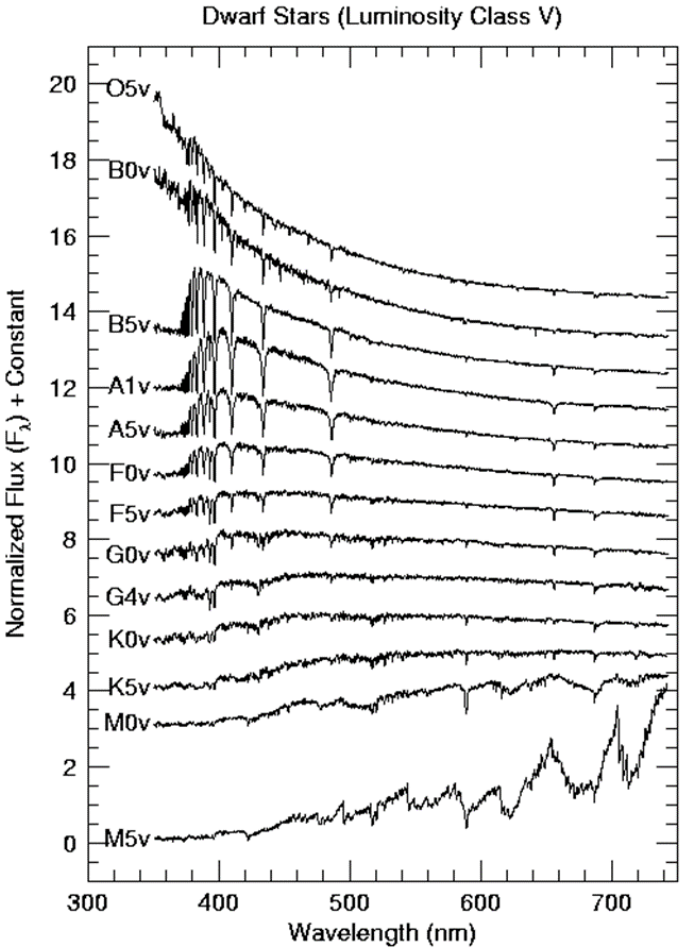
   A region has a surface brightness of M if the total flux emitted from 1 square arcsecond is equivalent to the flux emitted from a point source of magnitude M. Hence, we divide the luminosity by the apparent surface area of the nebula and get the magnitude equivalent to such

point source, from which we get:

$$m' = 13.8 + 2.5\log(3.16) \approx 15.1$$

Additionally, since the nebula is larger than 1 square arcsecond, the average surface brightness should be "dimmer" than the nebula's apparent magnitude, leaving $m = 15.1$ as the only possible answer.

10. In the spectra of B to F type stars, one can typically observe a sudden increase in the emission continuum around the 364.5 nm wavelength as one goes from shorter to longer wavelengths. This sudden increase is often known as the 'Balmer Jump'.



Which is the best explanation for this phenomenon?

(A) The Balmer Jump reflects the onset of fusion dominated by the CNO-cycle in stars hotter than G-type stars.

(B) At sufficiently high temperatures, singly ionized Hydrogen produces strong emission at the 364.5 nm wavelength, which is partially absorbed and re-emitted at lower wavelengths by metals in the stellar atmosphere.

(C) Light with wavelengths shorter than 364.5 nm have sufficient energy to ionize electrons from the first excited state of Hydrogen and is thus absorbed, thus producing a continuum absorption feature left of the jump.

(D) At sufficiently high temperatures, ionized metals such as Calcium and Titanium Oxide causes light with wavelengths shorter than 364.5 nm to be absorbed through Compton scattering.

(E) None of the above.

Solution:

A) is nonsensical since the presence of the CNO-cycle varies by different stellar mass, and not within the spectra of a single star.

B) is incorrect because the absorption of light by metals produces a continuum absorption along a range of wavelengths and will not produce the jump feature shown

D) is similarly incorrect since Compton scattering does not have a discrete wavelength cutoff needed to produce the jump feature.

11. Read the following passage and identify objects A and B.

With an integrated apparent magnitude of 1.9, **Object A** is one of the brightest star clusters in the night sky. As such, it can be seen with the naked eye in Singapore and at least most of the Southern Hemisphere on a good night. It forms one of the vertices of a famous asterism.

Object A's visibility makes it a convenient starting point for starhopping to Object B, which is less than 5 degrees north of Object A. **Object B** is a vast region of star formation and contains several star clusters within its boundaries. However, its most notable feature is that it contains a highly luminous and variable star at its center. This star experienced a significant outburst in the 19th century, briefly becoming the second-brightest star in the night sky

(A) Pleiades (M45) and Crab Nebula (M1)

(B) Pleiades (M45) and Flaming Star Nebula (C31)

(C) **Southern Pleiades (C102) and Carina Nebula (C92)**

(D) Southern Pleiades (C102) and Sagittarius Star Cloud (M24)

(E) 47 Tucanae (C106) and Tarantula Nebula (C103)

Solution:

The only star clusters listed that are visible in most parts of the Southern Hemisphere are Southern Pleiades and 47 Tucanae. Among these two, C102 is a part of a famous asterism - the Diamond Cross. Among C92 and C103, C92 is well known to have many star-forming areas and star clusters. The highly luminous star in question is Eta Carinae, which "erupted" in 1837 to form the now-Homunculus Nebula,

12. Capella is 46 light years away from Earth. Boss Rightyear is travelling on a spaceship to Capella at a speed such that the journey will take 20 years as measured by the clock on his spaceship. How long will his trip to Capella approximately take, as measured by clocks on Earth?

(A) 20 years

(B) 46 years

(C) **50 years**

(D) 64 years

(E) 82 years

Solution:

The length-contracted distance to Capella in Boss Rightyear's frame at a speed β (in terms of fraction of the speed of light v/c) is given by:

$$L = \frac{46}{\gamma}$$

where γ is the Lorentz factor $\dfrac{1}{\sqrt{1-\beta^2}}$ and L is in light-years.

The time of travel in years is thus:

$$T = \frac{L}{\beta} = \frac{46}{\gamma\beta} = 20$$

Solving for β,

$$\gamma\beta = \frac{\beta}{\sqrt{1-\beta^2}} = \frac{46}{20}$$

$$\frac{\beta^2}{1-\beta^2} = \left(\frac{46}{20}\right)^2$$

$$\beta = \sqrt{\frac{(46/20)^2}{1+(46/20)^2}} = 0.917$$

Thus time taken to travel to Capella in the earth frame is given by $\dfrac{46ly}{0.917c} \approx 50$ years.

13. Distant stars observed around the galactic center often appear redder than what might be expected from their spectral classes. Which of the following best explains the cause of this reddening?

(A) Starlight is redshifted due to the expansion of the universe

(B) Starlight is scattered by interstellar gas and dust in the galactic plane, with blue light being preferentially scattered

(C) Starlight ionizes interstellar hydrogen gas, stimulating emission in the H-alpha wavelength

(D) Stars near the galactic center orbit in smaller and faster orbits, thus their light is Doppler shifted

(E) Stars near the galactic bulge tend to be older and thus redder

Solution:

As we discuss about stars that appear redder than they seem, the description in Option E cannot be correct. As we get closer to the galactic center, more interstellar matter scatters starlight - most of which preferentially scatter blue light. Stars within the Milky Way are not subject to Hubble expansion, hence not experiencing any Hubble redshift.

A: Stars within the same galaxy would not experience the effects of universe's expansion due to the galaxy's own gravity, which means it cannot be redshift.

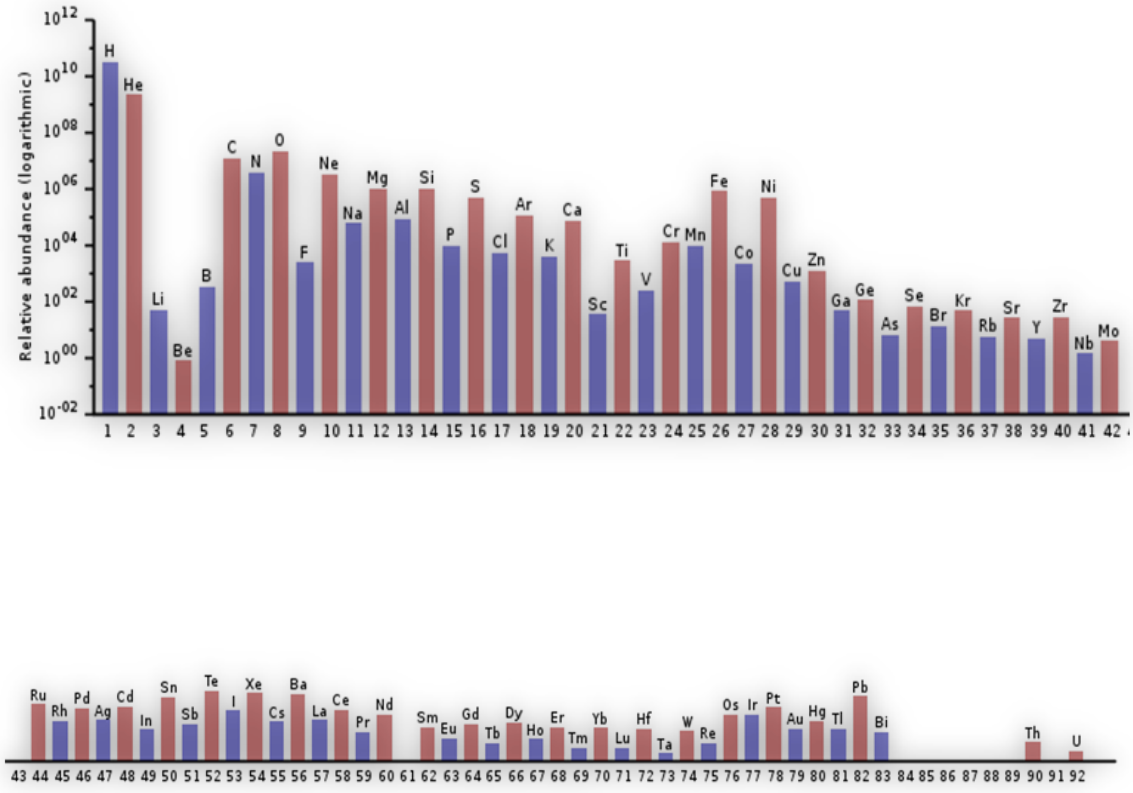
B: This option is correct. Light of shorter wavelengths are preferentially scattered by interstellar gas and dust. Since there are much more interstellar gas and dust as we got closer to the galactic centre, the more blue light is scattered and the star become redder than it should be.

C: The inner mechanism that makes stars red is not relevant as we are discussing when stars seem redder than they should be at rest.

D: The Doppler shift caused by this cannot be purely redshift in nature. There will be periods when the star will have its radial velocity pointing at us.

E: Similar to C, the rest color of the stars is irrelevant as we discuss stars that seem to be redder than they are at rest radially for us.

14. Figures below show the relative abundance of different elements in the universe according to our best estimates from the composition of the Solar System. Lighter and heavier elements have been separated for visibility.



Patterns in this abundance graph can be useful for astronomers in studying nucleosynthesis in stars and in the early universe. Which of the following is **not** true?

(A) The relative abundance of even-numbered proton nuclei in the graph is due to the mechanism involved in forming them mainly involving helium nuclei.

(B) **The relative absence in the amount of Beryllium observed in the universe is an indication of the abundance of Beryllium burning stars that might be too dim to be observed by current technology.**

(C) The relative abundance of Iron and Nickel in the universe is due to them being the end point for massive star's stellar nucleosynthesis.

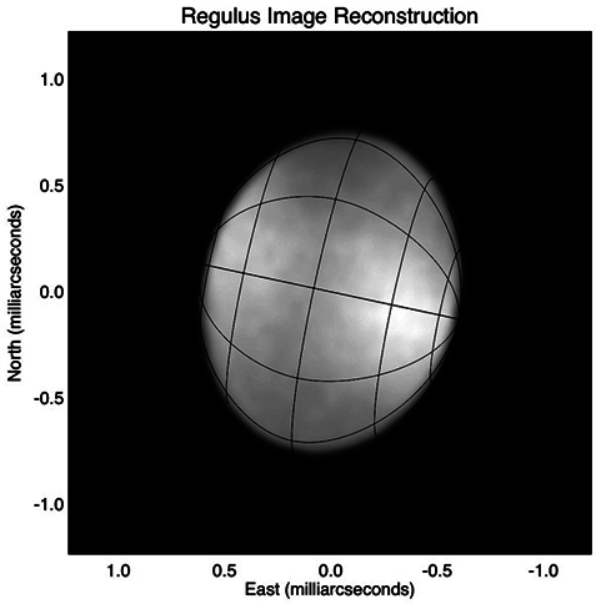
(D) The gap between Bismuth and heavy radioactive elements (e.g., Th and U) is mainly due to the radioactive decay since their formation in supernovae as most of these elements have a short half-life time.

(E) None of the above.

Solution:

Option B is wrong because the lower abundance of Beryllium is generally attributed to the relative efficiency of the Triple Alpha Process/Helium Burning at producing Carbon rather than Beryllium as its product. There is no star that predominantly burn beryllium as a source its nuclear energy.

15. Regulus is a bright star in the constellation of Leo. It is notable for spinning so fast that it almost spins itself apart. Shown below is an image of Regulus taken with the help of interferometry, where the whiter parts of the image indicate higher temperatures.



Which of the following reason best accounts for the difference in brightness observed?

(A) Doppler shift results in less photons received around the equator

(B) Convection near the equator is disrupted and energy cannot be transported to the surface from the core to heat up the gas at the star surface efficiently

(C) **The surface gravity at the poles is higher and hence requires higher hydrodynamic pressure to balance it out, causing the poles to be hotter and brighter**

(D) The gas at the equator emits photons more directly aimed at us

(E) The larger radius at the equator reduces the rate of heat transfer to the surface and hence makes the equator of the star cooler and dimmer

Solution:

The lower distance between the poles and the core of the star increases the surface gravity at the poles, resulting in a higher hydrodynamic pressure needed to support the star at the poles and hence the star being hotter and brighter at the poles. This phenomenon is the consequence of gravitational darkening. Doppler shift alone is not sufficient to explain such difference in brightness.

16. If the Crab Pulsar has a periodicity of 33ms, what is its minimum density (in kg/m^3) such that it will not rip itself apart from centrifugal force? You may assume the density within the pulsar is uniform.

(A) **1.3×10^{14}**

(B) 1.3×10^{8}

(C) 3.9×10^{6}

(D) 3.6×10^{3}

(E) None of the above

Solution:

We equate the outward centripetal force to balance the inward gravitational force on the surface of the pulsar. Let R be the radius of the pulsar.

$$m\omega^2 R = \frac{GMm}{R^2}$$

$$\omega = \sqrt{\frac{GM}{R^3}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{G\rho 4\pi}{3}}$$

Re-arranging and substituting T = 33 ms, we obtain the following ρ:

$$\rho \approx 1.3 \times 10^{14} kg/m^3$$

17. Astronomers often observe the 21 cm emission line produced by the spin-flip transition of neutral hydrogen in order to map out the large scale distribution of cosmic gas and dust. Which of the following are reasons for why the 21 cm emission wavelength is used?

    i The universe is predominantly filled with cold neutral hydrogen that produces 21 cm emission

    ii Radiation with a 21 cm wavelength is easily scattered by cosmic dust, therefore allowing for the structure and distribution to be clearly mapped

    iii 21 cm emission has a narrow spectral width which allows us to accurately determine velocities through Doppler shift

    iv 21 cm emission is not absorbed by the atmosphere and thus can be observed by ground-based radio telescopes

    (A) i, iii and iv

    (B) i, ii, and iii

    (C) ii and iv

    (D) ii, iii, and iv

    (E) All of the above

    Solution:

    I is true.

    III is true as narrow emission lines allow for subtle shifts of peak positions due to Doppler shift to be easily distinguished and detected.

    IV is true as the radio window (parts of the electromagnetic spectrum where it is transparent to radio waves) allowed us to receive the signal using radio telescopes.

    II is false; 21cm radiation is useful precisely because it is not scattered by interstellar gas and dust, and thus can allow for accurate mapping of large-scale distributions of molecular hydrogen.

18. Detecting a binary star with an elliptical orbit can be much tougher than circular ones as it involved many more parameters to be solved and could only be done by a computer. Which of the following parameters is not one of these new parameters that is required for determining the semi-major axis and masses ratio of the orbit?

    (A) Position of Perigee

    (B) Eccentricity

    (C) Angle position of Vernal Equinox

    (D) Argument of Perigee

    (E) Radial velocity

    Solution:

    While radial velocity is a required parameter for elliptical orbit analysis, they are not exclusive to elliptical orbits and are also required in the circular orbits. The other four parameters are

integral variables in determining the geometry of the elliptical orbit, hence we can determine the position and velocity of the object.

19. Suppose our Sun is replaced by a red dwarf of 0.4 solar masses. Estimate the apparent magnitude of the "new Sun" as seen from Earth. You may assume Earth does not change its orbit. Take the solar apparent magnitude as -26.74.

(A) -30.27

(B) -23.26

(C) -18.63

(D) -13.22

(E) -4.76

Solution:

For main-sequence stars, the following relation approximately holds:

$$L \propto M^{3.5}$$

$$L_{dwarf} = 0.4^{3.5} L_{sun}$$

$$L_{dwarf} = 0.0405 L_{sun}$$

Using this ratio, we can calculate the new star's apparent magnitude.

$$m = -26.74 - 2.5 \log(0.0405)$$

$$m \approx -23.26$$

Note that the mass-luminosity relation may not hold precisely as stated for red dwarfs as the exponent for the mass-luminosity relation will be slightly different than 3.5. Nonetheless, we may assume the same relation applies to obtain an approximated answer.

20. A star has a special classification of B8Ia. We can thus conclude that

(A) The radius of this star is much larger than our Sun

(B) The star would appear very red to the naked eye

(C) This star has 8 times the luminosity of our Sun

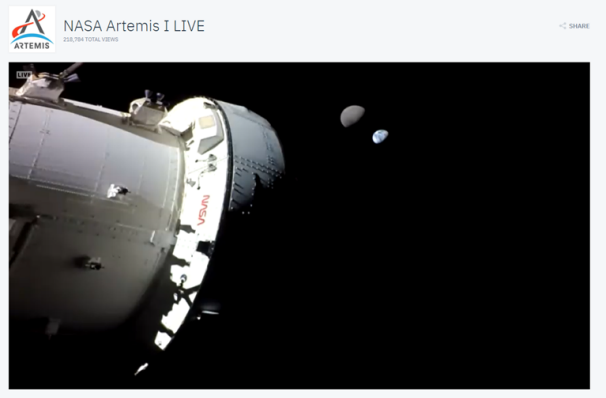
(D) This star has 8 times the mass of the Sun

(E) This star contains an unusually high abundance of A-group elements such as aluminum, argon, and arsenic

Solution:

In Harvard classification, "B8" implies that the star seems blue-white and the number does not directly imply to the luminosity and mass of the star itself in comparison to the Sun. While "Ia" denotes a luminous supergiant in the Yerkes classification system. With this information, Option A is the best descriptor of a blue-white luminous supergiant.

21. At one point during its journey, NASA's Artemis I live camera captured a moment in which the Moon's radius appeared 25% larger than the Earth's radius as seen below.



Assuming that the Earth and Moon are perfectly spherical and sufficiently far away from Artemis I such that the small angle approximation is valid, how much further was the Earth-Artemis I distance compared to the Moon-Artemis I distance? In other words, calculate the ratio of distances between Earth-Artemis and Moon-Artemis.

(A) 5.74

(B) 4.58

(C) 3.67

(D) 3.42

(E) 2.93

Solution:

Using the small-angle approximation:

$$\theta_{Moon} = 1.25\theta_{Earth}$$

$$\frac{R_{moon}}{x} = 1.25\frac{R_{Earth}}{x + d_{Earth-Moon}}$$

Where x is the distance from Artemis to the Moon.

$$1 + \frac{d}{x} = 1.25\frac{R_{Earth}}{R_{moon}}$$

$$1 + \frac{d}{x} \approx 4.58$$

22. A star cluster consists of 5 1-mag stars and 10 2-mag stars. What is the integrated magnitude of the star cluster?

(A) **-1.38**

(B) $+1.38$

(C) -5.38

(D) $+5.38$

(E) None of the above

Solution:

Note that a 1-mag star is $10^{\frac{2}{5}} \approx 2.52$ times more luminous than a 2-mag star. Hence, using Pogson's modulus:

$$m_{int} = 2 - 2.5\log(5 \times 2.52 + 10)$$

$$m_{int} \approx -1.38$$

23. One of the implications of a flat galaxy rotation curve is the winding problem. If stars in a spiral galaxy all mostly move at around the same velocity, stars near the galactic core will make many more orbits than stars in the outer regions of the galaxy, as their orbital radii are smaller. Over time, this would cause the spiral arms of the galaxy to wind up, destroying the spiral structure.

What is currently believed to be the resolution for this winding problem?

(A) Dark matter holds the star in place relative to the spiral arms.

(B) Dark energy causes the space between spiral arms to expand. However, the extra mass within the spiral arms counteracts the expansion, thus keeping the stars within the spiral arms.

(C) **The spiral arms are actually maintained by density waves. Gas clouds enter the density wave and are compressed by them, triggering star formation which creates numerous luminous young stars - creating the spiral arms.**

(D) Supernovae shockwaves propagating throughout the galaxy undergo constructive interference at the spiral arms, leading to more star formation within these arms.

(E) The winding problem does not exist.

Solution:

Dark matter is the resolution for why we observe flat galaxy rotation curves. It is also believed to be concentrated in the outer halo of the galaxy, not within spiral arms. Similarly, dark energy only has significant impacts at distances larger than that of individual galaxies.

The coincidences required for D mean that it is not a viable explanation, especially given how numerous spiral galaxies are. Finally, not only is E contradicted by observed galaxy rotation curves, the proposed resolution actually makes the problem worse. If the outer reaches of the galaxy rotate slower than the inner regions, spiral arms would wind up even faster than they would under the original winding problem.

24. A group of astronomers was chilling under the night sky when one noticed a peculiar supernova. Its brightness, B declined exponentially. In other words,

$$B \propto e^{-t/\tau}$$

where τ is a constant with a value of 30 minutes.

At its brightest, the supernova has an apparent magnitude of +4. Assuming the maximum magnitude that human eyes can see is +6, how long can they observe the Type 2L Supernova until it became invisible to them?

(A) 40 mins

(B) 45 mins

(C) 50 mins

(D) 55 mins

(E) None of the above

Solution:

The ratio between a 6-mag and 4-mag star is given by:

$$6 - 4 = -2.5log(B)$$

$$B = 10^{-\frac{2}{2.5}} \approx 0.1585$$

From the calculated ratio, we can calculate for t:

$$0.1585 = e^{-t/\tau}$$

$$t = -30 \times ln(0.1585)$$

$$t \approx 55mins$$

You are provided with 3 telescopes with the following specifications. Answer question 25 and 26 based on the information given.

Name:	Telescope A	Telescope B	Telescope C
Type:	Newtonian Reflector	Schmidt Cassegrain	APO Triplet Refractor
Aperture:	203mm	127mm	107mm
Focal Ratio:	F3.9	F10	F7
Weight:	7.94kg	2.72kg	6.9kg

25. Which of the following statements is/are true given no accessories are used?

i Telescope A contains 2 mirrors

ii Telescope B is best for observing large Deep Sky Objects (DSO) as it has a long focal length

iii Telescope C contains a mirror and a few lenses

iv Telescope A has the shortest focal length

(A) **i only**

(B) i and ii

(C) i, ii, and iv

(D) ii and iv

(E) All of the above

Solution:

Newtonian telescopes contain 2 mirrors.

Schmidt Cassegrain telescopes' focal length are much more suitable for planets.

Refractors, as the name suggests, do not have mirrors. Telescope C has the shortest focal length.

26. Given that an eyepiece of 15mm focal length and Apparent Field-of-View (AFOV) of 70° is used on Telescope B. Calculate the True Field-of-View (TFOV) of the setup.

(A) 0.432°

(B) **0.827°**

(C) 1.43°

(D) 2.04°

(E) 2.89°

Solution:

The magnification ratio of Telescope B is:

$$M = \frac{f_{ob}}{f_{oc}} = \frac{127 \times 10}{15}$$

$$M \approx 84.67$$

Using the known magnification, we get the true FOV of the telescope:

$$TFOV = \frac{AFOV}{84.67}$$

$$TFOV \approx 0.827°$$

27. A comet of mass m with a parabolic orbit passes its perihelion at distance p. If the mass of the Sun is M, what is the angular momentum of the comet?

(A) $\sqrt{GMmp}$

(B) $\sqrt{2GMmp}$

(C) $\sqrt{GMm^2p}$

(D) $\sqrt{2GMm^2p}$

(E) $2\sqrt{GMm^2p}$

Solution:

In a parabolic orbit, we can express the angular momentum as:

$$L = mvr$$

Since the comet in a parabolic orbit, it moves at its escape velocity at all points along its orbit. Hence, at perigee:

$$L = mp\sqrt{\frac{2GM}{p}}$$

$$L = \sqrt{2GMmp}$$

28. The no-hair theorem states that a stationary black hole can be completely described by three quantities. What are the three properties?

(A) Mass, momentum, Schwarzchild radius

(B) Momentum, angular momentum, entropy

(C) Surface area, energy, entropy

(D) Surface area, energy, electric charge

(E) Mass, angular momentum, electric charge

Solution:

The no-hair theorem states that a stationary black hole can be completely described by its mass, angular momentum, and electric charge,

29. Low mass stars of spectral types K and M tend to show more prominent metal absorption lines as compared to O, B and A spectral type stars. Which of the following statements is the best explanation?

(A) **O, B, and A spectral type stars have higher temperatures in their outer atmospheres which causes metals to be ionized. Ionized metals produce weaker absorption lines.**

(B) K and M-type stars tend to be metal-rich as the presence of metals lowers the equilibrium temperature of stellar atmospheres, which is reflected in its spectral classification.

(C) The initial mass function of molecular clouds with high metallicities is skewed towards the lower-mass range, therefore metal-rich stars tend to be low-mass stars.

(D) Lower mass stars tend to be older, and therefore are usually population II stars which are defined by having excess metals in their atmosphere.

(E) In the Harvard spectral classification system, K and M types stand for Potassium and Magnesium, which are metals. On the other hand, O, B, and A stand for Oxygen, Boron and Argon which are not metals.

Solution:

High temperatures of O, B, A-type stars are enough to ionize metals within the stars. Electrons of ionized metals are free and hence thus cannot be excited. This directly causes the weaker absorption lines.

30. A hypothetical three-star system consists of three component stars with identical apparent magnitudes but different sizes and their orbits are coplanar with our line of sight to the system. Star A and B are 4 and 2 times larger in radius than Star C. Assuming they are visible but not resolvable with the naked eye, in which case will the system appears the dimmest?

(A) All stars do not occult each other

(B) **Star A occults Star B and C**

(C) Star B occults Star A and C

(D) Star C occults Star A and B

(E) Star A occults Star C, but not Star B

Solution:

The dimmest possible configuration of a three-star system is when the integrated magnitude is equal to the dimmest star's luminosity. In the case of this star system, this occurs when the largest star (i.e., Star A) occults the equally bright but smaller counterparts.

31. It is November 2022 at 3 am and Mars just crossed the meridian a few minutes ago. At the same time and date in 2023, will Mars still be visible in the night sky?

(A) Yes, Mars is around inferior conjunction

(B) Yes, Mars is around superior conjunction

(C) No, Mars is around inferior conjunction

(D) No, Mars is around superior conjunction

(E) Not enough information to tell

Solution:

Since Mars crosses the meridian at 3am at the 2022 observation, Mars has passed Eastern Quadrature and is approaching opposition. Taking into account Mars' synodic period at 687 days, we can estimate that Mars should be roughly behind the Sun at the same time and date next year (365 days later). Since, Mars is an superior planet, we call the configuration a superior conjunction.

The following table contains information regarding a Cube-Satellite in Low Earth Orbit. Answer questions 32-34 with the information provided in the table.

Name	Eclipse-SAT
Mass	22.15 kg
Orbital Height	800 km
Orbital Type	Circular Sun-synchronous
Orbital Inclination	98.67°

32. Calculate the orbital period of the Eclipse-SAT satellite.

(A) 89 minutes

(B) 95 minutes

(C) 101 minutes

(D) 109 minutes

(E) 120 minutes

Solution:

We use Kepler's third law and calculate its orbit against Moon's orbit.

$$T_{SAT}^2 : T_{moon}^2 = (h_{SAT} + R_{earth})^3 : a_{moon}^3$$

$$T_{SAT} = (27.322 days)\sqrt{(\frac{6370 + 800}{3.843 \times 10^5})^3}$$

$$T_{SAT} = 0.06963 days \approx 101 minutes$$

Conversely, you may calculate the orbit velocity using $\sqrt{\frac{GM}{R+h}}$ to obtain the same result.

33. Calculate the orbital speed of the Eclipse-SAT satellite.

(A) 6.34 km/s

(B) 7.46 km/s

(C) 8.98 km/s

(D) 9.49 km/s

(E) 11.2 km/s

Solution:

Using your answer from the previous question,

$$v = \frac{2\pi(R_{earth} + h_{SAT})}{T_{SAT)}}$$

$$v = \frac{2\pi \times 7170}{101 \times 60}$$

$$v \approx 7.46 km/s$$

34. A communication center on ground last communicated with the Eclipse-SAT satellite directly overhead at midnight (0000 hrs) of 21 June 2023. If the satellite orbits in the equatorial plane on a prograde motion (i.e., same as Earth rotation's direction), when would the satellite be directly overhead the center again? Take the length of one sidereal day to be 86164 seconds.

(A) 0014 hrs

(B) 0029 hrs

(C) 0142 hrs

(D) 0145 hrs

(E) 0149 hrs

Solution:

The question implicitly asked us to find the synodic period from Earth's rotation and the satellite's orbit revolution.

$$\frac{1}{T_{syn}} = \frac{1}{T_{SAT}} - \frac{1}{T_E}$$

where T_{SAT} is the satellite's orbital period (as found in Question 42) and T_E is Earth's sidereal day.

$$\frac{1}{T_{syn}} = \frac{1}{6060} - \frac{1}{86164}$$

$$T_{syn} \approx 6518s \approx 1h49m$$

35. Which of the below coordinate systems are **not** used for finding Deep Sky Objects regardless of location and time on Earth?

(A) Equatorial coordinate system

(B) Ecliptic coordinate system

(C) Galactic coordinate system

(D) Alt-azimuth coordinate system

(E) None of the above

Solution:

The alt-azimuth coordinate system depends on the observer's time and location, hence its implausibility for use in DSO listing. The rest of the coordinate systems are based of fixed planes on the night sky.

36. An alien civilization is more technologically advanced than the human civilization. An alien astrologer (*yes, astrology is a science for them*) observed the time evolution of the solar system and tabulated the distance between the aphelion and perihelion of each planet and the time each planet takes to orbit the Sun.

What can the alien astrologer know about the Sun from the gradient of the straight line with the highest accuracy? Assume their line of sight aligns with our ecliptic.

(A) Sun's radius

(B) Sun's luminosity

(C) Sun's mass

(D) Earth's radius

(E) Earth's mass

Solution:

Assuming the Earth's mass is negligible relative to the Sun, one can calculate the orbital velocity from the information given. Since $v = \sqrt{\frac{GM}{r}}$ and r is known, one can calculate Sun's mass with highest precision.

37. Recently, astronomers discovered that the dwarf planet Quaoar possesses a ring system outside of its Roche limit. Why is this unusual?

(A) Dwarf planets are too small to support a ring system

(B) Given enough time, this ring should coalesce into moons

(C) A ring system requires multiple moons in order to be stable, which Quaoar does not possess.

(D) Due to the presence of numerous other bodies in the Kuiper belt, these rings would be easily destabilized by the gravitational influence of other bodies

(E) This situation is not unusual at all

Solution:

Roche limit is defined as the smallest orbit where a smaller celestial body would be able to sustain itself from a larger celestial body (i.e., planets or dwarf planets) before being disintegrated through the latter's tidal force. Rings usually form as the disintegrated materials coalesced within the Roche limit. Otherwise, ring-like materials outside the Roche limit would instead coalesce into a singular celestial body (i.e., a moon)

38. The James Webb Space Telescope is one of the most anticipated space telescopes with an immense budget. Because it is primarily designed for near-infrared astronomy, it must be positioned at the L2 Lagrange point of the Earth-Sun system. Which of the following statement(s) is/are true?

i At $L2$, its instruments can be kept cold and thermally stable

ii It can only observe objects in certain angles at any instance of time due to its sunshield

iii It orbits around the Earth with the same period as the Moon

(A) i only

(B) i and ii

(C) ii and iii

(D) All of the above

(E) None of the above

Solution:

Note that Earth's L2 is located behind the Earth as seen from the Sun. At L2, the largest sources of infrared radiation (i.e., Sun, Earth, and Moon) are all in the same direction and thus can be easily blocked by a sunshield. Note that the infrared radiation from these objects can severely impair the telescope's optical capability in Infrared.

39. The Arecibo Telescope was a large radio telescope built in the mountains of Puerto Rico, US. Unlike conventional radio telescopes which have parabolic reflectors, the Arecibo Telescope had a spherical primary reflector. Which of the following reasons best explains why?

(A) A spherical shape matched best the natural sinkhole it was built on

(B) A spherical reflector allowed the telescope to collect more light given the same area

(C) A spherical reflector allowed the telescope to only focus on very specific radio wavelengths

(D) A spherical shape allowed the detector to re-orientate and focus on any part of the primary reflector to look at different parts of the sky

(E) A spherical reflector does not suffer from chromatic aberration

Solution:

The spherical shape means that the receiver can be oriented at any part of the dish, hence processing images of different parts of the night sky.

40. Unlike planets like Earth and Uranus, Jupiter does not experience significant seasonal changes. This is primarily because:

(A) Jupiter is tidally locked to the Sun

(B) Jupiter moves too slowly along its orbit

(C) The axial tilt of Jupiter is negligible

(D) The weather on Jupiter is driven by tidal forces exerted by the Galilean moons

(E) The weather on Jupiter is driven by deuterium fusion within its core

Solution:

Seasonal changes as seen on Earth and Uranus are direct consequences of the planet's axial tilt as different hemispheres faces the Sun for different periods of time at any instance. With its 3°tilt against the normal of the orbital plane, seasonal changes in Jupiter is barely noticeable.

41. In the equatorial coordinate system, the 00hrs RA meridian is defined to be at the First Point of Aries, with RA increasing in the west-to-east direction. Which of the following statement(s) is/are true about RA-Dec coordinates? Note that we take the first point of Aries as the start of the tropical year.

    i The RA of the Sun increases throughout a tropical year

    ii The RA of stars increase throughout a sidereal year

    iii Celestial objects with a later RA rise later in the night

    iv Stars with the same RA always rise and set at the same sidereal time

    (A) i only

    (B) i and ii

    (C) i and iii

    (D) i, iii, and iv

    (E) All of the above

    Solution:

    I is true as the Sun moves from West to East along the ecliptic over the course of a year.

    II is false as the Right Ascension of stars is fixed and does not change without any peculiar motions.

    III is true as the sky rotates from East to West, thus stars rising in the East have a later RA than stars setting in the West.

    IV is false as the statement is only true at the equator.

42. Which of the following about Saturn's moons are **false**?

    (A) Saturn has many shepherd moons which help to confine particles within the rings and prevent them from spreading out into space

    (B) Enceladus is thought to have a sub-surface ocean as it can get energy from tidal heating

    (C) Orbital resonance between the moons keeps their orbits stable

    (D) Titan is located in between the B Ring and C Ring, thus making it a shepherd moon as well

    (E) None of the above

    Solution:

    A shepherd moon is a natural satelite orbiting around a planet with rings (such as Saturn) from which gravity keeps the materials that comprising rings from breaking apart. Titan is one of the biggest moons of Saturn. It cannot be a shepherd moon as it is too big. It has an orbit that is much further out than the rings.

43. Which of the following statements are **incorrect**?

I Due to libration and other orbital effects, we see a bit more than half of the Moon's surface from Earth despite the Moon being tidally locked to Earth

II Ancient Greek philosophers cannot see some stars modern Greek astronomers can

III Protostars, despite their lack of fusion, are always more luminous than when they reach the Main sequence.

IV A future inhabitant of Mars will observe Earth go through different phases

(A) None of them

(B) I

(C) II

(D) **III**

(E) IV

Solution:

I: Moon's libration moves the Moon's axial tilt slightly which allows us to see more than half of the lunar surface

II: This is true due to precession, where the former are able to see some of the now-Southern stars.

III: Protostars of lesser mass usually become less luminuous directly as it condenses into a main-sequence star (see: Hayashi track). However, as protostars become heavier, they take a less linear track across the Hertzprung-Russel diagram such that some of them ignores this trend and eventually become more luminuous (see: Heyney TRACK)

IV: Straightforward answer.

44. The Cosmic Microwave Background Radiation (CMBR) is an important tool in understanding the early universe. Which of the following description of CMBR is **incorrect**?

(A) The CMBR is the light coming from the surface of last scattering, which is the earliest time light can freely travel in the universe

(B) The CMBR is dated to about 380,000 years after the Big Bang

(C) The effective blackbody temperature of CMBR is redshifted to about 2.7 K due to the expansion of the universe

(D) **The CMBR is emitted from the edge of the observable universe 46.5 billion light years away and only reaches us due to the expansion of the universe**

(E) The main variation of effective blackbody temperature of CMBR is due to Earth's relative velocity with respect to CMBR

Solution:

CMBR is emitted from every point in space from the light that escaped from the last scattering 380,000 years after the Big Bang. What we are observing is the light from the early universe

that have just reached us due to cosmic expansion.Due to the finite age of the universe we are unable to observe the entire observable universe form Earth at the current moment.

45. The Milankovitch cycle attempts to account for long-term cyclical patterns in the Earth's climate by considering the effects of astronomical cycles and how Earth's rotation changes over time. Which of the following astronomical cycles are included in the Milankovitch cycles?

i The Precession of the Equnioxes

ii The Solar Cycle (sunspot cycle)

iii Apsidal Precession of Earth's orbit

iv Nutation of Earth's orbital axis

v The Saros Cycle

(A) i and iii

(B) ii and iv

(C) i, iii, and iv

(D) ii, iv, and v

(E) All of the options are correct

Solution:

From the description given in the question about the Milankovitch cycle, those stated in i), iii), and iv) are related to changes in Earth's rotational axis over long periods of time (i.e., thousands of years) and has a profound impact on Earth's climate. The Solar and Saros cycles are way too short (each cycle lasts for only 10-20 years) and its effects are relatively negligible to have a profound effect on our climate.

46. Suppose it is the March Equinox and you heard that there will be a meteor shower tonight. The meteor radiant lies close to the First Point of Aries. What can you do to maximize the number of shooting stars that you can visually observe from this meteor shower at night?

(A) You should move north of the Arctic Circle to ensure that you enjoy 24 hours of night

(B) You need to be on a spacecraft to observe these meteors burn up

(C) You should only start observing after local midnight, to ensure that you are on the side of the Earth directly facing the meteor shower

(D) There is nothing you can do as the radiant is only above the horizon in the daytime

(E) You can simply wait a few months, and observe at the night in which the radiant rises at local sunset

Solution:

By definition, the Sun will be at the First Point of Aries during March Equinox. Hence, the observer would not be able to see the meteor shower's radiant at the night sky since it will be below the horizon with the Sun.

47. Scientists can approximate densities of matter (including dark matter) of a dust cloud variable to its radius $\rho(r)$. Assuming that the matter moves in circular orbits with radius r from the center of the cloud, determine the relationship between $\rho(r)$ and r so that the orbital velocity still be the same for all values of orbital radius r

(A) $\rho(r) \propto r^{-4}$

(B) $\rho(r) \propto r^{-3}$

(C) $\boldsymbol{\rho(r) \propto r^{-2}}$

(D) $\rho(r) \propto r^{-1}$

(E) $\rho(r) \propto r$

Solution:

Note the formula for orbital velocity:

$$v = \sqrt{\frac{GM}{r}}$$

Since we need the velocity to be constant, we get the relationship between mass and radius as:

$$M \propto r$$

Consider the portion of dust mass dM with thickness dr at distance r,

$$dM = 4\pi\rho r^2 dr$$

Since $dM = dr$, we can rearrange to get:

$$\rho = r^{-2}$$

48. You observe a galaxy emitting a H-alpha line at a wavelength of 785 nm. The rest-frame H-alpha emission wavelength is 656 nm. What is the radial velocity of the galaxy?

(A) 0.549c, receding

(B) 0.549c, approaching

(C) **0.178c, receding**

(D) 0.178c, approaching

(E) 0.243c, approaching

Solution:

The H-alpha line line is redshifted, which means that the galaxy is receding away from us as the observer. The redshift observed is:

$$1 + z = \frac{\lambda_{obs}}{\lambda_{rest}}$$

$$z = \frac{785}{656} - 1 \approx 0.1966$$

We use the redshift obtained to get the receding velocity of the galaxy.

$$z = \sqrt{\frac{1+\beta}{1-\beta}} - 1$$

$$1 + 0.1966^2 \approx \frac{1+\beta}{1-\beta}$$

$$1.432(1-\beta) \approx 1 + \beta$$

$$0.432 \approx 2.432\beta$$

$$\beta = \frac{v}{c} \approx 0.178$$

49. Which of the following is/are true about eclipses from an observer on the Moon?

(A) At any given location, a solar eclipse on the Moon is rarer than on Earth

(B) **During a purely penumbral lunar eclipse on Earth, every part of the nearside of the Moon sees a partial solar eclipse**

(C) A solar eclipse (total or partial) can occur anywhere on the Moon

(D) During a total solar eclipse on Earth, Earth would appear reddish from an observer on the Moon

(E) None of the above

Solution:

A lunar eclipse on Earth is equivalent to a solar eclipse on Moon, hence making a solar eclipse more common on the Moon than on Earth. The farside of the Moon would not experience any solar eclipse. The Moon would only cast a "small" black shadow on the surface on the Earth.

A penumbral lunar eclipse means that the Earth only partially blocks out Sun's disk as seen from the Moon. This translates to a lunar partial solar eclipse.

50. In the northern hemisphere during winter, in which directions do we expect the Sun to rise and set respectively?

(A) NE, NW

(B) NE, SW

(C) **SE, SW**

(D) SE, NW

(E) Trick question. The Sun will always be below the horizon during winter.

Solution:

The Sun will definitely be south of the celestial equator during winter in the Northern Hemisphere. Since the Sun always intersects the horizon from due east to due west regardless of latitude, the correct direction of Sunrise and Sunset would be South-East and South-West, respectively.

[End of paper]