ASTROCHALLENGE 2016 DATA RESPONSE QUESTIONS

JUNIOR ROUND

INSTRUCTIONS

- This paper consists of 14 printed pages, excluding this cover page.
- Do **NOT** turn over this page until instructed to do so.
- You have 2 hours to attempt all questions in this paper.
- At the end of the paper, submit this booklet together with your answer script.
- Your answer script should clearly indicate your school (and team number) on **EVERY** page, as well as the individuals in the said team on the first page.
- It is your team's responsibility to ensure that all pages of your answer script have been submitted, including pages to be detached from this booklet.

DRQ 1: The Galaxy Cake [20 marks]

(Disclaimer: Any resemblance of the following fictional galaxy to Messier 100/ NGC 4321 in Coma Berenices is NOT coincidental but based on actual observed data)

Once upon a time, there were two young Scientists: Carrie Alpha, and Ken Epsilon. They discovered a new galaxy simultaneously. As they were both unable to decide on a name, they decided to temporarily name it Cα-Kε, after their initials. To decide who will have the honour of deciding on the permanent name of the newfound galaxy, they both decided to challenge each other to calculate the distance of the newfound galaxy from Earth.

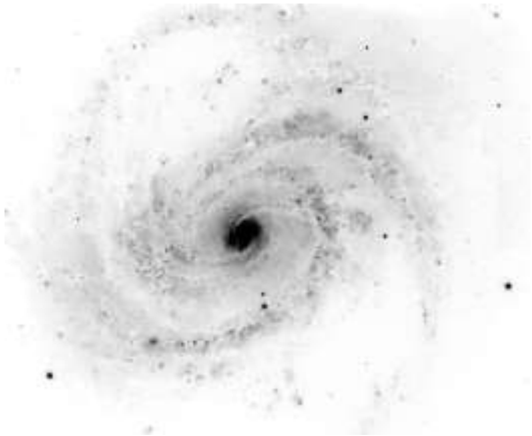


Figure 1: Diagram showing newly discovered galaxy, temporarily designated Cα-Kε. Standard candles included. Actual candle brightness varies depending on type and period.

Ken devoted 3 years to observing Type 1a Supernovas within Cα-Kε itself. He was fortunate enough to observe a total of 3 events consecutively year after year, and he jotted down the apparent magnitude of each supernova as precisely as he could from his location on an Earth-based laboratory. He was confident that because of the way Type 1a Supernovas worked, he'll obtain a reasonable estimate for the distance to Cα-Kε.

Unfortunately, his colleague, Igor Nance, has many gross misconceptions about the way Type 1a supernova works. Help Ken write an article to his colleague on the mistakes in this excerpt (3):

"Type 1a supernovas usually occur between binary stars, of which one of the stars already became a white dwarf and the other star is a massive Supergiant. The massive Supergiant will engulf the white dwarf as it slowly spirals into its companion. When this happens, the white dwarf breaks up once it crosses the Chandrasekhar Limit, without electron degeneracy pressure to maintain its orbit. It then fuses with the supergiant and causes a giant supernova, leaving only a black hole in the aftermath."

Answer: Other star can be ANY star; mass from other star spirals into the white dwarf; Chandrasekhar limit is not Roche limit; white dwarf reaches critical mass to explode; electron degeneracy pre45`4ssure prevents collapse of itself, not orbit; companion star is ejected; no black hole is formed

Observed supernova	Date of observation	Location	Apparent Magnitude
SN 2014D*	14/02/14**	Galactic Nucleus	15.6
SN 2015R*	25/12/14**	Near Galactic Bulge	15.7
SN 2016Q*	31/10/14**	Galaxy Arm	15.3

*Disclaimer 1: real designations are SN1901B, SN1914A, SN2006X, found on those years.

**Disclaimer 2: Do not expect to always observe supernovae on Valentine's Day, Christmas or Halloween.

Estimate the distance from Earth to Cα-Kε, using only information from the above given data. Provide your answer in parsecs. (3)

m – M = 5 lg (d/ 10)   M = -19.3 m + 19.3 = 5 lg (d/ 10)

$(0.954993 \times 10^8 + 1 \times 10^8 + 0.831764 \times 10^8)/3 = 9.289 \times 10^7$ parsecs

Comment on two possible sources of error which could have affected Ken's estimate of the distance between Earth and Cα-Kε, using Type 1a supernovas. (2)

Light of supernova has Extinct-ed/ diminished/ etc.; Atmospheric turbulence on Earth; Redshift of galaxy; small sample size; Different location and thus different effect on light eventually; Observational difficulties; Rounding errors; also accepted for those sharp enough: Supernova observed in 1901, 1914 might not have the proper instrument for correction to determine magnitude well

The Chandrasekar limit is defined as the maximum mass of a white dwarf before it undergoes supernova. It is given by the following simplified equation for stars in the galaxy Cα-Kε, based on empirical data:

$$M_{limit} = \frac{2.02\sqrt{3\pi}}{2}\left(\frac{\hbar c}{G}\right)^{1.5}(1.525\ m_H)^{-2}$$

Where ħ is the Reduced Planck's Constant,

c is the speed of light,   G is the Gravitational Constant, m_H is the mass of a Hydrogen Atom;

All other constants not easily obtainable in the formula booklet are provided.

With this in mind, calculate the Chandrasekar limit for the white dwarf, in terms of Solar masses. (2)

h = $6.62606957 \times 10^{-34}$ rh = h/(2π)   c = 2.99792458×10^8 G = 6.67384×10^{-11}

mH = 1.672622×10^{-27} mHwe = 9.10938×10^{-31}+MH   Msun = 1.989×10^{30}

(2.02sqrt(3π))/2 ((rh*c)/G)$^{1.5}$ (1.525MH)$^{-2}$ = 4.91364×10^{30}

(2.02sqrt(3π))/2 ((rh*c)/G)$^{1.5}$ (1.525MHwe)$^{-2}$= 4.90829×10^{30} (without electron)

Ans/Msun = 2.47 solar mass, approximately.

Carrie, on the other hand, also spent 3 years analysing periods of countless stars within Cα-Kε, and identified many regular Cepheid variables near the centre of the galaxy. She worked aboard the International Space Station and had a much better view of Cα-Kε than Ken does. To adjust for issues with instrument sensitivity, she realised that **0.15 should be subtracted from the difference between apparent and absolute magnitude.**

Cepheid variable observed**	Luminosity (Apparent Magnitude)*			Period (days)	Location of Cepheid in Cα-Kε
	Maximum	Minimum	Average		
Cα – 1	23.8	26.2	25.0	53.1	Near top of Galactic Bulge
Cα – 2	25.1	26.5	25.8	43.2	Galaxy Arm
Cα – 3	25.9	27.1	26.5	30.4	Edge of Galactic disk
Cα – 4	25.4	26.4	25.9	26.2	Empty region between Arms

*Disclaimer 3: Remember to subtract 0.15 from the final value used for the *difference between apparent and absolute magnitude.*

**Disclaimer 4: These are fictional stars with data from actual stars

Help Carrie write a message to Ken on what Cepheid variables are, and how they are used to calculate distances in space (2).

Stars which varies in brightness, period proportional to luminosity, good for distance modulus

Using the above data, obtain a suitable estimation of the distance between Earth and Cα-Kε. Provide rationale for which data point(s) is/are used in obtaining your estimation. (Hint: Teamwork required) (4)

M = -2.76 lg P – 1.4 m + 19.3 = 5 lg (d/ 10)

PVa = {53.1, 43.2, 30.4, 24.4}; MVa = -2.76 Log[10, PVa] - 1.4; {-6.16126, -5.91394, -5.49273, -5.2292} (This is the absolute magnitude of each Cepheid variable)

$\mathrm{dVa} = \sqrt{100^{(\mathrm{mVa}-\mathrm{MVa}-0.15)/5}} * 10$; $\{1.593 \times 10^7, 2.055 \times 10^7, 2.336 \times 10^7, 1.570 \times 10^7\}$,

Using average magnitudes mVa = {25, 25.8, 26.5, 25.9}; Mean distance is 1.889×10^7 parsecs away.

Note that final answer may vary if the group discards certain data, or calculate a mean of all magnitudes.

Comment on two OTHER possible sources of error (besides Ken's and her instrument) which could have affected Carrie's estimate of the distance between us and Cα-Kε, using Celpheid variables. (2)

Cepheid relationship is empirical and might not apply to those in particular; Average magnitude between max and min luminosity is a good benchmark, but in reality light curve could skew average towards either end; Location might have diminished light; any other applicable answers from Ken's part

Comment on any discrepancy between the results from Ken's and Carrie's methods, and suggest other possible methods to verify the actual distance. (2)

Same order of magnitude, but Ken's value is about 5 times further; Use RR lyrae stars; estimation using nearby galaxy clusters; REJECT stellar parallax. Redshift/ Hubble: half

Plot Twist!

If you've followed the story thus far, the Astrochallenge 2016 team will like you to vote for your preferred plot twist and ending. Please indicate it EITHER next to your School and Team name, OR at the end of the answers to this question. Please refrain from writing a new plot twist unless your team is confident of finishing the rest of this paper.

The plot twists are as follows:

A. Igor Nance turned out to be a mad scientist, and the villain.

B. Cα-Kε turns out to be M100 all along, but somehow everyone forgot about it.

C. Carrie turns out to be an artificial intelligence who has Ken fooled.

D. The world is but one of many multiverses which will fail to exist.

E. Carrie and Ken reunite and live happily ever after.

F. Others (Create your own ending)

DRQ 2: Tabby's Star [20 marks]

KIC 8462852, otherwise known as Tabby's star, has been widely publicised for its unusual and erratic behaviour. In this question, we will try to dispel some erroneous myths that have been going around. Before we begin, here are some known properties of Tabby's Star

Mass	1.43 $M_\odot$
Radius	1.58 $R_\odot$
Bolometric Luminosity	4.7 $L_\odot$
Distance	454 pc
Apparent magnitude	11.7

While the age of the star is unknown, its spectral signature suggests that it is a main-sequence star. Furthermore, kinematic studies suggest that the star is not associated with any nearby newborn star clusters.

Kepler

A portion of the Kepler light curve for Tabby's Star is shown below. As we can see, the key feature of Tabby's star is that it displays large dips in its light curve that can last for several days. During the largest dip, the observed luminosity fell by 22%. To see how astounding that is, let us consider the following scenarios:

(a) If these dips are caused by a single body orbiting the star, its orbital period is around 750 days. What is the semi-major axis of the body? [1 mark]

Orbit Period = 750 days = 6.48x10^7 s

Using Kepler's 3rd Law and under the assumption that $m_1 >> m_2$, where m_1 is the mass of the Tabby Star and m_2 is the orbiting body, we get:

$$T^2 = \frac{4\pi^2}{G(m_1+m_2)}a^3 \approx \frac{4\pi^2}{Gm_1}a^3 \Rightarrow a = \sqrt[3]{\frac{Gm_1T^2}{4\pi^2}}$$

$$\therefore a = \sqrt[3]{\frac{(6.67\times10^{-11})(1.43\times1.99\times10^{30})(6.48\times10^7)^2}{4\pi^2}} = 2.7\times10^{11}m$$

(b) Suppose a planet was responsible for the dip seen here. Determine its hypothetical radius, in terms of Jupiter radii. [2 marks]

Observed luminosity is decreased by 22%

Understand Relationship between Luminosity, L and Area, A is $L \propto A$

Hence, observed area of Tabby star is also decreased by 22%. This area is blocked by the cross sectional area of the orbiting planet

$$\therefore A = 0.22\times\pi(1.58R_\odot)^2 = 2.66\times10^{17}m^2$$

$$\Rightarrow Radius\ \ of\ \ planet, r_p = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{2.66\times10^{17}}{\pi}} = 2.909\ldots\times10^8 \approx 7.2R_{♃}$$

(c) What would be the black-body temperature of this "planet"? [3 marks]

Assume that the orbit of planet has near 1 eccentricity so we can use the semi-major axis, *a* as a good approximation and that the Tabby star is considered isotropic

Flux of radiation at *a*, $f = \frac{4.7L_{\odot}}{4\pi a^2} = \frac{4.7(3.846\times10^{26})}{4\pi(2.7\times10^{11})^2} = 1973.1 \ldots Wm^{-2}$

Effective radiation absorbed by planet,

$P_{absorb} = f \times A = 1973.1 \ldots \times \left(2.66 \times 10^{17}\right) = 5.24 \ldots \times 10^{20} \approx 5.25 \times 10^{20} W$

Assuming that the planet is an ideal black body, using the Stefan–Boltzmann law with equilibrium conditions

$$P_{absorb} = 4\pi r_p^2 \sigma T_p^4$$
$$5.25 \times 10^{20} = 4\pi(2.909 \ldots \times 10^8)^2(5.67 \times 10^{-8})T^4$$
$$\therefore T = \sqrt[4]{8.70 \ldots \times 10^9} \approx 305K$$

(d) Given that its relatively cool, the large apparent size of this "planet" is probably not due to the presence of an escaping atmosphere.

Furthermore, planetary models suggest that the maximum size (by volume) of a planet is roughly the size of Jupiter. Why is this the case? In other words, why wouldn't adding a large amount of mass to Jupiter increase its volume significantly? Thus, explain why a planet is unlikely to have created this dip. [3 marks]

According to our models of planet formation, sufficiently massive planets are able to accrete large amounts of gas from the protoplanetary disk, and transform into gas giants. Due to their composition, adding mass to gas giants causes them to compress (increased gas pressure to counteract gravitational attraction). This explains why the volume of these gas giants is roughly constant.

Given the above information, we know that there's a natural size limit for planets, which is much smaller than the observed size of the "planet". This suggests that a planet cannot have caused this dip.

(e) Instead of a planet, suppose a dust cloud was responsible for the observed dip. For simplicity, assume the dust cloud is similar to that of the Martian atmosphere. The average column mass of the Martian atmosphere is 160 kg per square meter covered, and assume that the Martian atmosphere blocks off 40% of all incoming visible radiation.

Under these assumptions, what would be the mass of the dust cloud that lies in our line of sight? Express your answer in terms of Moon masses. [2 marks]

To reproduce a 22% overall dip, around $\frac{22}{40}$ of the apparent area of the star was obscured by the dust cloud. Thus, the apparent area of the dust cloud is:

$$A = \frac{22}{40}\pi(1.58R_{\odot})^2 = 2.091 \times 10^{18}\ \ m^2$$

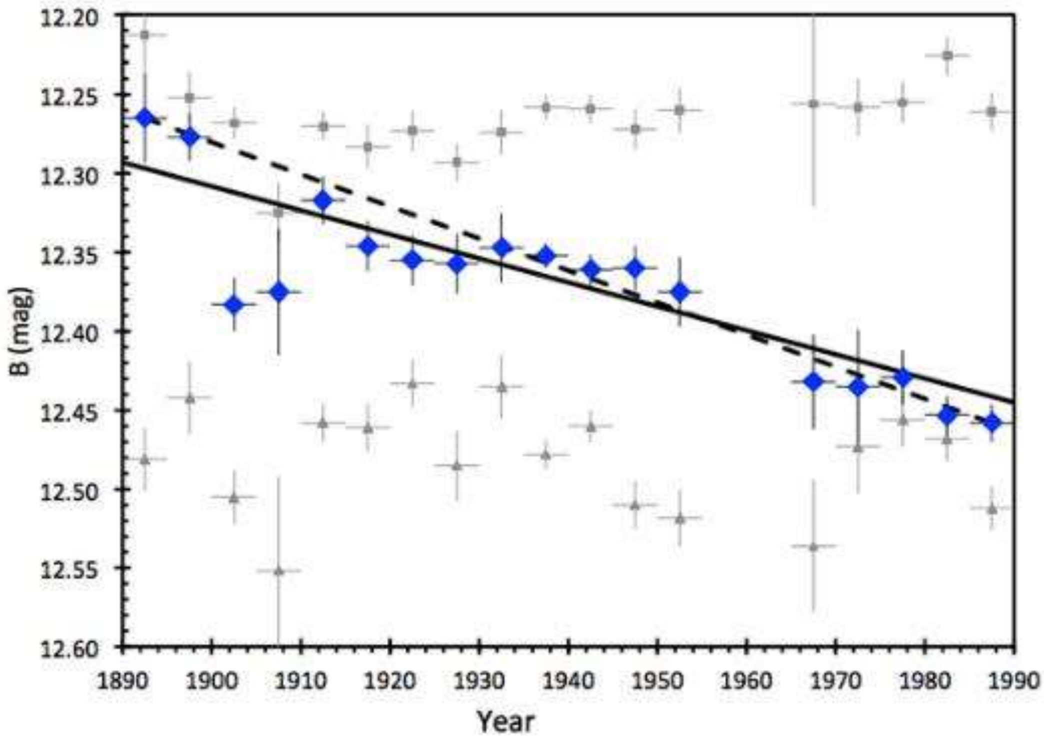
The mass of the cloud is then

$$m = 160A = 3.346 \times 10^{20}\ \ kg = \ \ 0.00455\ \ m_{☾}$$

For comparison, the total mass of the *entire* asteroid belt is around' 4% of the Moon. Unless we are really lucky to observe the full extent of the cloud, this suggests a highly violent star system that's filled with dust, just that most of it is not within our line of sight

Photometry with Harvard Archival Plates

Deepening the mystery, it turns out we do have long term photometric observations of the star, along with 2 nearby field stars. As we can see, Tabby' star (blue diamonds) displays a noticeable long-term dimming trend, especially compared to nearby stars.



(f) Between 1890-1895, Tabby's star had an average B magnitude of 12.265, while this fell to 12.458 between 1985-1990. Assuming this trend is repeated for the star's actual luminosity, what is the average change in the star's luminosity per year? [3 marks]

Use relation between apparent and absolute (bolometric) magnitude

$$m_1 - M_1 = 5\log\frac{d}{10pc}$$

In addition, the relation between absolute magnitude of a star and the Sun

$$M_1 = -2.5\log\frac{L_1}{L_\odot} + 4.7554$$

Assume that over 100 years, the distance from Tabby star and Earth remains constant. We then form 2 simultaneous equations

$$12.265 - \left(-2.5\log\frac{L_1}{L_\odot} + 4.7554\right) = 5\log\frac{d}{10pc}$$

$$12.458 - \left(-2.5\log\frac{L_2}{L_\odot} + 4.7554\right) = 5\log\frac{d}{10pc}$$

Combining the simultaneous equations, we get

$$-0.193 + 2.5\log\frac{L_1}{L_2} = 0$$

$$\therefore \frac{L_1}{L_2} = 1.1945\ldots \Rightarrow \frac{L_2}{L_1} \approx 0.837$$

Meaning that over 100 years, the observed luminosity dropped by 16.3%

Therefore the average decrease per year is 0.163%

(g) In the previous section, we assumed that the change in the star's m_B fully reflects changes in the star's luminosity. Is it possible for this assumption to be false? Explain using an example. [3 marks]

Yes. [1 mark] The simplest example would be the case of a transit across the star. [1 mark] The star appears to dim, but its luminosity remains unchanged [1 mark]

Original answer: The answer that we were looking out for was a case where the star's color changed (e.g. from a red supergiant to a blue supergiant). [1 mark] The luminosity of both stars may be very similar, but the red supergiant would be much dimmer through a blue filter, since it emits very little blue light. [1 mark]

(h) The current leading explanation for the large short term dips observed in the star is that we are observing a swarm of comets evaporating very near the star. Briefly explain why this hypothesis has difficulty explaining the long term trends in this light curve. [2 marks]

By Kepler's second law, comets spend very little time near their perihelion. This means that a large (and increasing!) stream of comets is required to create this observed dimming over an entire century. Referring to your answer in e, the required total mass of the comets involved would be tremendous, and could not be sustained over long timescales.

DRQ 3: The Hertzsprung-Russell (H-R) Diagram [20 marks]

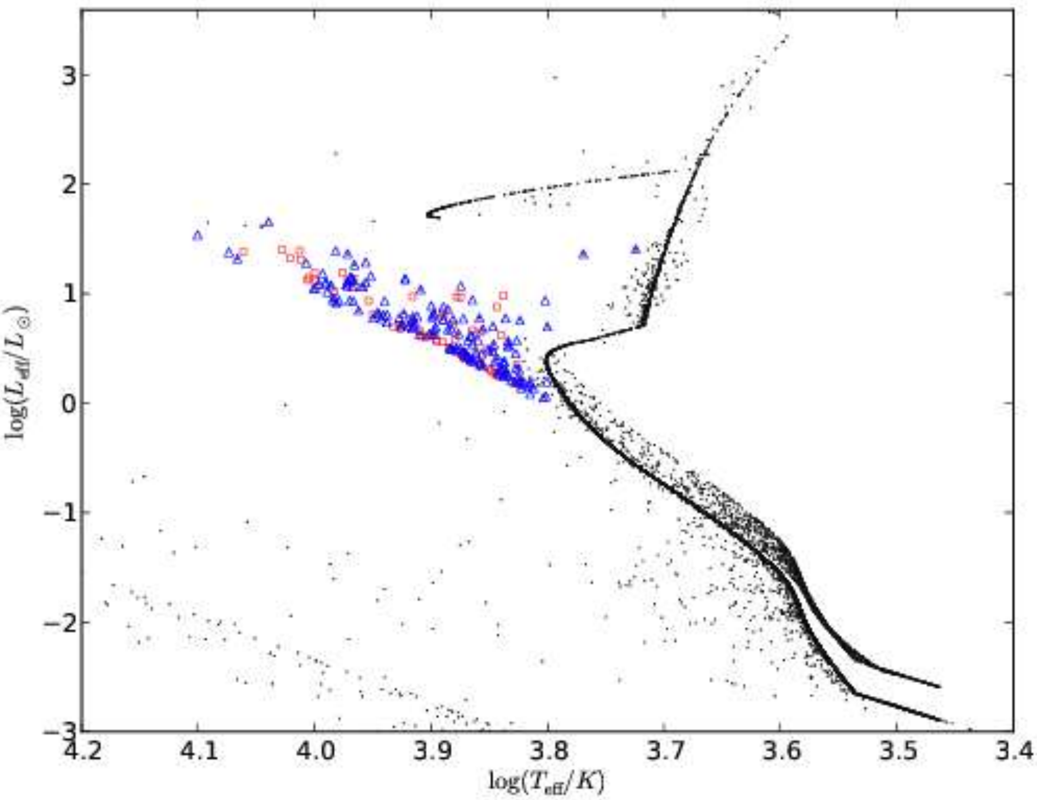


Figure 3.1 *shows the Hertzprung-Russell (H-R) diagram, plotting luminosity versus temperature, from a simulation. Squares and triangles denote blue-straggler stars in single and binary systems, respectively. Source: Chatterjee (2013)*

The existence of "blue stragglers" has confounded astronomers ever since their existence was first noted in Allan Sandage's 1953 paper. Although they are relatively rare stars, a few of them have been found in star clusters, where they clearly stand out because they occupy a region of the H-R diagram that should be completely empty. A star's age and initial mass uniquely determine its luminosity and temperature: stars of a given mass on the main sequence have a more or less fixed position on an H-R diagram. In a star cluster, all members formed at about the same time, and so we see a clear position on a cluster H-R diagram called the "turn-off" where stars with main sequence lifetimes equal to the age of the cluster starts to move off. This phenomenon occurs with the exception of a few interlopers affectionately dubbed as "blue stragglers".

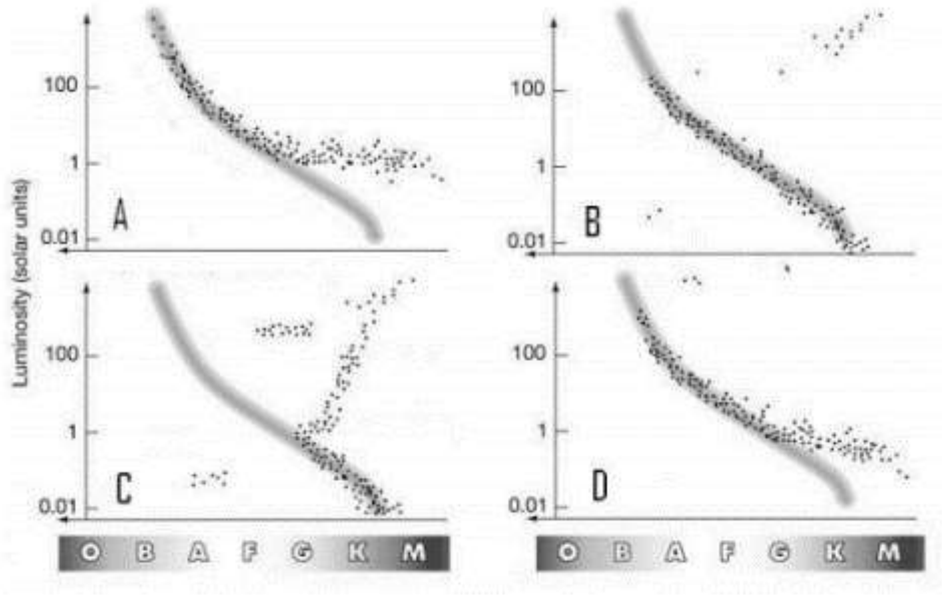


Figure 3.2 *4 H-R diagrams showing how stars of different masses (all starting from the same age) evolve over time. Each dot in the diagrams represents a star.*

<u>Question I</u>

(a) From Figure 3.1, determine the absolute bolometric magnitude of the main-sequence turnoff point. [2 marks]

Measuring with a ruler, L is around $10^{(6/14)}$ solar luminosities
Accept values between $10^{(4/14)}$ to $10^{(8/14)}$ solar luminosities

Plug in the best answer into the absolute bolometric magnitude formula, to get 3.68

(b) It is believed that the most luminous red giant stars serve as a standard candle. Define what a standard candle is. [1 mark]

A standard candle is a class of astrophysical object with known luminosity which may be used to determine its distance.

(c) Using Figure 3.1, determine the luminosity and absolute bolometric magnitude of the brightest red giant. [2 marks]

Measuring with a ruler, L is around $10^{3+(5/14)}$ solar luminosities
Accept values between $10^{3+(3/14)}$ to $10^{(7/14)}$ solar luminosities

Plug in the best answer into the absolute bolometric magnitude formula, to get -3.64

(d) Rank the H-R diagrams in Figure 3.2 according to increasing age. Explain the motivation for your choice. [3 marks]

A, D, B, C
As the cluster evolves with age, the heavier stars evolve off the main sequence branch into the red giants, while the lesser mass stars evolve into white dwarfs.

(e) Suggest why stars of spectral class M lie above the main-sequence in diagrams A and D [3 marks]

These are protostars which have yet to achieve hydrostatic equilibrium with masses that are significantly lower. These stars are still undergoing gravitational freefall before hydrostatic equilibrium and hence are larger and far more luminous than their main sequence counterparts

(f) Why is it surprising for astronomers to find "blue stragglers" beyond the 'turn-off point'? [2 marks]

The main sequence turn-off point indicates where stars in this cluster with larger mass and hence greater luminosity stars beyond this turn-off have evolved off the main sequence into red-giants. However, presence of these blue stragglers suggesting that these stars were either formed later or maintain their luminosity through other means.

(g) Explain how blue stragglers are formed. [2 marks]

These blue stragglers are known to be formed due to mass transfer in a binary star system where the companion star is most commonly a white dwarf. By mass transfer, the star is provided additional nuclear fuel which allows it to maintain its luminosity and hence appear 'bluish' as compared to the other stars in the cluster.

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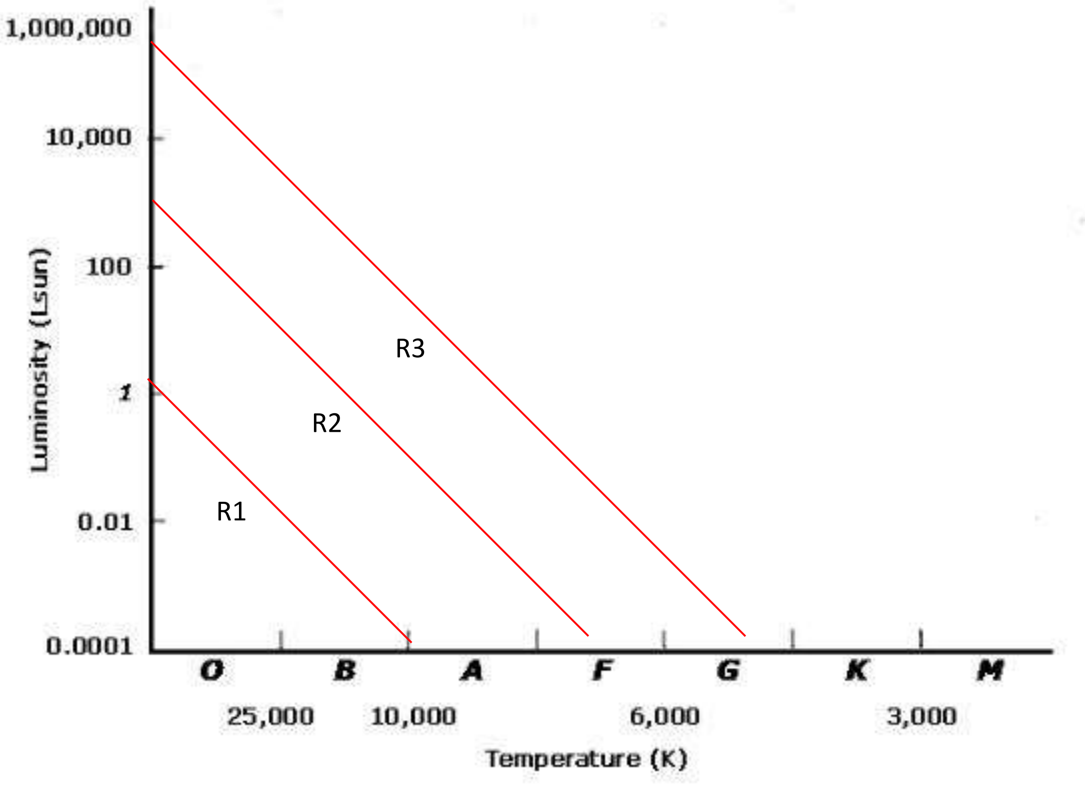


Figure 3.3 *H-R Diagram for Luminosity against Temperature*

<u>Question II</u>

Find an expression for Log_{10} (L/L_{sun}) in terms of radius R/R_{sun} and T_{eff} and other constants. [2 marks]

$$\mathrm{Log(L/Lsun)} = \mathrm{Log}\left(4\pi(\frac{\mathrm{R}}{\mathrm{Rsun}})^2\sigma\right) + 4\mathrm{Log(Teff)}$$

(a) Sketch the equation found in (a) on Figure 3.3 for three different scenarios, namely R = R1, R2 and R3, where R1 < R2 < R3. Label your sketches. [3 marks]

Detach this page and attach it to your answer script

DRQ 4: The Martian [20 marks]

In the movie “The Martian”, *Hermes* took on itself to return to Mars to pick up astronaut Mark Watney who had been stuck on Mars for 47 Sols. *Hermes* did so using the so-called “Rich Purnell manoeuvre”. Similarly, we are on a space mission from Earth to Mars that is not too far different from the said manoeuvre. Our mission is to launch a spacecraft from Earth to Mars in the period of 2016-2020. The Hohmann transfer orbit method is used for this mission. In this question, you will be investigating whether the mission will be successful.

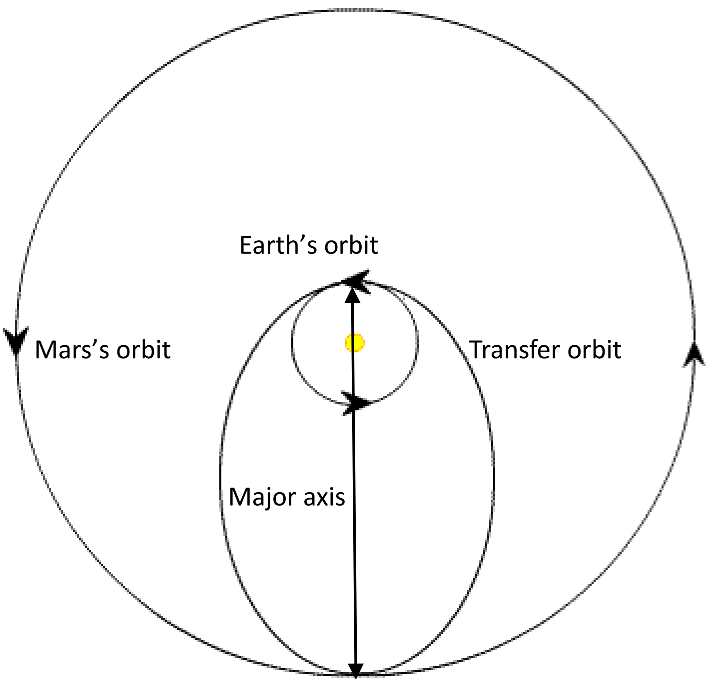


Figure 4.1 *This diagram shows Sun in the center of it all, the inner orbit represents in the orbit of Earth and the outer orbit that of Mars.*

The Hohmann transfer orbit is an **elliptical orbit** used to transfer between two circular orbits of different radii on the same plane. To perform the Hohmann transfer, 2 engine impulses (instantaneous velocity changes, Δv) are required. The first impulse moves the spacecraft onto the elliptical (intermediate) orbit and the second impulse moves it off into the orbit of the other planet/object.

In general, the total energy of a spacecraft is the sum of its ***kinetic energy*** and ***gravitational potential energy***, and this total energy also equals ***half the gravitational potential energy at the “average” distance*** a (the semi-major axis of the elliptical orbit).

Question I

(a) Write down the equation in relation to the statement above and hence or otherwise express v^2, where v is the velocity of the orbiting body, in terms of the radius of earth’s orbit (R_E), the mass of the sun (M_{sun}) and the semi-major axis, a and any other relevant constants. [2 marks]

$$\frac{1}{2}mv^2 - \frac{GM_{\text{sun}}m}{R_E} = -\frac{GM_{\text{sun}}m}{2a}$$

$$v^2 = GM_{\text{sun}}(\frac{2}{R_E} - \frac{1}{a})$$

The equation found in (a) is always otherwise known as the vis-viva equation. This provides us with an expression that relates the total energy of the body on the elliptical orbit at the point of perigee (the point closest to the Sun) in relation to its kinetic energy

(b) In order to find the first instantaneous change in velocity Δv_1, we will need to find the semi-major axis, a. It is known that for an ellipse, the semi-major axis is half the distance of the major axis, or the distance between the perigee and the apogee as shown in the diagram below. Hence, express semi-major axis, a as a function of the earth's orbit, R_E, as well as Mar's orbit, R_M. [1 mark]

$$a = \frac{R_E + R_M}{2}$$

(c) Before launch, the initial velocity of the spacecraft is equal to the Earth's orbital velocity. From the information above, show that the first instantaneous change in velocity Δv_1 required to transfer into the Hohmann orbit is:

$$\Delta v_1 = \sqrt{\frac{GM_{Sun}}{R_E}}\ (\sqrt{\frac{2R_M}{R_E + R_M}} - 1)$$

[4 marks]

$$v_i = \sqrt{\frac{GM_{\text{sun}}}{R_E}}$$

$$v_f = \sqrt{GM_{\text{sun}}(\frac{2}{R_E} - \frac{2}{R_E + R_M})}$$

$$\Delta v_1 = v_f - v_i$$

After simplification,

$$\Delta v_1 = \sqrt{\frac{GM_{Sun}}{R_E}}\ (\sqrt{\frac{2R_M}{R_E + R_M}} - 1)$$

For the spacecraft to leave the Hohmann transfer orbit onto Mars' orbit, a second instantaneous change in velocity Δv_2 is required. The idea is similar as found in the previous steps (a) – (c) except that we are moving OUT of the Hohmann transfer orbit. This yields:

$$\Delta v_2 = \sqrt{\frac{GM_{Sun}}{R_M}}\ (1 - \sqrt{\frac{2R_E}{R_E + R_M}})$$

(d) To calculate fuel requirements (and associated initial mass of the spacecraft, m_i). I use two different methods.

- Apply the rocket equation twice, using Δv_1 and Δv_2 as required, and changing the final mass of the spacecraft for each stage.
- Apply the rocket equation once, but using $\Delta v_T = \Delta v_1 + \Delta v_2$ instead.

Show that both methods will lead to the same value for m_i [2 marks]

From formula book,
The rocket equation is given as

$$\Delta v = v_{\text{exh}} \ln(\frac{m_i}{m_f})$$

$$\Delta v_1 = v_{\text{exh}} \ln(\frac{m_i}{m'_f})$$

$$\Delta v_2 = v_{\text{exh}} \ln(\frac{m'_f}{m_f})$$

Where m'_f is the final mass after the first impulse and m_f is the final mass after the second impulse.

$$\Delta v_1 + \Delta v_2 = v_{\text{exh}} \ln\left(\frac{m_i}{m'_f}\right) + v_{\text{exh}} \ln\left(\frac{m'_f}{m_f}\right)$$

$$\Delta v_1 + \Delta v_2 = v_{\text{exh}} \ln\left(\frac{m_i}{m_f}\right)$$

After some manipulation,

$$m_i = m_f * e^{\frac{\Delta v_1 + \Delta v_2}{v_{\text{exh}}}}$$

(e) Hence or otherwise, given that $v_{\text{exh}} = 2989.1\text{m/s}$, find the percentage loss in mass for the 2 impulses (instantaneous change in velocity), Δv_1 and Δv_2. [2 mark]

$$\Delta v_1 + \Delta v_2 = v_{\text{exh}} \ln\left(\frac{m_i}{m_f}\right) = 8794.28 + 5643.86$$

$$\Delta v_T = 14438.14 \ \text{m/s}$$

Given that,

$$\Delta m = m_i - m_f$$

After some manipulation gives,

$$\frac{m_f}{m_i} = e^{-\frac{\Delta v_1 + \Delta v_2}{v_{\text{exh}}}}$$

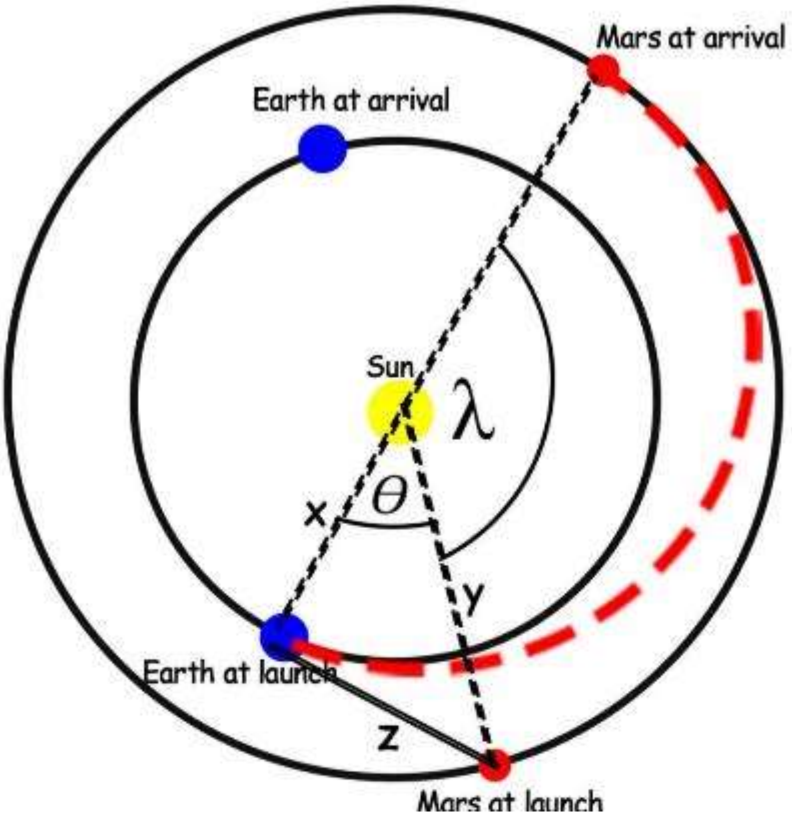
$$1-\frac{m_f}{m_i}=1-e^{-\frac{\Delta v_1+\Delta v_2}{v_{exh}}}$$

$$\frac{\Delta m}{m_i}=1-e^{-\frac{\Delta v_1+\Delta v_2}{v_{exh}}}$$

Substituting the relevant values gives a 99.2% loss in mass.

Some details of the initial plan are as follow:

- The spacecraft is to be launched on 8th June, 2016.
- On that day, the distance between Earth and Mars is 1.05×10^8 km.



(f) Given the information provided, find angle θ and hence angle λ as shown in the diagram above. [2 marks]

Hint: Triangle XYZ is generally NOT a right-angled triangle

$$z^2 = x^2 + y^2 - xy\mathrm{Cos}(\theta)$$

Substituting the relevant values in the Cosine rule as shown above (given in the formula booklet) and making θ the subject, you will find θ to be 21.8˚.

$$\lambda = 158.2^\circ$$

(g) Using Kepler's Third Law, find how many days it will take for the spacecraft to reach Mars (or reach the apogee), supposing that it was launched on 8th June 2016. State any assumptions used. [2 marks]

K3L:

$$T^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

Since the mass of the spacecraft is much smaller than Sun, $m << M_{\text{sun}}$

$$T^2 = \frac{4\pi^2}{G(M_{\text{sun}})} a^3$$

And that

$$a = \frac{R_E + R_M}{2}$$

Substituting the relevant values would hence give us,

$$T = 4.47 \times 10^7 s$$

Or

$$518 \text{ days}$$

But, we are only interested in half the period, so we would hence need **259 days** to reach the apogee.

(h) Given that the period of Mars orbit is 687 Earth days, suggest if the spacecraft will reach Mars in time if it was launched on 8th June 2016. [3 marks]

We know that the period of orbit for Mars is 687 Earth days and that for it to move by and angle λ = 158.2˚, it would require 301.898 Earth days. Yet, the time required for the apogee is 259 days which means by then Mars would still be behind the spacecraft in position. We hence know that the spacecraft will not be able to reach Mars if it was launched on 8th June 2016. It has in fact likely missed its launch window.

DRQ 5: Observation Question [20 marks]

One fateful Friday evening, Ivan decided to do a sidewalk on the Engineering Bridge, Faculty of Engineering, National University of Singapore. He brought along a 5" Newtonian with a focal length of 700mm and sadly only an eyepiece with a focal length of 30 mm and a true field of view of 55 degrees.

Ivan only realised it was a full moon after setting up and hence grudgingly pointed at the full moon for viewing.

Question I

(a) What is the angular diameter of the Moon? Assume circular orbits. [2 marks]

Solving for

$$\theta = \text{ArcSin}[\frac{1.738 * 10^6}{3.843 * 10^8}]$$

Gives, $\theta = 0.259^\circ$.

(b) Determine the apparent field of view of the eyepiece, and hence determine if the Moon fills the frame of the eyepiece with this setup. [3 marks]

Magnitude is above $\frac{70}{3}$ or 23 times.

Which then gives aFoV for the eyepiece of 2.36°

The Moon will not fill up the field of view sadly.

(c) During the sidewalk, a student asks what a zodiac constellation is. Explain. In other words, what property makes the zodiac constellations different from the rest? [1 mark]

A zodiac constellation is defined such that it lies on the ecliptic where the Sun can be found in that constellation during the Earth's orbit sometime in a year. Whereas other constellation may not be necessary found on the ecliptic.

Despite the full moon, Ivan decided he would give a Stellarium show of "What-Could-Have-Been-Seen", if the moon won't there. He then orientated the view on Stellarium to this particular part of the night sky as shown on the next page.

Question II

(a) Identify the cardinal points on the diagram on the next page [2 marks]

i) Circle Polaris (α UMi) [1 mark]

ii) Trace out the 'Southern Cross' and label it accordingly. [1 mark]

iii) Trace out the 'Summer Triangle' and label it accordingly. [1 mark]

iv) Trace the 'Big Dipper' and label accordingly [1 mark]

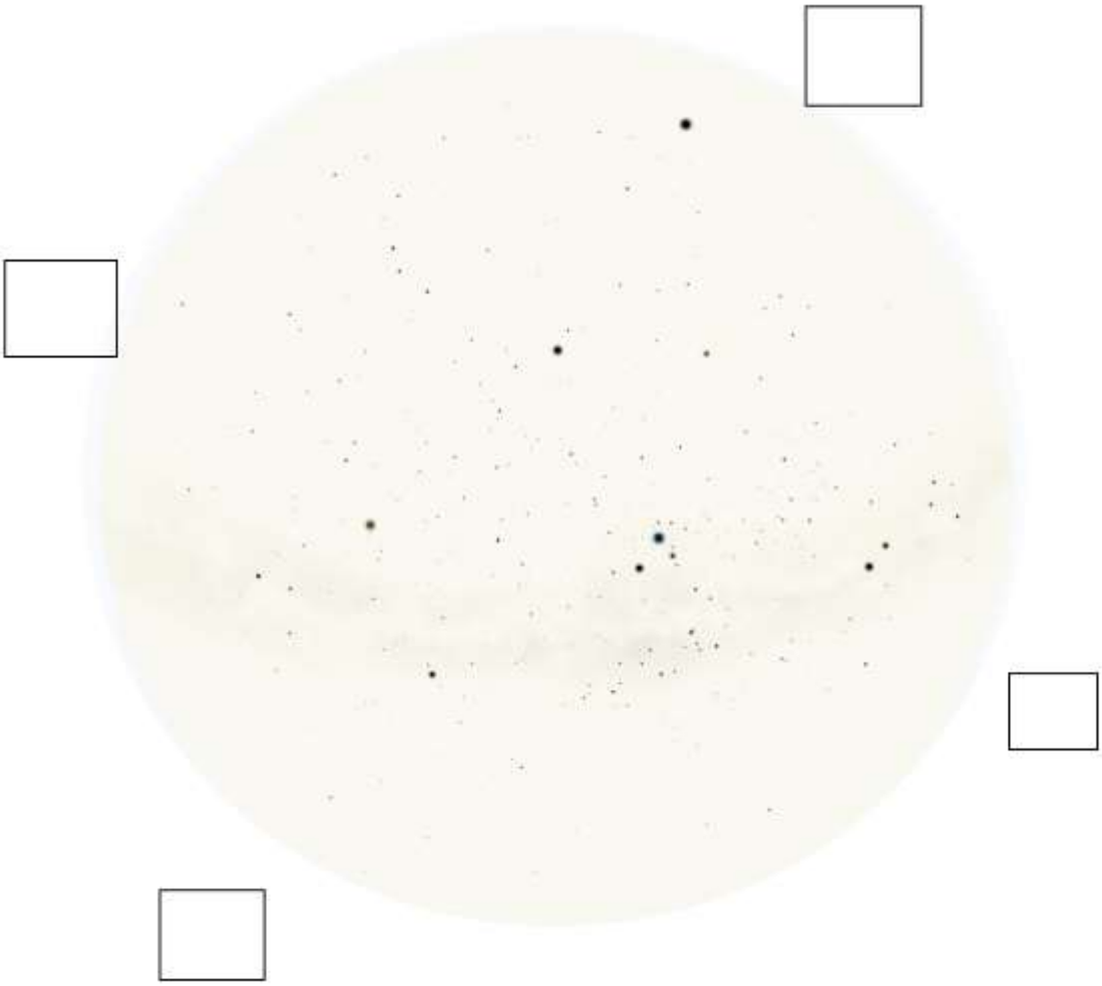
(b) List out 2 major IAU constellations (other than Crux, Ursa Major and Ursa Minor) that can be seen in the diagram. [2 marks]

(c) List down 2 prominent nebulae that are visible, and mark their positions on the diagram. [2 marks]

(d) List down 2 prominent open clusters that are visible, and mark their positions on the diagram. [2 marks]

(e) List down 2 prominent globular clusters that are visible, and mark their approximate positions on the diagram. [2 marks]

Detach this page and attach it to your answer script



The figure shows approximately half of the celestial sphere, but which half?

Detach this page and attach it to your answer script