AstroChallenge 2023
Junior Team Round

SOLUTIONS

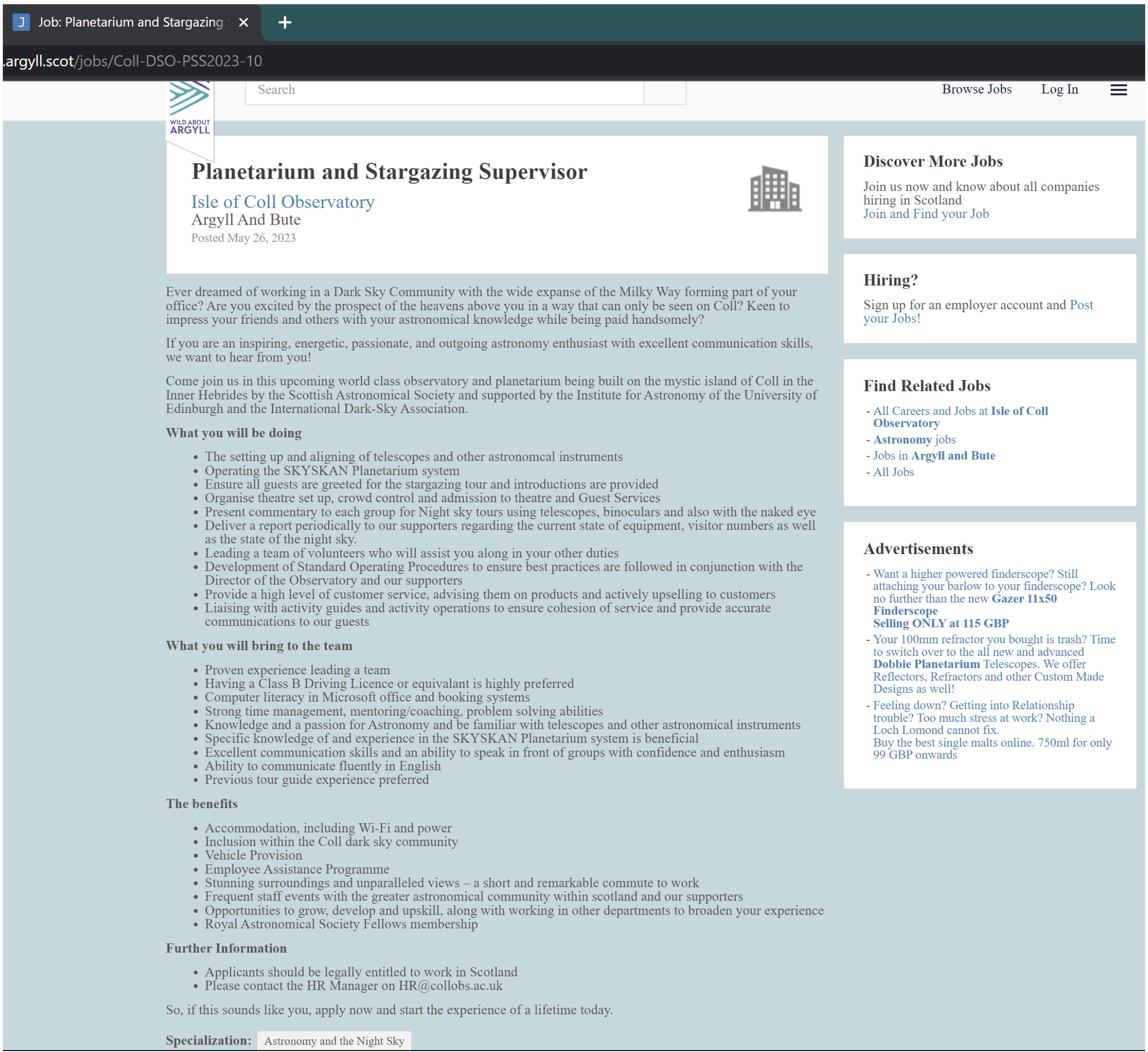
Saturday 27th May 2023

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. This paper consists of **41** printed pages, including this cover page.
2. Do **NOT** turn over this page until instructed to do so.
3. You have **2 hours** to attempt all questions in this paper.
4. At the end of the paper, submit this booklet together with your answer script.
5. Your answer script should clearly indicate your name, school, and team.
6. It is your responsibility to ensure that your answer script has been submitted.
7. The marks for each question are given in brackets in the right margin, like such: [**2**].

Question 1 A Job Interview

After about a year of sadness and depression, Jack came across this job advertisement on the internet courtesy of his friend Asher.


Job: Planetarium and Stargazing
.argyll.scot/jobs/Coll-DSO-PSS2023-10
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WILD ABOUT ARGYLL

Planetarium and Stargazing Supervisor
Isle of Coll Observatory
Argyll And Bute
Posted May 26, 2023

Ever dreamed of working in a Dark Sky Community with the wide expanse of the Milky Way forming part of your office? Are you excited by the prospect of the heavens above you in a way that can only be seen on Coll? Keen to impress your friends and others with your astronomical knowledge while being paid handsomely?

If you are an inspiring, energetic, passionate, and outgoing astronomy enthusiast with excellent communication skills, we want to hear from you!

Come join us in this upcoming world class observatory and planetarium being built on the mystic island of Coll in the Inner Hebrides by the Scottish Astronomical Society and supported by the Institute for Astronomy of the University of Edinburgh and the International Dark-Sky Association.

What you will be doing

- The setting up and aligning of telescopes and other astronomcal instruments
- Operating the SKYSKAN Planetarium system
- Ensure all guests are greeted for the stargazing tour and introductions are provided
- Organise theatre set up, crowd control and admission to theatre and Guest Services
- Present commentary to each group for Night sky tours using telescopes, binoculars and also with the naked eye
- Deliver a report periodically to our supporters regarding the current state of equipment, visitor numbers as well as the state of the night sky.
- Leading a team of volunteers who will assist you along in your other duties
- Development of Standard Operating Procedures to ensure best practices are followed in conjunction with the Director of the Observatory and our supporters
- Provide a high level of customer service, advising them on products and actively upselling to customers
- Liaising with activity guides and activity operations to ensure cohesion of service and provide accurate communications to our guests

What you will bring to the team

- Proven experience leading a team
- Having a Class B Driving Licence or equivalant is highly preferred
- Computer literacy in Microsoft office and booking systems
- Strong time management, mentoring/coaching, problem solving abilities
- Knowledge and a passion for Astronomy and be familiar with telescopes and other astronomical instruments
- Specific knowledge of and experience in the SKYSKAN Planetarium system is beneficial
- Excellent communication skills and an ability to speak in front of groups with confidence and enthusiasm
- Ability to communicate fluently in English
- Previous tour guide experience preferred

The benefits

- Accommodation, including Wi-Fi and power
- Inclusion within the Coll dark sky community
- Vehicle Provision
- Employee Assistance Programme
- Stunning surroundings and unparalleled views – a short and remarkable commute to work
- Frequent staff events with the greater astronomical community within scotland and our supporters
- Opportunities to grow, develop and upskill, along with working in other departments to broaden your experience
- Royal Astronomical Society Fellows membership

Further Information

- Applicants should be legally entitled to work in Scotland
- Please contact the HR Manager on HR@collobs.ac.uk

So, if this sounds like you, apply now and start the experience of a lifetime today.

Specialization: Astronomy and the Night Sky

Discover More Jobs
Join us now and know about all companies hiring in Scotland
Join and Find your Job

Hiring?
Sign up for an employer account and Post your Jobs!

Find Related Jobs
- All Careers and Jobs at **Isle of Coll Observatory**
- **Astronomy** jobs
- Jobs in **Argyll and Bute**
- All Jobs

Advertisements
- Want a higher powered finderscope? Still attaching your barlow to your finderscope? Look no further than the new **Gazer 11x50 Finderscope** **Selling ONLY at 115 GBP**
- Your 100mm refractor you bought is trash? Time to switch over to the all new and advanced **Dobbie Planetarium** Telescopes. We offer Reflectors, Refractors and other Custom Made Designs as well!
- Feeling down? Getting into Relationship trouble? Too much stress at work? Nothing a Loch Lomond cannot fix. Buy the best single malts online. 750ml for only 99 GBP onwards

Figure 1: An advertisement for a job at a public observatory

At this point, not thinking exactly straight, Jack happily agreed and wrote an email on the spot to sign up for it. The email reply invited him for a physical interview on the island. It was located at 56,62° N; 6,62° W.

It was a windy day when Jack stepped into the RSPB Totronald headquarters building being used as a temporary office while the observatory was being built. He had to take a flight to Manchester and change trains at Glasgow. When he finally arrived at the port town of Oban, he had to stay overnight before the ferry early the next morning. Then was the long walk from the pier to where Jack currently was. Fortunately, someone gave him a lift partway.

Well, here he was faced by 3 interviewers. There was one civil servant of some sort, a professor of some sort and a younger person who was the vice president of something or the other.

After getting through some small talk during which Jack mixed up William Farquhar and Stamford Raffles, they got to the questions proper.

“You are from Singapore. Do tell us what astronomical wonders are visible in Singapore but not here."

(a) Name 2 objects (stars or DSOs) that can never be seen from the Isle of Coll but visible in Singapore. Ignore the effects of light pollution. **[2]**

> **Solution:**
>
> **Any prominent objects at far south declinations. Examples:**
> **Jewel Box**
> **Carina Nebula**
> **47 Tucanae**
> **Southern Pleiades**
> **(1m for each correct answer)**

Jack answered as you have and a follow up was asked.

(b) "Observers on the Isle of Coll can only see a smaller part of the celestial sphere as compared to observers in Singapore." Explain if this statement is true or false, and why. **[1]**

> **Solution:**
>
> **This statement is true. [0.5m]**
>
> **Observers on the Isle of Coll cannot see objects with declination south of -33deg, while observers in Singapore, being nearly on the equator, can see nearly the entire celestial sphere. [0.5m]**

Jack could not tell if the interviewers were impressed at that answer. They just hummed. Without giving Jack some time to take a break, the younger person spoke up.

“Not all of this job is about your astronomical knowledge. We also desire some people with the ability to teach to the masses.” He paused. “While I was working in the Observatory in Galloway, we were asked the following question: I’ve read up about these things called open clusters and globular clusters. However, I do not understand how are they different even after reading the article. Could you enlighten me?”

(c) State two differences between open clusters and globular clusters. **[2]**

> **Solution:**
>
> **Globular clusters are far more massive than typical open clusters. [1m]**
>
> **Stars in globular clusters tend to be composed of older-low mass stars as compared to open clusters. [1m]**

The same person immediately followed up.

“With the advent of semi-realistic games like Herbal Space Programme, a solar system spaceflight simulator of sorts, in the past few years or so, more and more people have started asking us how far away is the nearest star cluster and if humans could ever travel there...”

“I am not sure I remember the exact distance...”

“That is fine, give an estimate."

(d) Estimate the distance to the nearest star clusters. **[1]**

Solution:

Any answers on the order of 10^2 light years are accepted. [1m]

Hearing this, Jack received some nods from the interviewers. It was then that the civil servant spoke up.

(e) "Singapore's light pollution is rather infamous, I'm sure. Name a common form of light pollution and explain how it can affect astronomy." **[2]**

Solution:

Possible answers: 1) Light Trespass. Light trespass affects us by affecting our dark vision by having unwanted light shine on where you are stargazing.
2) Skyglow. Makes the sky brighter than it usually is, hence reducing the contrast with which we are able to see stars and other objects.
3) Glare. Light from nearby sources interfere with our vision.
(1m for example, 1m for explanation)

After Jack answered, a follow up question was asked.

"Given the skies around here are solidly Bortle 2, what deep sky objects would you expect to be able to see here with your naked eye which you would not be able to back in Singapore?"

(f) Name 2 Deep Sky Objects visible to the naked eye on the Isle of Coll that would otherwise be impossible to see in Singapore. Note that the objects named should at least be above the horizon at some point in a year at both locations. **[2]**

Solution:

Easiest answers would be the Triangulum Galaxy and Andromeda Galaxy, which are common references for determining the Bortle scale of a site. Any other possible answers are also accepted.

After this answer, it was then that the Professor, who has so far remained silent, spoke up: "For our observatory we have decided to go with a 350mm f/3 reflecting telescope. It will be mounted on a L600 Direct Drive mount, which is an alt-azimuth mount. What do you think of our setup?"

(g) State one advantage and one disadvantage of a reflecting telescope as opposed to a refracting telescope. **[2]**

Solution:

Pro: Newtonian reflector do not have lenses which may cause chromatic aberration. Lighter, less costly etc all acceptable. [1m for any correct answer]
Con: Central obstruction, maintenance needed, more exposed elements. [1m for any correct answer]

(h) Given that the telescope mount is computerized with tracking capabilities, explain if the Professor ought to have opted for an equatorial-style mount instead. **[2]**

Solution:

Any reasonable yes/no answer.

Equatorial mounts require counterweights which add to the weight of the entire mount. For larger set-ups, this is not optimal because the telescope is already incredibly heavy and would require a very large and sturdy equatorial platform. [1m]

However, equatorial mounts are more optimal for astrophotography as it mitigates field rotation. If an Alt-azimuth is used, a camera or telescope rotator is required. [1m]

For about an hour, the interviewers asked Jack about various other related questions. To his surprise, Jack felt that he was able to answer most of the questions.

“We have went on for some time about what we can observe in the sky and what we can use to observe the sky. However, we had been asked this question before: There was this photo of the black hole some time back published by NASA[1]. How was it possible for a photo of a black hole to be taken?”

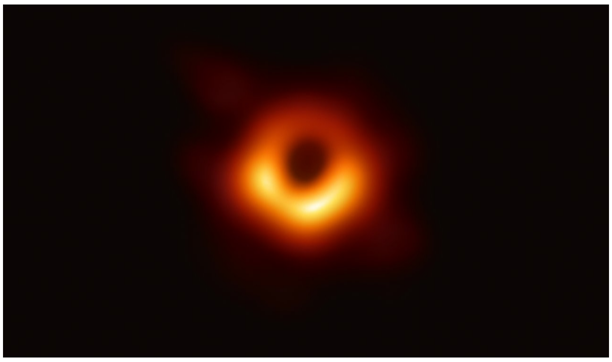

Figure 2: The EHT image of the M87 supermassive black hole

(i) Briefly explain how was the image of the M87 supermassive black hole taken by the Event Horizon Telescope produced, and why it is not possible to take a similar photo through a typical optical telescope. **[2]**

Solution:

The M87 black hole has a very smal angular size and a typical optical telescope will not have sufficient angular resolution to image the black hole. [1]

The image of the M87 black hole was produced through Very Long Baseline Interferometry (VLBI) with multiple radio observatories over the world in order to achieve a sufficiently high angular resolution. [1]

The interviewers nodded. “Thank you for answering our questions for about an hour...”

“It is my pleasure!” Jack could only nod and smile. There was nothing else he could say. He did enjoy the interview though.

“We just have 2 more questions for you.”

(j) "Explain why stars often seem to have different colours." **[2]**

[1]The famous photo of the M87 black hole was actually taken by the Event Horizon Telescope project, which was an international collaboration by various scientific institutions and radio observatories. NASA has no direct involvement in the project.

Solution:

Stars emit at different wavelengths according to their blackbody temperature. This difference in peak emission results in different colours seen by human eyes. [2m]

(k) **"Another question we are frequently asked is: I am interested in stargazing and astronomy. How should I start?."** [1]

Solution:

Any reasonable answer.

After answering the last questions, the interviewers invited Jack to pose some questions to them. After asking a few, Jack declined to ask any more.

"Thank you once again for coming all the way from Singapore. We really appreciate it." Said one of them with a smile. "You will be staying on the island for a while, I gather?"

"Ah yes, dark skies are hard to come by over there." Jack relaxed slightly. "I hope to take a few photos and actually stargaze... It has been a while since I have last seen such pristine skies. Plus I need to unwind, life in Singapore is too stressful..."

The Interviewers smiled. "That indeed. We will let you know the results soon."

(l) **Do you think Jack will get the job? Explain why or why not.** [1]

Solution:

Any answer.

Question 2 Asteroid-Centered Direct-Redirection Quest

Part I Prologue: A (Nasty) Discovery

It is 2am - the air is cold and dry in the control room of the Steward Observatory's Catalina Station in Arizona. The quiet hum of electronics fill the background while LED indicators twinkle in the peripheral. Alvarez wonders what luck he had, picking the short straw and working on this lovely Christmas night.

Whilst combing through recent images, a peculiar point has been raised by the computer. Alvarez pulls up an archive image and overlays it with the image that the telescope just took. It's a newly discovered Near-Earth Object (NEO). As protocol dictates, the ephemeris must be run to predict the path of the NEO.

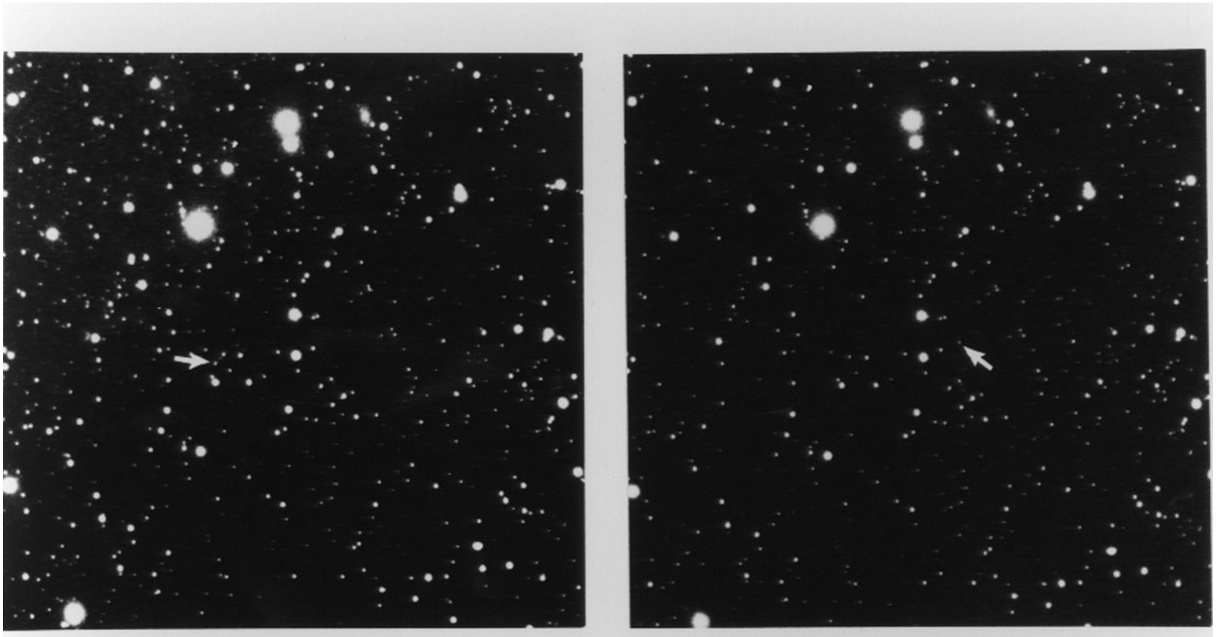

Figure 3: A Christmas Discovery[2]

"Hmm, the computer must be bugging out. The distance becomes zero in 2 years." However, despite multiple runs on multiple computers, Alvarez confirms the worst - we really have a NEO on a collision course with Earth. We have exactly two years till impact (25 Dec 2025). It's ironic really, being named after the scientist that came before that proposed the dinosaur-killing asteroid hypothesis as we face plausible extinction.

[2]This is actually the photographic plates for the discovery of Pluto.

Part II So... what's the plan?

Today, you'll be part of the team that will be planning the Asteroid-Centered Direct-Redirection Quest. Thanks to further observation brought about that nasty Christmas surprise, we know the following details about Alvarez's asteroid.

Mass	df29848f
Diameter	7-10 km
Semi-Major Axis	1.6 AU
Orbital Period	ff2341e1
Type	C-Type

Table 1: Information on *Alvarez's* Asteroid

(a) Given the available information, calculate the Orbital Period of Alvarez's Asteroid in Earth years. **[1]**

Solution:

The semi-major axis is given in Table 1. We can then use Kepler's 3rd Law to get the orbital period:

$$T^2 = \frac{4\pi^2}{GM}a^3$$

$$T = 6.40 \times 10^7\text{s} = 2.02 \textbf{ Earth Years}$$

Alternative:
Another way is to make use of the K3L relation for Solar System objects. By using units of AU and Earth Years, K3L reduces to:

$$T^2 = a^3$$

This gives us the same result:

$$T = 2.02 \textbf{ Earth Years}$$

Both versions will be accepted.

(Marker's Remark) A few scripts instead cited the full form of K3L:

$$T^2 = \frac{4\pi^2}{G(M+m)}a^3 \quad (1)$$

Where m is the mass of the asteroid. Given that the mass of the asteroid was not given, one has to make the assumption that $m \ll M$ and therefore $M + m \approx M$ to proceed. This solution was also accepted.

We can plot out the asteroid's orbital part together with Earth's. We can see that the asteroid's perihelion point coincides with Earth. That is where the asteroid will collide with Earth.

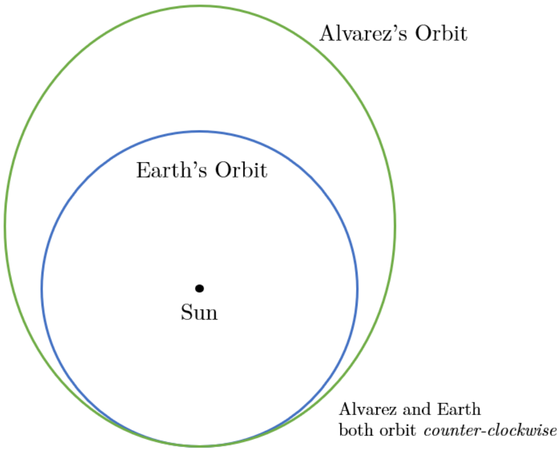

Figure 4: A view of Alvarez and Earth's orbit from the north ecliptic pole.

Both objects orbit *counter-clockwise* around the Sun as viewed above the ecliptic north pole. Additionally, Alvarez's orbital plane aligns with the Earth's orbital plane - the two orbital planes are co-planar.

This is when you wonder, what exactly is our plan to stop the asteroid? Your mind immediately goes towards the recent Double Asteroid Redirection Test (DART) that NASA conducted. This is your game plan:

Figure 5: Game Plan 1

Part III Smash It

Going along the direction of smashing it with an object like the DART mission, the plan is to send a probe up to meet it along its orbit and then hit into it. You, knowing that failure is not an option, want to explore this idea and test its feasibility first. This is where your underpaid team comes in to help you.

A member suggests, for argument, that the probe be sent in a retrograde orbit which matches the orbit of Alvarez.

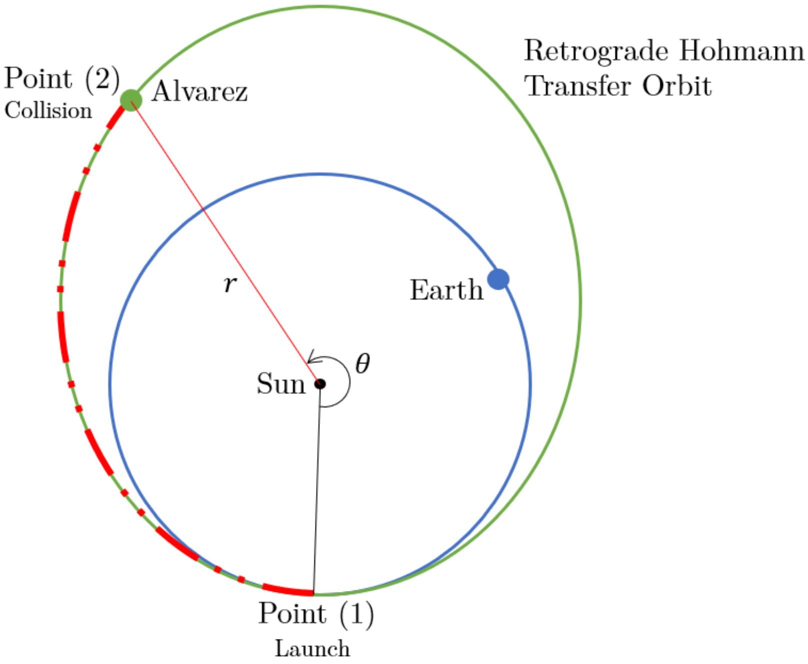

Figure 6: Collision Course

To simplify calculations, Earth's orbit is treated as circular. The probe that we send will already have left the sphere of influence of the Earth and is now orbiting the Sun along the same orbit as Earth's. We need to change the probe's velocity to change the probe's orbit.

For an elliptical orbit of semi-major axis a around a large central mass M, when the orbiting object is at a distance r from the central mass, it has a velocity v given by the **Vis-Viva** equation:

$$\boxed{v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)} \tag{2}$$

(b) Calculate the $|\Delta v|$ needed to change the probe's orbit. **[3]**

Solution:

As per the description given, we will ignore all gravitational effects by the Earth.

Starting as a circular orbit around the Sun, the probe's orbit is circular with a radius of 1 AU. We can get its initial velocity from:

$$v_0 = \sqrt{\frac{GM_\odot}{D_\oplus^2}}$$

Where $D_\oplus$ is the radius of Earth's orbit and $M_\odot$ is the mass of the Sun. We get that:

$$v_0 = 29.82 \text{ km/s}$$

The next thing is to find the final orbital velocity. We know the final orbit of the probe is the same as Alvarez. So it has an orbital semi-major axis of $a = 1.6$ AU with a perihelion distance of $r_p = 1$ AU.
When we change the probe's velocity, it is at Earth's orbit and at perihelion. The

time taken for this orbital manoeuvre is very small compared to the orbital period. Hence it can be assumed to be instantaneous. Using the vis-viva equation given:

$$v_f^2 = GM_\odot \left(\frac{2}{r_p} - \frac{1}{a}\right)$$

$$v_f = 34.97 \text{ km/s}$$

Take note that v_0 and v_f are in opposite directions. Hence, the $|\Delta v|$ is:

$$|\Delta v| = 64.8 \text{ km/s}$$

Note: This is an extremely large Δv and is extremely infeasible in real life. (Marker's Note) A lot of participants forgot that the probe will be sent into *retrograde* orbit around the Sun to rendezvous with the asteroid. This led to multiple teams citing the answer as $v_f - v_0 = 5.15$km/s which is incorrect. It is important to distinguish the direction when calculating the Δv of an orbital transfer.

We can find the point of impact. At impact, the asteroid will be this distance from the Sun, the computer tells us:

```
solution» r = 1.80519 AU
```
(3)

We will want to have a sense of how much the impact will change the asteroid's trajectory to be sure of the efficacy of this plan.

To model the impact, we turn to our trusty friend: Conservation of Angular Momentum. Together with some simplifying assumptions, we can model the impact as a *completely inelastic collision*. This would mean that after the collision between the probe and the asteroid, the two masses stick together and move as a single entity. We can then show that the change in the asteroid's velocity is roughly given by:

$$\Delta V \approx -\frac{2mu}{M} \tag{4}$$

m and M are the masses of the probe and the asteroid respectively, with u being the orbital velocity of the probe with respect to the Sun at the point of impact.

(c) For a medium-sized probe of $m = 1000kg$ and using the mass of the asteroid as 2×10^{15}kg, show that the change in velocity is $\Delta V = 2.07 \times 10^{-8}$m/s. **[2]**

Solution:

We can use the impact parameters known to us so far. We know the radial distance from the Sun is 1.80519AU. Using the Vis-Viva equation, we can get that:

$$v^2 = GM_\odot \left(\frac{2}{r_p} - \frac{1}{a}\right)$$

Using the equation derived in the earlier part:

$$\Delta V = \frac{-2mu}{M+m} \approx \frac{2mu}{M}$$

$$= \frac{2(1000)(20670)}{2 \times 10^{15}}$$

$$= 2.07 \times 10^{-8} \text{ m/s}$$

(Marker's Note) The negative sign in the equation is due to the way u is defined, where positive indicated a prograde orbit. This is a remnant of an old draft. As such,

leniency was given when participants ignored the negative sign and the negative value of the velocity u. The way the equation was derived was through the Conservation of Momentum. These points were introduced during the Post Mortem presentation.

This value looks too small. Recalling the DART mission, the value we get in this analysis are orders of magnitude **smaller** than what reality seems to suggest. This means that the collision has a lot more *impact* than we expect in our current analysis.

(d) Which assumption did we make in our analysis that caused such a underestimation in our theoretical prediction? [2]

Solution:

The main culprit is the assumption that the collision was *completely inelastic*. In reality, the energy from the impact will lift a lot of debris from the surface in various directions. A better approximation would be to treat it as a *partial disintegration* process. However, such a process is very hard to model.

(Marker's Note) Any correct mention that the inaccuracy came from us physically modelling it as a completely inelastic collision was accepted.

A few groups cited that we have neglected the gravitational interaction between the asteroid and the probe which led to an underestimation. This was not accepted as it is not a significant contributor. We can use the following back-of-the-envelope calculation to see.

We shall consider the gravitational potential energy of the Asteroid-Probe system:

$$U_{int} = -\frac{GMm}{r} \tag{5}$$

where M is the mass of the asteroid and m is the mass of the probe, with r being their separation. At collision, the surfaces of the two bodies touch. As the radius of the probe is much smaller than the radius of the asteroid, we can approximate the separation of their centers to be the radius of the asteroid. Refering back to Table (1), we use the smaller end of 7km in diameter.
We get the GPE to be:

$$U_{collision} = -\frac{GMm}{R} = -\frac{(6.67 \times 10^{-11})(2 \times 10^{15})(1000)}{3500} = 3.81 \times 10^5 J \tag{6}$$

In the most extreme case where we allow the probe to fall unto the asteroid from infinity, the change in GPE will be converted to Kinetic Energy:

$$\Delta U = \Delta KE$$

$$U_{collision} - U_{\infty} = \frac{1}{2}mv^2 = 3.81 \times 10^5 J$$

Comparing this to the Kinetic Energy of the system due to their orbital motion about the Sun:

$$KE = \frac{1}{2}Mu^2 + \frac{1}{2}mu^2 = 4.27 \times 10^{23} J \tag{7}$$

The extra energy is not even an additional 1% and is thus negligible.

Extra Note.
What researchers do is to model it as how we did in the question, but include an extra "enhancement factor" β. This has the added benefit of encompassing material effects in the collision. Modelling such a collision is still an active area of research. Refer to Stickle (2022) [2], Stickle (2017) [3], and Herberling (2017) [6] for more

details.

For more details about the DART mission, refer to Atchison (2016) [4] and Naidu (2020) [5].

Part IV Blow it Up

Despite the underestimation, the uncertainty comes with the material of the object and might cause it to have a very small *impact* towards the redirection of the asteroid. We head back to the drawing board.

This is when your team mate suggests another method, the one taken by the film *Armageddon (1998)* took - blow it up.

Figure 7: BOOOOM

To blow it up, we need to add energy into the asteroid to overcome the gravitational forces holding it together. Well, technically, we also need to break the rock by overcoming the rock's elastic properties but then we would have to hire an engineer and complicate matters. So let's just consider the gravitational aspect only.

As mentioned, we need to overcome the gravitational forces holding the asteroid together. The amount of energy needed is given by the Gravitational Binding Energy of the body:

$$U = -\frac{3GM^2}{5R} \tag{8}$$

The magnitude of $|U|$ is the minimum energy required.

(e) Given the same mass of the asteroid as 2×10^{15}kg, and a radius of 5.4km, show that $|U| = 2.96 \times 10^{16}$J. **[2]**

Solution:

We plug in the values into the equation given

$$\begin{aligned} |U| &= \frac{3GM^2}{5R} \\ &= \frac{3(6.67 \times 10^{-11})(2 \times 10^{15})^2}{5(5400)} \\ &= 2.96 \times 10^{16} \text{ J} \end{aligned}$$

(Marking) Free two marks. M1 for correct use of formula, A1 for final answer.

We have a lot of nuclear weapons in the world. As of early-2023, the world has around 13000 nukes[1]. The thermonuclear weapons of today have around 100-kiloton yield. 1 kiloton of TNT equivalent is around 4.184×10^{12}J.

(f) Given the worlds' current nuclear stockpile, is it enough to gravitationally unbind the asteroid in one singular (spectacular) blast? [2]

Solution:

The total yield of all the nukes:

$$E_{nukes} = 13000 \times 100 \times 4.184 \times 10^{12}$$
$$= 5.44 \times 10^{18} \textbf{ J} > |U|$$

So, just considering it as a gravitationally bound system of loose rocks, then yes, we have more than enough energy to blast it apart.

(Marking) Another free 2 marks. M1 for correct conversion and to find the energy, A1 for correct conclusion.

This idea seems like it will work, but you have some concerns. The most glaring is a political one - that we would be able to get the world to agree to collectively empty out their nuclear reserves to destroy the asteroid. But there are also other physical concerns regarding the feasibility of the plan.

(g) List two other possible (physical) pitfalls with this plan of using nukes, or other high-yield explosives, in saving the Earth from the asteroid impact. [2]

Solution:

We have several to choose from:

1. **We did not account for the material properties of the asteroid. To be more sure, we will need to account for that.**
2. **The launching of nukes is very dangerous and risky. If it fails, then we just exploded the nuke in the atmosphere. Potential fallout to humans of Earth.**
3. **Without modification, nukes are unable to travel into a solar-centric orbit. Detonation will have to take place very close to the Earth which gives rise to potentially radioactive debris raining down.**
4. **The asteroid might break into multiple, still-large, chunks of radioactive mini-asteroids that impact Earth.**
5. **Most of a nuke's energy output is it's light and heat. The pure concussive pressure blast we associate with nukes is only due to the presence of the atmosphere. It would be rather ineffective in breaking the asteroid up unless the explosion occurs inside the asteroid.**

(Marking) M1 each for any plausible and correct response.

Part V Shine Bright like a Diamond

Back to the drawing board, the recent *Rihanna* performance at the Superbowl inspired you with one last plan. We make the asteroid *shine bright like a diamond.* Well, again technically, diamonds don't shine but rather reflect.

Figure 8: Reduce, Reuse, Reflect

Your plan is to make the surface of the asteroid completely reflective. Then, because light has momentum, the sunlight will reflect off the surface of the asteroid and deflect it. We will proceed with a thought experiment to determine the feasibility of the plan.

For this to work, we must first figure out how much light the asteroid receives along it's journey. We need to find the intensity of the sunlight when the asteroid is a distance d from the sun.

(h) Given the luminosity (power output) of the Sun is $L_\odot$, show that the intensity of light received at a distance d from the sun b (in units of W/m^2) is given by the following equation.

$$b = \frac{L\odot}{4\pi d^2}$$

[1]

Solution:

The sun's light spreads out over the surface area of a sphere as it expands, so:

$$b = \frac{P}{A} = \frac{L_\odot}{4\pi d^2}$$

The next part is calculating the rate of momentum transfer from the sun's light to the asteroid. From Quantum Physics™, we know that the momentum of light is given by:

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \tag{9}$$

Where h, f, and λ are Planck's constant, the frequency, and the wavelength of light. For a reflective sphere, we can consider how the light will reflect of it. We model parallel rays coming from the sun and lighting up half the surface of the sphere before reflecting off the surface.

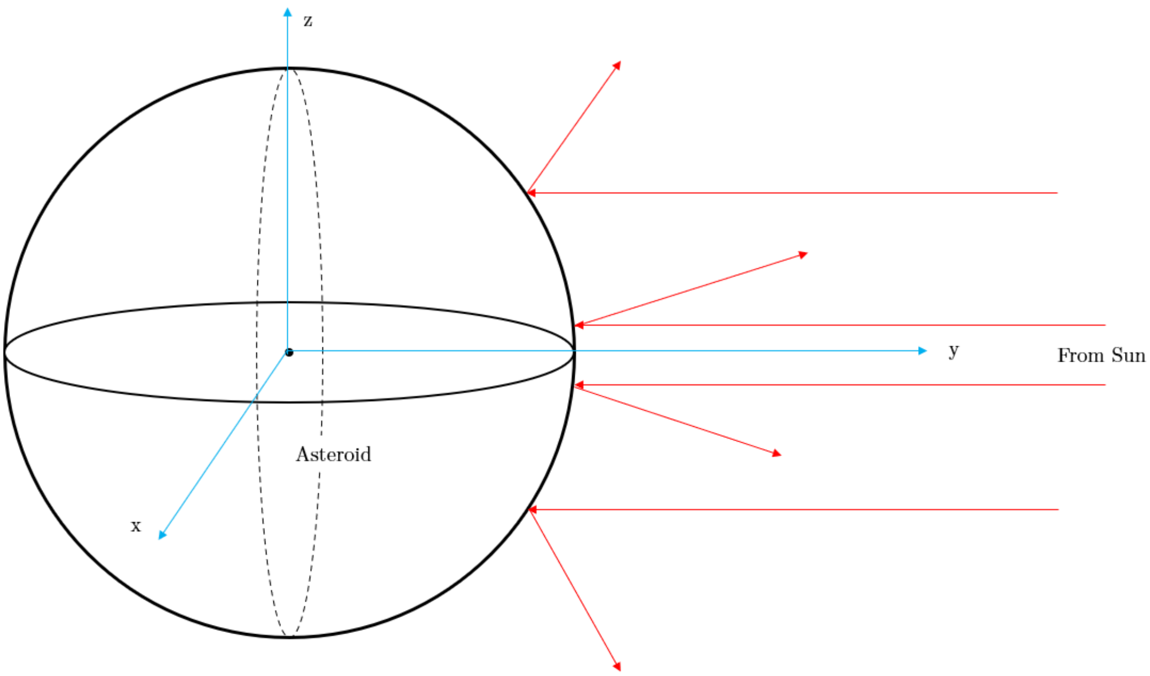

Figure 9: Spherical Reflections

Because of the spherical symmetry, the momentum transfer in the x and z directions balance. The only net momentum transfer to the asteroid in the negative y-direction due to only half of the face being lit. Let us consider the surface of the asteroid that is in the y-z plane, such that any observer on the asteroid at that line of points will have the Sun on their local meridian. We will further define additional angles for a spherical coordinate system (r, θ, ψ).

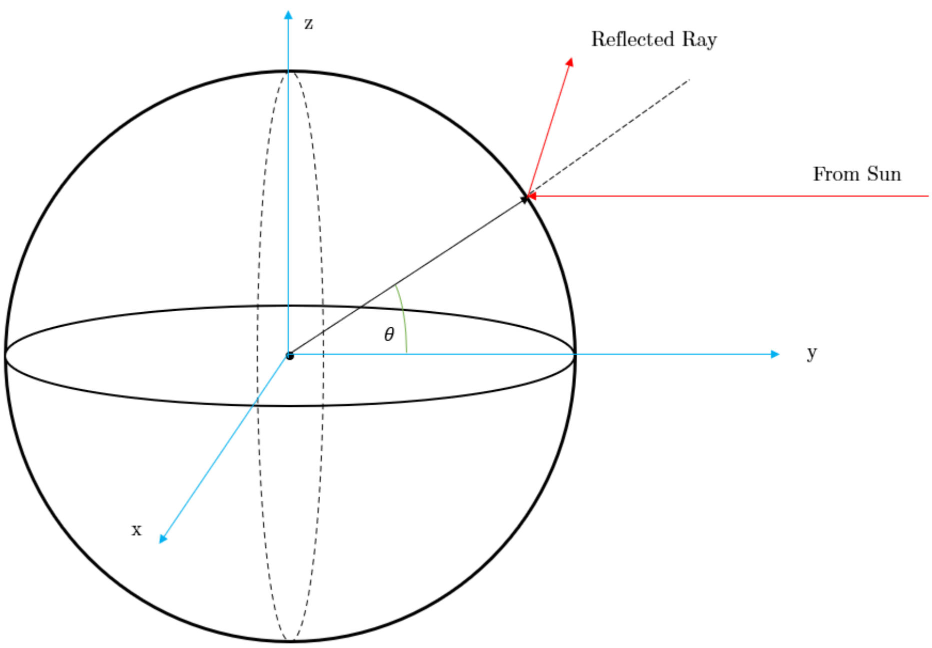

Figure 10: Reflected Ray

(i) Show that the change in the y-component of the momentum by this light ray shown in Figure 10 is as follows:

$$\Delta p = p_\gamma(1 + \cos 2\theta)$$

where p_γ is the momentum of the incoming photon. **[1]**

Solution:

It is simple geometry:

$$\Delta p = p_f - p_i = p_\gamma \cos 2\theta - (-p_\gamma)$$
$$= p_\gamma(1 + \cos 2\theta)$$

(Marking) M0.5 for the correct expression for $\Delta p = p_f - p_i$. Correct geometry M0.5

We find a similar variation for other points on the sphere to give us the momentum imparted by the photon when it hits a random spot (r, θ, ψ) on the sphere's surface to be:

$$\Delta p = p_\gamma(1 + \cos 2\theta \cos 2\psi) \tag{10}$$

We can then sum up the momentum change across the whole hemisphere to get the total momentum imparted:

$$\Delta p = \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} p_\gamma(1 + \cos 2\theta \cos 2\psi) d\theta d\psi \tag{11}$$

$$= p_\gamma \pi^2 \tag{12}$$

We can then use Newton's 2nd Law to find the force exerted by this radiation in the negative-y direction:

$$F = \frac{dp}{dt} = \Delta p \times N \tag{13}$$

Where N is the number of photons hitting the surface of the asteroid per second. If we assume that the photons from the sun are of a specific wavelength λ, then we can find N.

(j) Show that the expression for N is:

$$N = \frac{b\pi R^2 \lambda}{hc}$$

where R is the radius of the asteroid. **[2]**

Solution:

We can find the incident power on the surface by multiplying the intensity of the sun by the cross-sectional area of the asteroid.

$$P_i = b\pi R^2$$

Then, the number of photons that hit per second is the incident power divided by the energy per photon:

$$E_\gamma = \frac{hc}{\lambda}$$

Combining the two gives us the desired result:

$$N = \frac{P_i}{E_\gamma} = \frac{b\pi R^2 \lambda}{hc}$$

(k) Hence or otherwise, show that the acceleration experienced by the asteroid is given by:

$$a = \frac{L_\odot \pi^2 R^2}{4Mcd^2}$$

where M is the mass of the asteroid. State where the direction of acceleration is pointed towards. **[2]**

Solution:

We go back to Newton's 2nd Law:

$$F = Ma = \frac{dp}{dt}$$
$$a = \frac{F}{M} = \frac{\Delta p \times N}{M}$$

From earlier, we can sub in:

$$\begin{aligned} a &= p_\gamma \pi^2 \frac{b\pi R^2 \lambda}{hc} \frac{1}{M} \\ &= p_\gamma \pi^2 \frac{L_\odot}{4\pi d^2} \frac{\pi R^2 \lambda}{hc} \frac{1}{M} \\ &= \frac{L_\odot \pi^2 R^2 p_\gamma \lambda}{4Mhcd^2} \\ &= \frac{L_\odot \pi^2 R^2}{4Mcd^2} \end{aligned}$$

The direction of acceleration points radially outward from the sun. (Marking) M1 for correct expression for incident power, M0.5 for correct expression for energy of photon, M0.5 for $N = P/E$.

But for a good sensing, we can find out it's acceleration

$$\boxed{\texttt{solution» a = 5.36E-13 m/s}^2} \tag{14}$$

That's the acceleration you feel from a 5kg bag of rice standing 630m away. So is Earth saved? Well, it depends on how early we saw it coming and whether there is enough time for these effects to build up. A combination of these techniques could be used one day in the (hopefully distant) future. Nonetheless, it's nice to look back at what we have achieved together.

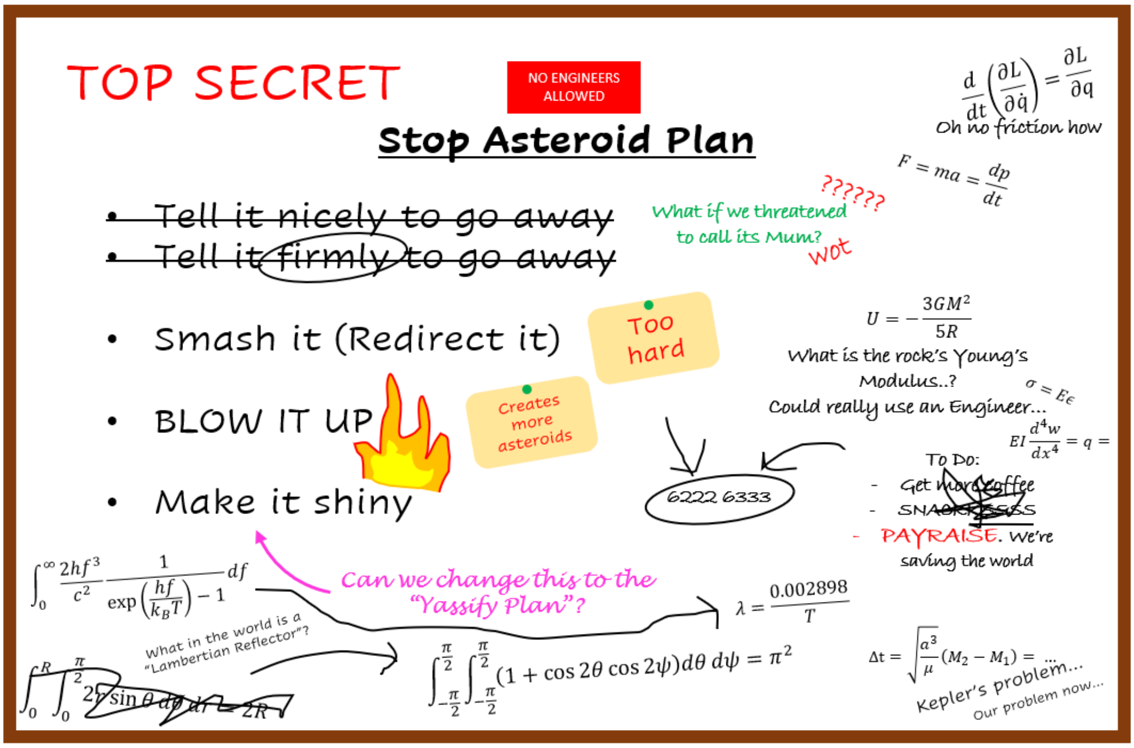

Figure 11: Hard Work

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Question 3 Exoplanets and Exo-Life

Part I Detecting Exoplanets

While stars are easily seen, exoplanets are not since they are so small and so far. They do not emit as much radiation as stars that let us detect them easily. Finding exoplanets is the first step in the search for life, looking for places that can possibly harbor life elsewhere.

The most common way of detecting exoplanets is using the transit method. In fact, the majority of exoplanets discovered thus far have been detected by the Kepler satellite and TESS (Transiting exoplanet survey satellite) via the transit method. When an exoplanet passes in front of its parent star, it blocks out some of the light, thus reducing the brightness of the star. If we plot out the apparent brightness of a star with an exoplanet over time (called a light curve), we see periodic dips in the curve, from which we can derive information about the orbital characteristics of the system.

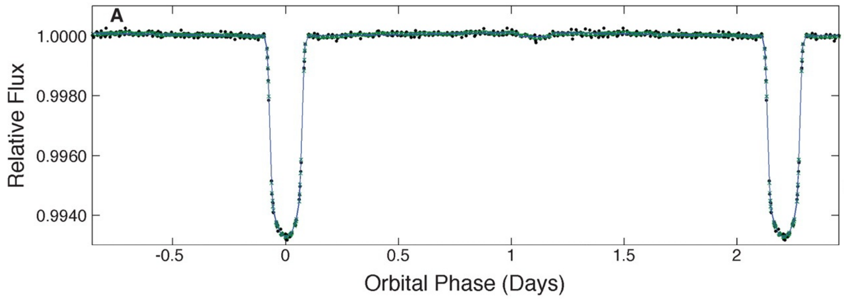


Figure 12: The light curve of a star with an exoplanet.

(a) Name 2 other methods of detecting exoplanets and briefly describe how they work. [4]

> **Solution:**
>
> **Direct imaging: Planets reflect starlight from their star or emit thermal radiation Gravitational microlensing: Light from a background star is lensed due to GR. When a star has a planet, the lensing effect can be seen Radial velocity: A star with a planet will have detectable doppler shifts as the star orbits the barycentre Transit timing: If a star has multiple planets, the planets will affect each other's orbit and cause a slight variation in the orbital period**
>
> **[1 per name, 1 per description. If the name does not match the description, only marks for the name is given]**

(b) The transit method is only able to detect a small fraction of all potential exoplanets, even if they are large enough to cause a detectable dip in the parent star's light curve. Briefly explain why. [1]

> **Solution:**
>
> **Transits are only observable if the plane of the orbit passes through the Earth./ The planet has to pass in front of the star as seen from Earth.**

(c) A Jupiter-sized planet was found to transit a sun-sized star. Estimate the expected percentage decrease in luminosity of the star. [1]

> **Solution:**
>
> **Assuming a homogenous surface brightness:**
> $r_{jupiter}/r_{sun} \times 100\% = (7.149 \times 10^7)/(6.963 \times 10^8) = 1.05\%$

(d) **The semi-major axis of the planet is 1.44 times that of Jupiter's. What is the orbital period of the planet in days? The mass of the star is 1 solar mass.** [1]

Solution:

Using Kepler's third law in years and AU:

$$\left(\frac{T_{planet}}{T_{jupiter}}\right)^2 = \left(\frac{a_{planet}}{a_{jupiter}}\right)^3$$

$$T_{planet} = (1.44)^{3/2} \times T_{jupiter} = 1.728 \times 11.86 \times 365.25 = 7485 \text{ days}$$

(e) **In the search for extraterrestrial intelligence (SETI), Astronomers typically only look for exoplanets orbiting spectral classes of stars type F to M. Suggest a why.** [1]

Solution:

Massive stars are short-lived and alien civilizations are unlikely to develop in such short spans of time.

Part II Exoplanet Conditions: Atmospheric Composition

After detecting exoplanets, we need to figure out if the planet is hospitable to life. The planet should have compounds that are needed for essential processes in life, and not contain any compounds that can cause problems for life. If a planet has no oxygen or has high amounts of sulfur dioxide, we know that it is incredibly unlikely to host any form of life. By studying the atmospheric composition of the planet, astronomers can deduce the likelihood of the planet having life.

In November 2022, the JWST was used to conduct spectroscopy on WASP-39b. Astronomers were able to identify certain compounds in the atmosphere of WASP-39b, such as sulfur dioxide, sodium, potassium, and water vapour.

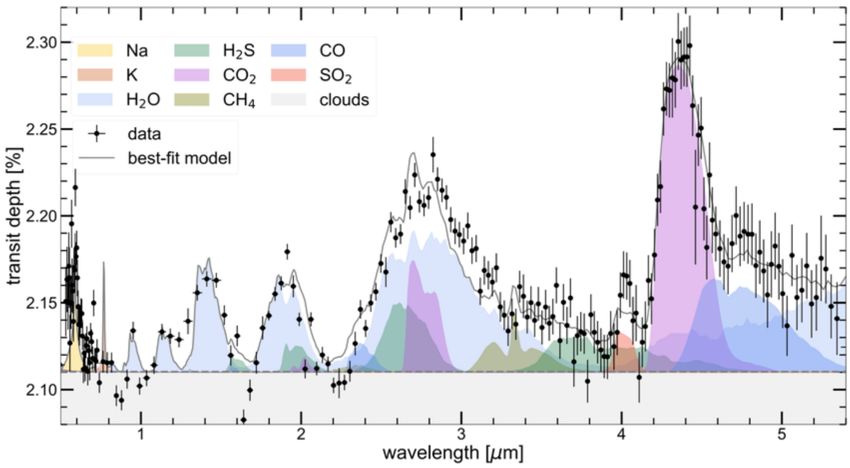


Figure 13: The spectrum of the exoplanet WASP-39b.

(f) **Name 2 kinds of planetary spectra that can be acquired by astronomers, and briefly explain how are they produced.** [2]

Solution:

Any two of the following examples (1m per correct answer)
Transmission: Light from the star passes through the atmosphere of a planet during

a transit
Reflection: Light from the star is reflected by the planet
Emission: Blackbody radiation from the planet

(g) **Explain how the spectrum of a planet can be used to determine the chemical composition of a planet.** [1]

Solution:

The presence of absorption lines [1m] in the continuous spectra of a planet indicates the presence of a particular molecule absorbing that wavelength of light [1m].

Part III Exoplanet Conditions: Temperature

Other than the atmospheric composition of planets, another important factor to look out for is the temperature of the exoplanets. Temperature is important in determining the habitability of a planet. The Goldilocks zone or the habitable zone around a star defines the range of orbital distances that a planet can orbit around a star while still possessing liquid water on its surface. It cannot be too near the star as water would vaporize, nor too far where the water would freeze.

(h) **Kepler-10b, an exoplanet orbiting close to its parent star Kepler-10, has surface temperatures ranging from -223°C to 1560°C. Suggest and explain how this is possible.** [3]

Solution:

The planet is tidally locked to the parent star. [0.5 mark]

One side of the planet always faces the star while the other side never faces the star, causing heating on only 1 side. [1 mark]

The planet does not have an atmosphere. [0.5 mark] An atmosphere can reduce temperature fluctuations and transfer energy to the cooler parts of the planet. The lack of an atmosphere means that higher temperature ranges are possible. [1 mark]

(While this is an extreme example of a planet that is clearly inhabitable as it is too close to its parent star, planets within the habitable zone that experience this effect would still have large temperature variations that still cause it to be inhabitable)

The equation for the equilibrium temperature of a planet is given by

$$T = \sqrt[4]{\frac{I(1-A)}{4\sigma}} \tag{15}$$

Where T is the temperature in Kelvins, I is the flux density of starlight incident on the planet in W/m^2, A is a dimensionless ratio called the albedo, and σ is the Stefan-Boltzmann constant.

(i) **Examine the given equation. Briefly explain the physical meaning of albedo, and how does having a higher albedo affect the equilibrium temperature of a planet.** [2]

Solution:

A higher albedo means more starlight is reflected and not absorbed by the planet. [1 mark] In equilibrium, less incoming radiation implies less outgoing radiation, which is achieved by a reduction in equilibrium temperature of the atmosphere. [1 mark]

(j) **Show that $T = \sqrt[4]{\frac{L(1-A)}{16\sigma\pi a^2}}$, where L is the luminosity of the parent star and a is the semi-major axis of the planet.** [1]

> **Solution:**
>
> **Intensity falls off proportionally to inverse square. Substituting $I = L/4\pi a^2$, we obtain the expression as above.**

(k) **Given that the sun's surface temperature is 5770K and Earth's albedo is 0.3, find the expected surface temperature of the Earth.** [2]

> **Solution:**
>
> **By the Stefan-Boltzmann law, luminosity of the sun $L_{sun} = 4\pi\sigma R_{sun}^2 T^4$. Substitute L into the given formula to obtain the equilibrium temperature.**

(l) **The expected surface temperature is lower than actual surface temperatures on the Earth. Suggest a reason why this is so.** [1]

> **Solution:**
>
> **Greenhouse gases in the atmosphere trap heat.**

Question 4 Red Spiral Galaxies at Cosmic Noon

Part I The James Webb Space Telescope

The James Webb Space Telescope (JWST) is widely touted as the successor to the celebrated Hubble Space Telescope. However, unlike Hubble, which primarily observes in the visible wavelength band of light, the JWST primarily observes in the infrared region of light. As a result, while Hubble could only observe light from the early universe up to a billion years into the past, the JWST can look back up to 12 billion years in the past, which is very close to the start of the big bang.

Unlike Hubble, which orbits the earth at low-earth orbit, the JWST is parked much further out in an orbit around the sun at the Lagrange 2 point of the earth-sun system. At Lagrange points, gravitational forces acting on an object is zero. This applies when the object in question (i.e., JWST) is relatively small compared to the two massive objects (i.e., Sun and Earth). The positions of Lagrange points of any two-body system is given below:

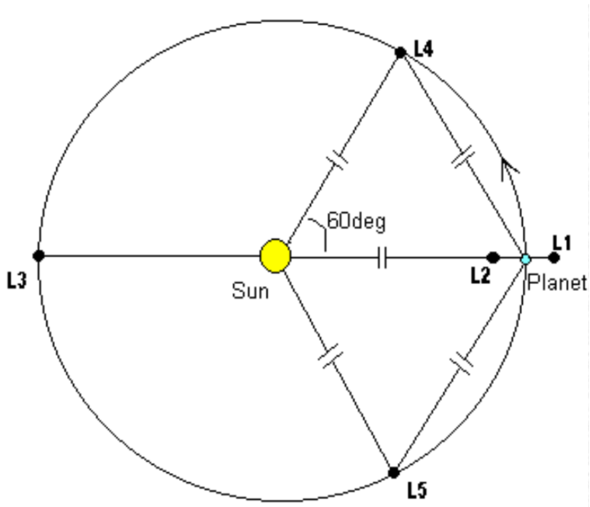


Figure 14: A diagram showing the position of the lagrange points in the Earth-Sun frame of reference

Another property of L2 and L1 is that the period is that they have the same period of revolution around the Sun as the Earth, making the distance between us and the telescope constant at all times.

(a) Given the information above, derive an equation on how we can find the distance between Earth and JWST. You do not need to make numerical calculations nor provide a neat RHS equation (i.e., r = f(x,y,z)) [2]

Solution:

Errata: Figure 14 incorrectly shows the L2 point between the Sun and Earth when it should actually be the other way round. As a result, full credit is given to answers that write the force equation according to the L2 point defined in the diagram instead.

Since Lagrange points have zero net force, we define the outwards centrifugal force as equal to the inwards gravitational force from both the Sun and Earth.

$$F_c = F_{g,earth} + F_{g,sun} \text{ [1m]}$$
$$m\omega^2(R+r) = Gm\left(\frac{M_{sun}}{(R+r)^2} + \frac{M_{earth}}{r^2}\right)$$

All of the variables in the above equation are known except for m, which is JWST's satellite. We divide both sides with the satellite mass, we get:

$$\frac{4\pi^2}{T^2}(R+r) = G\left(\frac{M_{sun}}{(R+r)^2} + \frac{M_{earth}}{r^2}\right) \text{ [1m]}$$

With T as the period of the Earth, R as Earth-Sun distance, G as the gravitational constant, and Ms and Me being the mass of the Sun and Earth, respectively.

Marker's Note: Many participants equated $F_{g,earth}$ to $F_{g,sun}$ instead. This is the equation for the equipotential point, which is entirely different from a Lagrange point.

In July 2022, NASA released one of the very first deep field images taken by the JWST of the galaxy cluster SMACS 0723. One of the first things that astronomers noticed about the image is the unexpected presence of a large numbers of red spiral galaxies.

But what exactly is unexpected about finding so many red spiral galaxies?

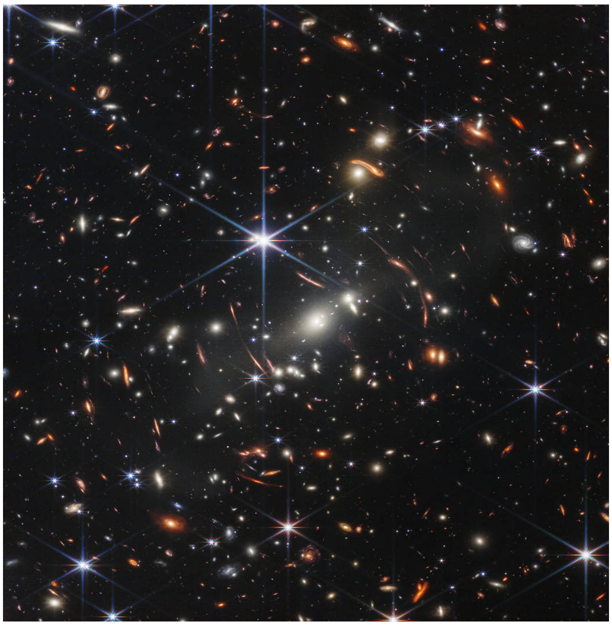


Figure 15: A deep-field image by the James Webb Space Telescope of the SMACS 0723 galaxy cluster.

Part II Seeing Red, Seeing Blue

Due to the cosmological expansion of the universe, galaxies at a further distance appear to recede from us at a faster speed. This relation between the apparent recessional velocity of a galaxy and its distance is captured by Hubble's Law:

$$V = H_0 D \tag{16}$$

Light from a receding galaxy undergoes redshift due to the Doppler effect, and the faster the galaxy recedes, the more the light is redshifted. The definition of relativistic redshift Z is given by:

$$z = \sqrt{\frac{c+v}{c-v}} - 1 \tag{17}$$

(b) Explain why the JWST is able to observe further back in time compared to Hubble by observing in the infrared. **[2]**

Solution:

Light from the distant universe takes a long time to travel and reach the earth, corresponding to looking back further in time. [1m] By Hubble's law, the high recessional velocity of distant galaxies means that light has been redshifted out of the visible wavelengths into the infrared. [1m] Therefore light from distant galaxies are only observable in the infrared.

In order to measure the redshift of distant galaxies, astronomers must know how the light from the galaxy 'looks' like in its own rest frame. One way to characterize how the light from a galaxy (or any astrophysical source) 'looks' like is to split the light into its component wavelengths, also known as its spectrum (Figure 16).

Galaxies can emit strongly at characteristic wavelengths due to interactions between light and specific atoms or molecules present. This produces characteristic 'spikes' in spectra associated with particular elements, and they are known as emission lines. Astronomers can use emission lines to determine the redshift of a star or galaxy. Redshift measurements obtained through such methods are called spectroscopic redshifts.

(c) Why can emission lines be used to measure redshift? **[1]**

Solution:

Emission lines are emitted at a particular known rest wavelength. Therefore, comparing the wavelength of redshifted emission lines in galactic spectra to the rest wavelength, we can calculate to a high degree of accuracy the amount that the galaxy was redshifted by.

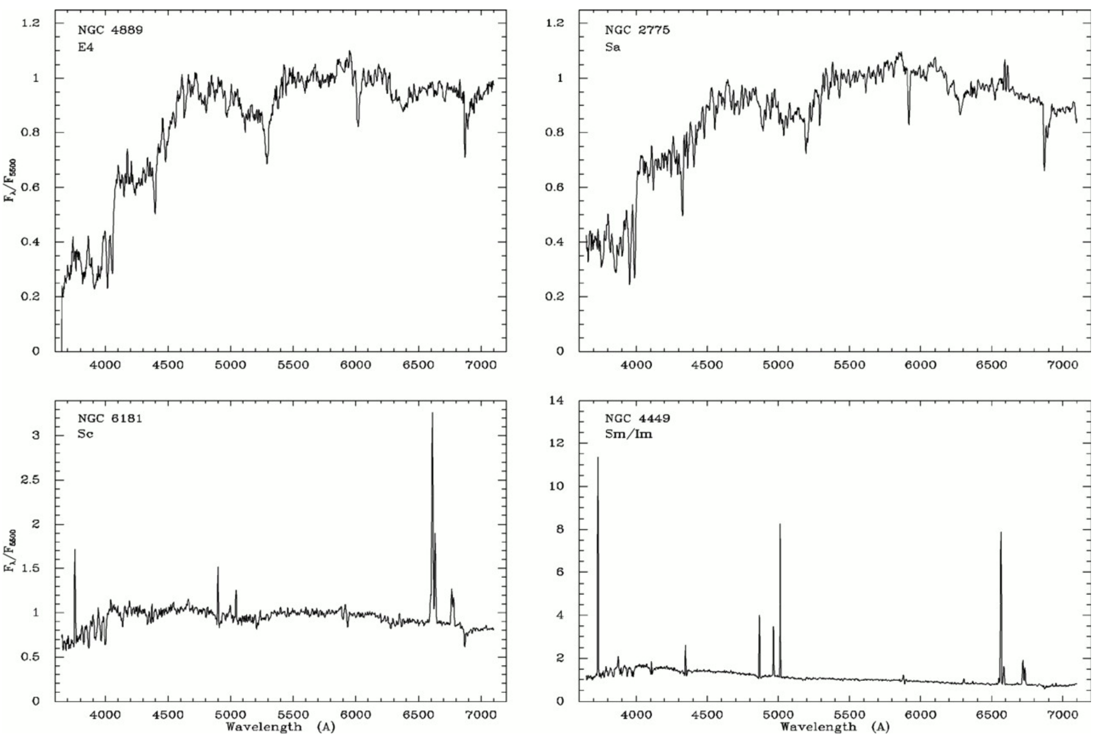


Figure 16: A typical galactic spectra for galaxy morphology types E4 (Elliptical), Sc (gas-rich spirals), Sa (gas-poor spirals), Sm/Im (irregular galaxies).

When we observe the spectra of elliptical galaxies such as in Figure 16, we find that they tend to not exhibit strong emission lines like in most spiral galaxies. Instead, we mostly see the 'reverse'; strong characteristic dips corresponding to known elements which can also be used to measure redshift. To understand this, it might be helpful to recall how the spectrum of a star looks like (Figure 17).

(d) Explain how are the absorption lines of a galactic spectra mainly produced. **[1]**

> **Solution:**
>
> **Light from a galaxy is the sum total of the light from its stars. Since starlight exhibits absorption lines, these absorption lines overlap and are reproduced in the overall spectra of a galaxy.**

(e) Therefore, suggest why spiral galaxies exhibit emission lines in addition to absorption lines in their spectra, but not elliptical galaxies. **[2]**

> **Solution:**
>
> **Spiral galaxies contain many large emission nebulae (HII regions) associated with star-forming regions, which produce the characteristic emission lines. [1m] Elliptical galaxies however usually contain no active star-forming regions and have little cold gas and dust that can be ionized into emission nebulae. [1m]**

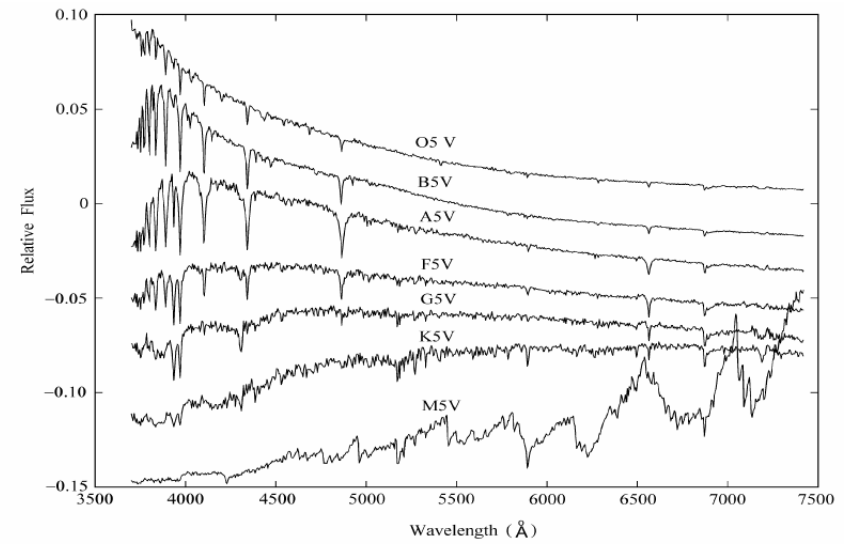


Figure 17: Examples of typical stellar spectra labeled by their Harvard spectral classification.

Another way that the redshift of galaxies can be investigated is by examining the spectral energy distribution (SED) of a galaxy (see Figure 18). In the spectra of a galaxy, we see that the emission and absorption lines are superimposed on a broader curve. This curve is made of the sum total of light from the things that make up the galaxy (think stars, nebulae, dust clouds, active galactic nuclei etc.), which is what we call the spectral energy distribution.

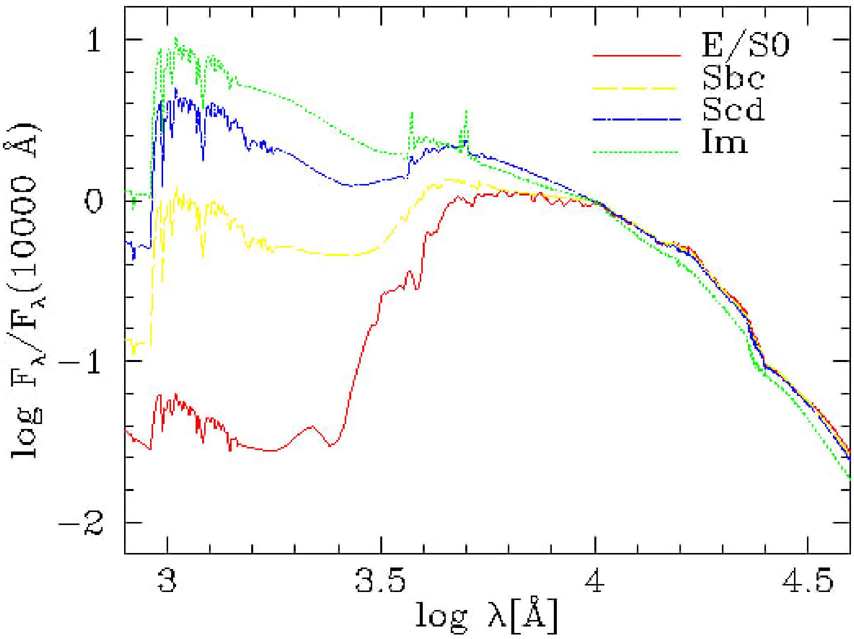


Figure 18: Synthetic SEDs produced by averaging spectra from local galaxies classified by morphology (Bolzonella et. al 2011).

If we have a model of what makes up a particular type of galaxy, we can reproduce the theoretical SED of a galaxy by combining the SED of its individual constituents. In particular, the SED of a star can be approximated as a blackbody curve. This means that hotter and short-lived stars emit more strongly in blue wavelengths while cooler and longer-lived stars emit more strongly in red wavelengths (see figure 17).

(f) Consider two contrasting cases of a recently active star-forming galaxy and a passive galaxy that has long ceased star formation. Suggest what differences we might expect to see in their SEDs and briefly explain why. W [2]

Solution:

An active star forming galaxy contains more young blue O-type and B-type stars, while an older galaxy without active star formation will be mainly made of longer-lived and redder M and K-type stars. [1m] Therefore, the spectral energy distribution of an active star forming galaxy will show stronger emission in shorter blue and ultraviolet wavelengths, and vice versa for old non-star forming galaxies. [1m]

When we observe the spectra of a distant galaxy, it is thus possible to estimate its redshift by comparing its observed spectral energy distribution to the spectral energy distribution predicted by a best-fit SED model, and estimating how much was the overall SED curve shifted laterally.

Part III Uncertain Identities

In our modern-day universe, galaxies can be broadly classified as either spiral or elliptical. Spiral galaxies often contain rich reservoirs of gas and dust, which fuel sites of ongoing star-formation galaxies. They are thus 'blue'. Elliptical galaxies are stripped of gas and dust, and rarely contain on-going star formation. They thus mainly contain long-lived low mass stars and are 'red and dead'. Astronomers believe that elliptical galaxies are formed as a product of galactic collisions and mergers over time.

(g) Based on their formation history, explain why elliptical galaxies are 'red and dead'. [2]

Solution:

When galaxies merge, much of the interstellar gas and dust in the galaxy are flung out

into space. The merger also triggers a sudden burst of star formation, which quickly exhausts the galaxy's supply of cold gas. [1m] Over multiple collisions and mergers, elliptical galaxies become devoid of cold gas and dust needed to fuel on-going star formation. [1m]

Marker's Note: Nearly all participants did not read the text carefully and broadly stated that elliptical galaxies are 'red and dead' because they have stopped star formation and are made of older stars (this was already explained in the text right before the question!). Nevertheless, half-credit was given to such answers out of mercy.

So far so good. Ellipticals are red, spirals are blue. But if that's the case, why are there red spirals too[3]?

Like elliptical galaxies, red spirals are red because they have largely stopped forming new stars. Unlike elliptical galaxies, red spirals still retain complex spiral structures, which means that they have not been disturbed by strong interactions with other galaxies during their lifetime. Red spiral galaxies are very rare in the universe, with a recent survey (Shimakawa et. al 2022) estimating that they make up only 2% of galaxies in our local universe. Some astronomers thus believe that red spiral galaxies are very old spiral galaxies that have exhausted their initial stock of gas and dust and were unable to accrete additional gas due to internal or environmental effects (Masters et. al 2010).

One difficulty with identifying red spirals from large-scale surveys of the sky is that it is often difficult to distinguish between red spirals without active star formation, and 'dusty spirals' that have high rates of ongoing star formation but have star-formation sites shrouded in dense gas and dust.

(h) Explain why 'dusty spirals' can appear similar to red spirals. **[1]**

Solution:

Dense clouds of gas and dust preferentially scatter short bluer wavelengths, and cause 'reddening' of the overall spectral energy distribution.

Part IV It's High Noon

Now we are ready to try to understand the red spirals in the JWST deep field image. In the paper 'Red Spiral Galaxies at Cosmic Noon Unveiled in the First JWST Image' by Fudamoto et. al (2022), the team handpicked a sample of visually red galaxies with spiral structures in the JWST data. Where available, the team used emission-line redshift measurements from other studies. Otherwise, redshift was obtained from a best-fit of SED models to the spectrum of the sampled galaxies.

SED-fitting is often used in place of spectroscopic techniques to obtain redshift data in large-scale surveys of distant high-redshift galaxies. In such surveys a small set of photometric measurements are taken at different wavelength ranges with filters. To reconstruct the SED and corresponding redshift, A set of model SEDs with variable parameters are passed through a set of filtering functions (e.g. the coloured envelopes labeled 'B', 'V', 'i' and 'z' in Figure 19) that mimic the spectral response of the real-life filters to obtain the corresponding synthetic photometric measurements (e.g. the coloured spots in Figure 19). The parameters of the model SED are varied until the difference between the synthetic photometric measurements and the actual photometric measurements are minimized.

[3]Yes, this was meant to be a poem

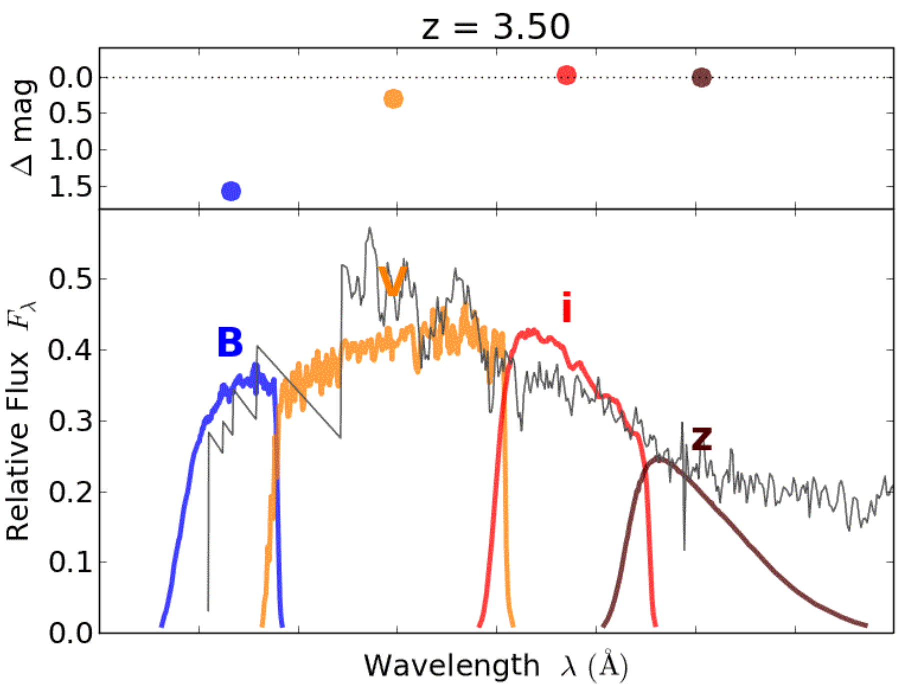


Figure 19: A graphical representation of the SED-fitting process

(i) **Based on what was discussed thus far, state the 3 factors that could plausibly explain the red appearance of the JWST galaxies. These factors need to be accounted for or be included as parameters in the model fitting process.** [3]

Solution:

1 mark per correctly identified factor:
1) Redshift
2) Presence of dust
3) Old stellar population

(j) **Explain why it might be preferable to obtain redshift measurements from SED-fitting rather than spectroscopy in the case of distant high-redshift galaxies.** [2]

Solution:

In order to obtain a detailed galactic spectra, there must be sufficient signal-to-noise (SNR) ratio in the data obtained. A higher SNR ratio can be achieved with brighter sources or longer exposure times. [1m]

Distant galaxies are typically very dim, and therefore it is difficult to obtain data with a sufficient SNR ratio that can be used to determine spectroscopic redshift. [1m]

Fudamoto et. al found that the best fit models for the red galaxies in the JWST image are either passive non-star forming galaxies, or heavily dust-obscured galaxies with redshifts in the range of $1 < z < 3$. The authors note that the range of redshifts roughly correspond to a time of the universe known as the 'Cosmic Noon'. What does this mean?

As we saw earlier, the spectra of a galaxy can give us a lot of information about what kind of galaxy, and whether there is recent or active star formation occurring in it. When astronomers look back in the distant past at old galaxies, they found that the rate of star formation across the universe reached its peak during the z=2 epoch, which astronomers call the Cosmic Noon. This result is often represented by plotting galactic star-formation rate (SFR) against redshift z in what's known as a Madau plot (figure 20).

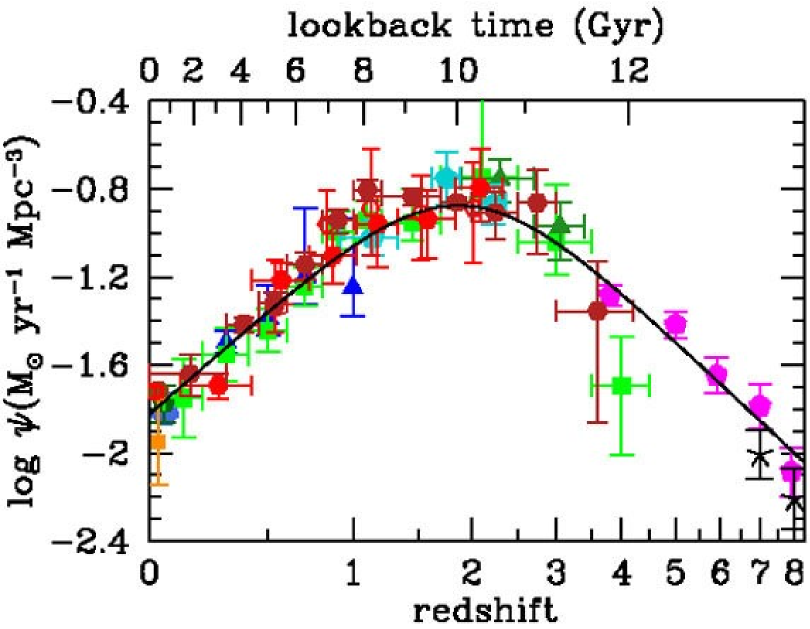


Figure 20: A madau plot which plots star formation rate against lookback time.

In summary, in one of JWST's earliest deep-field images, we have found an unexpectedly high proportion of visually red spiral galaxies present in the image. Model-fitting of the galactic spectra suggests that they might be passive red spiral galaxies with redshifts between 1 and 3, which puts them to the time of peak star formation during the 'Cosmic Noon'.

(k) Explain why this might be an unexpected result. **[2]**

Solution:

One or both of the following:

1) In general we expect there to be more passive and non-star forming galaxies in our present-day universe after the cosmic noon than in the early universe due to the lower overall star-formation rate, therefore it is surprising to find a high occurrence of red spirals in the early universe. [2m]

2) We also expect to see a smaller proportion of red spirals in the early universe as compared to the present era as it takes time for these spiral galaxies to exhaust their supply of gas and cease star formation. [2m]

Full marks for any other reasonable explanation.

Question 5 Arabic Astronomy

Part I Introduction

Fun fact: of all the star names approved by the IAU, approximately 2/3 of them have Arabic roots.

Why is this the case? Well, many of the IAU constellations and official star names are based on Ptolemy's Almagest written in the 2nd century. Starting from the 8th century, Arabic astronomers translated important astronomical texts like the Almagest into Arabic, helping to preserve these texts through the Middle Ages. These texts were then re-translated back into Latin in the 12th century, but many stars ended up maintaining their Arabic names.

However, if you think about this explanation for a bit, you might realize that this raises another question: why did the Arabs give those stars those names then? In some cases, these names were related to Ptolemy's 48 constellations. For example, the star Deneb derives its name from the Arabic phrase *dhanab al-Dajajah* (tail of the hen). Similarly, Denebola derives its name from the Arabic phrase *dhanab al-asad* (tail of the lion).

(a) Which IAU constellation is Deneb located in? **[0.5]**

> **Solution:**
>
> **Cygnus**

(b) Which IAU constellation is Denebola located in? **[0.5]**

> **Solution:**
>
> **Leo**

However, in other cases, Arabic astronomers assigned names based on their own unique starlore prior to the widespread adoption of Greek astronomy. Thus, while these star names do not make any sense in the context of Ptolemy's star catalogue, they reveal some of the key asterisms that ancient Arab astronomers used to describe the sky. The rest of this question will explore some of these asterisms and their cultural significance in the Arabic world during the Islamic Golden Age.

Part II Thuraya and The Follower

For instance, the star cluster Thuraya (figure 21) was highly significant in Arabic starlore, and was often referred to by its alternate name *an-najm* (The Asterism). Given that it is visible to the naked eye even in light polluted Singapore, perhaps it should not be surprising that Thuraya was held in such great esteem. Near Thuraya lies a bright red star known as *ad-dabaran* (The Follower).

(c) In English, what is Thuraya more often called? State its IAU home constellation. **[0.5]**

> **Solution:**
>
> **The Pleiades [0.25m], located in Taurus [0.25m]**

(d) What is the official IAU name for The Follower? State its IAU home constellation. **[0.5]**

> **Solution:**
>
> **Aldebaran [0.25m], located in Taurus [0.25m]**

(e) With reference to figure 21, suggest why did The Follower obtain its name. **[1]**

Solution:

Not only is it located near the Pleiades, it also rises shortly after the Pleiades. [1m]

Remarks: In practice, the mark is given if teams reference the image in a sensible/logical manner as part of their answer. Tautological answers did not receive marks (Answers along the lines of "The Follower obtained its name because it follows the Pleiades" were surprisingly common)

(f) The Follower was described by the Arab poet Dhu ar-Rumma as a red-turbaned camel herder with his crowd of camels. This crowd of camels is a reference to another famous deep sky object near The Follower. What proper name is associated with this crowd of camels? [1]

Solution:

The Hyades [1m]

(g) Stars undergo different phases as they age. What stage of life is The Follower at, and how does this lead to its observed color? [1]

Solution:

Red Giant [0.5m]

Remarks : Red supergiant is technically incorrect but will also be given full credit for the purposes of this question.

The core of Aldebaran has become depleted of hydrogen [0.5m], causing it to leave the Main Sequence. As it does so, the star's outer layers expand and cool dramatically [0.5m], causing the star to become redder (Wien's Displacement Law).

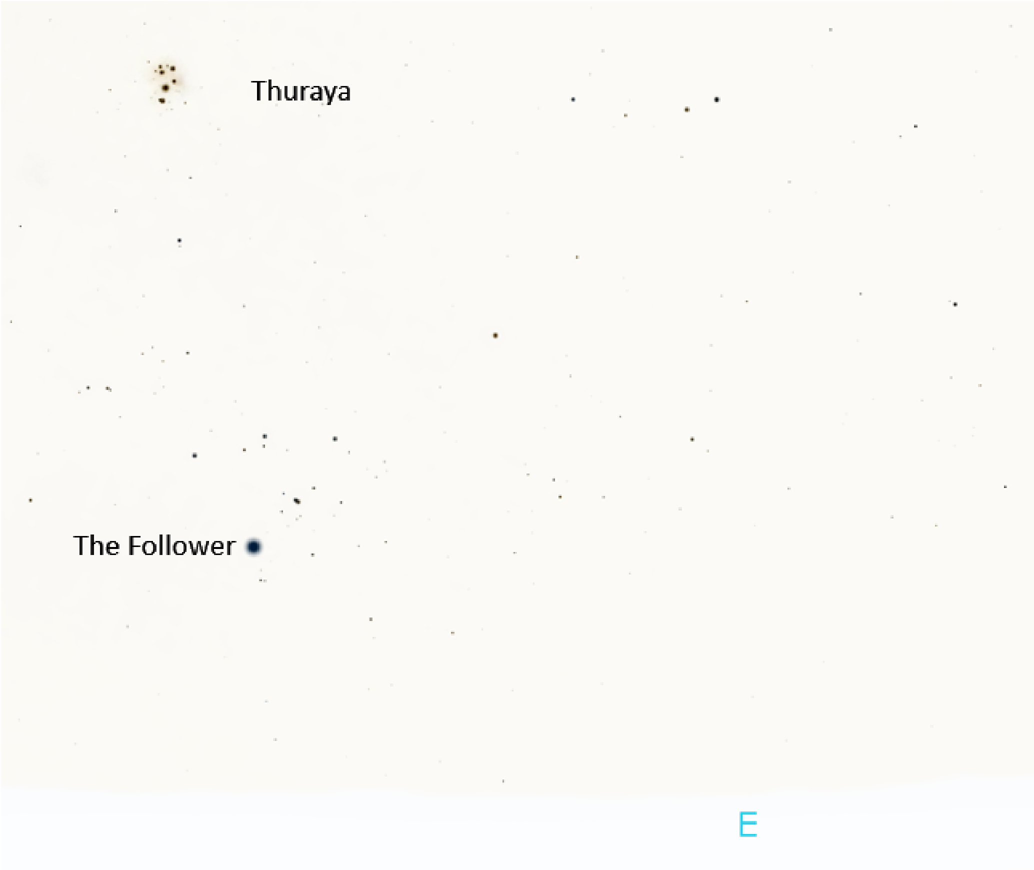


Figure 21: Thuraya and The Follower seen over the Eastern horizon, from the perspective of an observer in Medina, in what is now modern-day Saudia Arabia. The geographical coordinates of Medina are 24deg 28' N, 39deg 36' E.

Part III The Hands of Thuraya

Thuraya is often described as a woman with two hands stretching across the sky. As seen in the image below, these hands are much larger than Thuraya itself, and extend across a vast portion of the night sky.

To Thuraya's south, we have *al-kaf al-jadhma'* (The Amputated Hand). This asterism is short and contains relatively few stars. The Amputated Hand did not survive the adoption of Greek astronomy, and today its key stars lie within the IAU constellation of Cetus, the Sea Monster. However, its Arabic name was transliterated into Kaffaljidmah and assigned to Gamma Ceti. This is how Cetus the Sea Monster ended up with an amputated hand.

To Thuraya's north, we have al-kaf al-khadib (The Henna-Dyed Hand). Four stars in this region bear official IAU names that refer to their position within the Henna-Dyed Hand.

- Atik (from *al-'atiq*, the Shoulder Blade)
- Menkib (from *al-mankib*, the Shoulder)
- Mirfak (from *al-mirfaq*, the Elbow)
- Caph (from *al-kaf al-khadib*)

(h) Today, the Henna-Dyed Hand is largely contained within 2 IAU constellations. What constellations are they? In Greek astronomy, how are these two constellations linked? **[2]**

Solution:

Perseus and Cassiopeia [0.5m each]. Perseus saved Andromeda, the daughter of Cassiopeia [1m]

Remarks: The printed version of this question omitted the key word "largely". For this reason, Answers with Taurus as a response received a quarter mark.

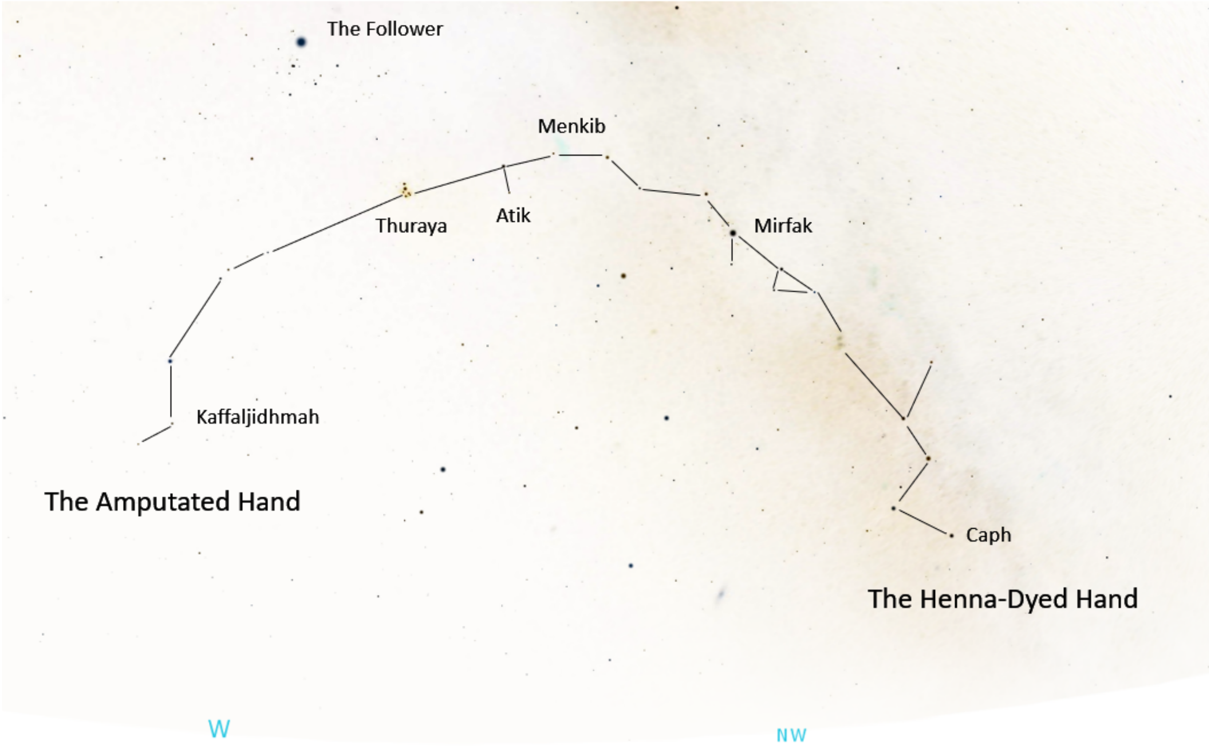


Figure 22: The Hands of Thuraya over the Western horizon, as observed from Medina. Key stars/objects have been labelled accordingly.

Part IV The Scorpion

To Arabic astronomers, the constellation al-'aqrab (The Scorpion) was associated with the intensifying heat of summer. Today, many of its stars have retained Arabic names that reference their positions within this constellation. For instance:

- Zubenelschmali (from *az-zubana ash-shamali*, the Northern Claw)
- Zubenelgenubi (from *az-zubana al-janubi*, the Southern Claw)
- Zubenelhakrabi (from *zubana'l-'aqrab*, the Claw of the Scorpion)
- Acrab (from *al-'aqrab*)
- Alniyat (from *an-niyat*, the Aorta)
- Shaula (from *ash-shawla*, the Raised Tail)

(i) What IAU constellation(s) contain the Scorpion? **[1]**

Solution:

Scorpius and Libra [0.5m each]

The Scorpion does not terminate at a single star. Rather, the Sting of the Scorpion is actually a star cluster (contained within the large circled area in figure 23). In dark skies, this star cluster is easily visible by eye as a large nebulous cloud. The nearby star Lesath is actually a reference to the Sting of the Scorpion (based on *al-latkha*, the Foggy Patch)

(j) Identify this star cluster by its common name **[0.5]**

Solution:

The Ptolemy Cluster [0.5m]

A prominent exception to this Arabic naming trend is *al-qalb*, The Heart of the Scorpion. The Heart of the Scorpion has been circled and highlighted in figure 23. The brightest star of the Scorpion instead retains its traditional Greek name.

(k) State the official IAU name of *al-qalb*. **[0.5]**

Solution:

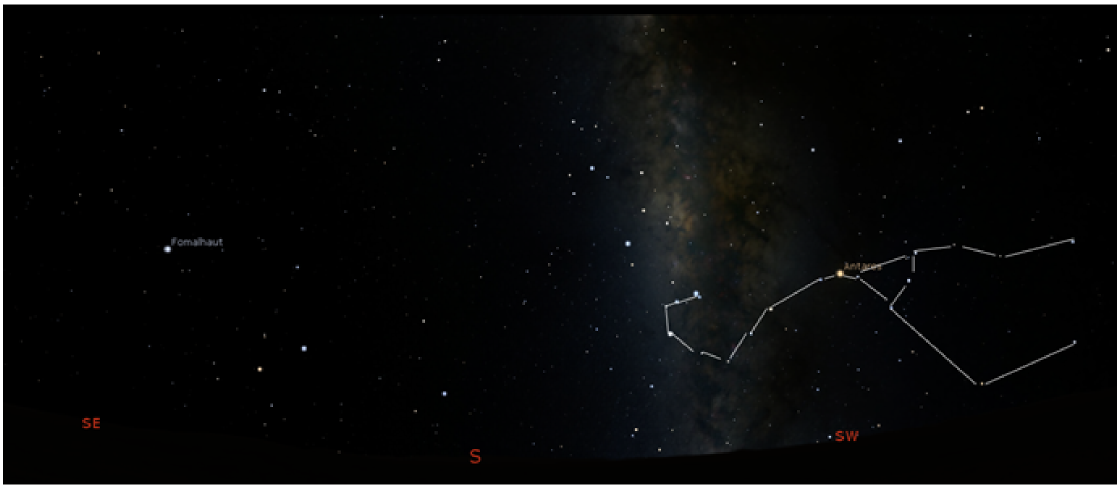
Antares [0.5m]

Traditionally, Arabic astronomers recorded their observations at a specific timeframe (approximately 1 hour before sunrise). After repeated measurements at this timeframe every day, they found that the Scorpion took about 2 months to completely rise and clear the southeastern horizon. However, the entire Scorpion took merely a few weeks to set completely in the southwest.

(l) Make a rough sketch in your answer script of the orientation of the Scorpion before it sets in the southwest. Hence or otherwise, explain why the Scorpion was quick to set and slow to rise. **[3]**

Solution:

Sketch marking points: Approximately correct relative orientation of the stars with respect to each other [1m] Approximately correct orientation of the stars with respect to the horizon: the constellation should appear to pivot around a point significantly below geographic South. [1m]



From an observer around the latitudes of the Middle East, the Scorpion appears almost level to the horizon as it sets (instead of jutting out of the horizon when it rises). Thus, the whole constellation rises slowly but sets rapidly. [1m]

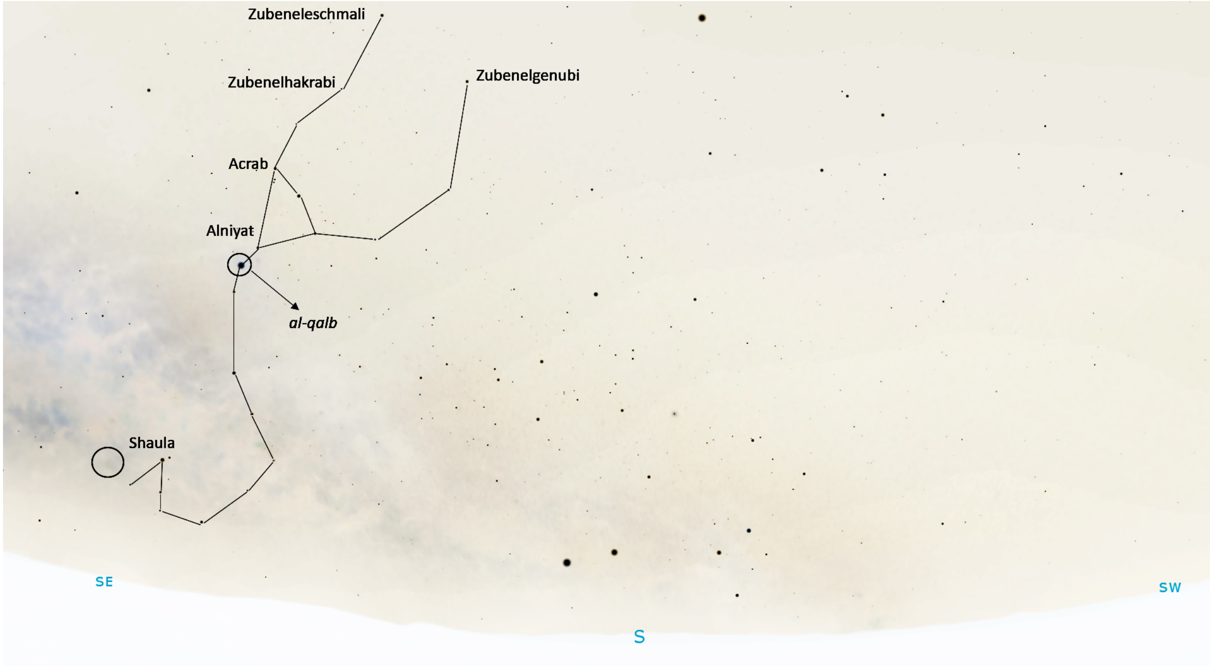


Figure 23: The Scorpion rising in the southeast from the perspective of an observer in Mecca, located in modern-day Saudia Arabia. The geographical coordinates of Mecca are 21° 25' N, 39° 49' E.

Part V The Two Vultures

Another key asterism in Arabic starlore was the Two Vultures. The setting of the Two Vultures in the predawn sky marked the beginnings of the rainy season of fruit harvest.

Each vulture contained an extremely bright star in its center, flanked by two dimmer but easily visible stars. The southern vulture was named the Flying Vulture (an-nasr at-ta'ir), for the three bright stars in a row were seen as a vulture with its outstretched wings in flight. The northern vulture was named the Alighting Vulture (an-nasr al-waqi'), because the stars were arranged in a V-shape, as though the vulture was preparing to land. Today, the central stars of each Vulture bear names that are transliterations of the names for each vulture.

(m) State the modern IAU name given to the central star of the Flying Vulture as well as the IAU constellation it is located in. [1]

> **Solution:**
>
> **Altair, in Aquila [0.5m each]**

(n) State the modern IAU name given to the central star of the Alighting Vulture as well as the IAU constellation it is located in. [1]

> **Solution:**
>
> **Vega, in Lyra [0.5m each]**

(o) A prominent modern-day asterism contains these two stars as part of its components. State this asterism and name the other star(s) that are part of this asterism. [2]

> **Solution:**
>
> **The Summer Triangle [1m]. Deneb is the last star that completes this asterism [1m].**

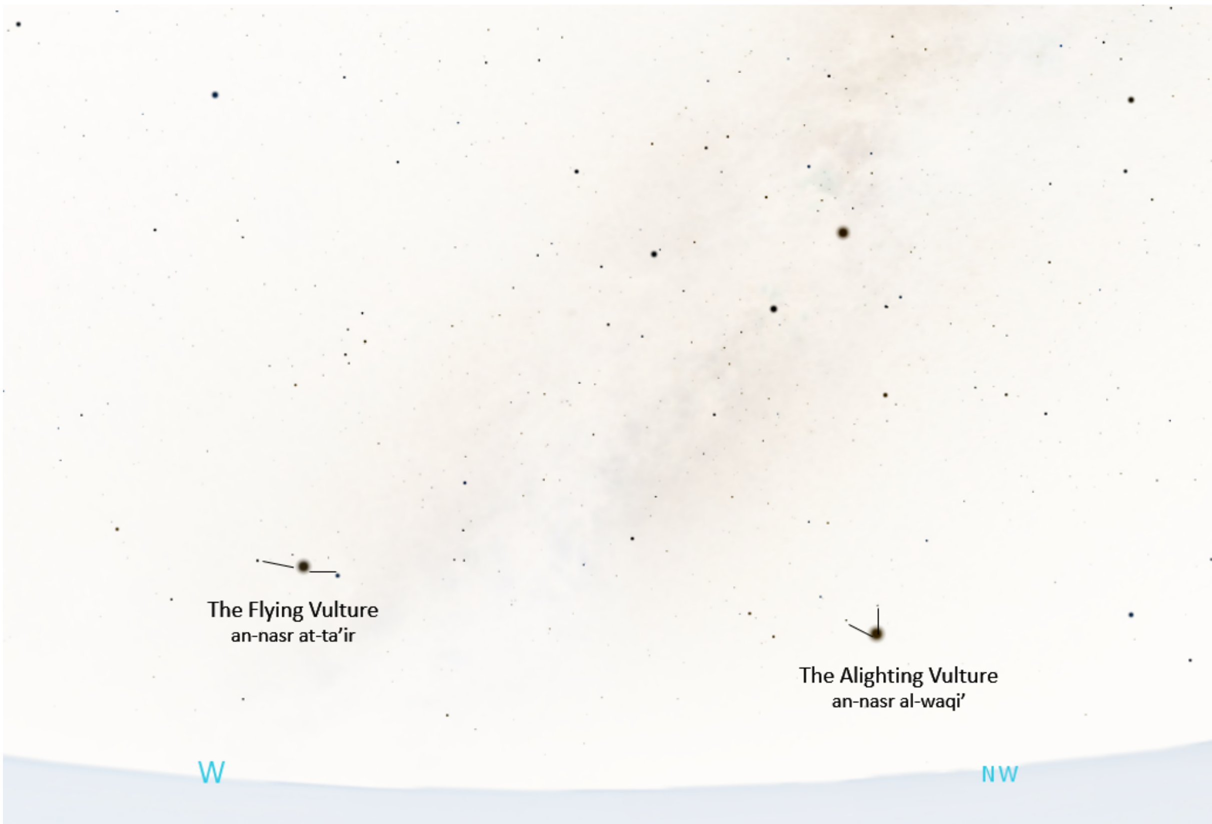


Figure 24: The Two Vultures setting in the northwestern horizon, as seen from Medina.

Part VI Star Chart Analysis

A 1-page sized replica of Figure 23 is attached on the next page. You are to mark your answers on the image directly. Remember to detach the page and staple it to your answer script as part of your submission.

(p) Circle and identify 2 bright stars in the image and state the IAU constellation that they belong to. These stars cannot be members of the Scorpion asterism. [1]

> **Solution:**
>
> **Excluding stars in The Scorpion asterism, the top 3 brightest stars are Rigil Kentaurus/Alpha Centauri (in Centaurus), Hadar/Beta Centauri (in Centaurus), and Spica (in Virgo). Accept other correct answers. [0.5m each]**

(q) Name and trace 1 asterism/constellation within the image (that is not part of the Scorpion). [1]

> **Solution:**
>
> **The Southern Cross. [0.5m for each name/correct trace]**

(r) Name and circle 2 deep sky objects within the image, other than the already circled deep sky object ("The Sting of the Scorpion"). [2]

> **Solution:**
>
> **The easiest remaining objects are probably the False Comet and the Jewel Box. Accept other answers. [1m each]**


Ophiuchus
Spica
Libra
Corvus
Crater
Blue Horsehead Nebula
Dschubba
Crab Globular Cluster
Dark Horse Nebula
M 23
Menkent
Webb's Cross
Herschel 36
Larawag
Scorpius
Lupus
η Cen
NGC 6124
Moth Wing Cluster
ω Cen Cluster
Centaurus
Antlia
Baade's Window
False Comet Nebula
Shaula
Ptolemy's Cluster
NGC 6322
NGC 6250
Norma
Gypsy Moth Cluster
Gacrux
Hadar
Rigil Kentaurus
NGC 5316
Little Scorpion Cluster
Mimosa
Crux
Queen's Cache Cluster
S Nor Cluster
NGC 6025
Circinus
Wishing Well Cluster
Pearl Cluster
Vela
SE
S
SW

A Works Cited

Question 4:

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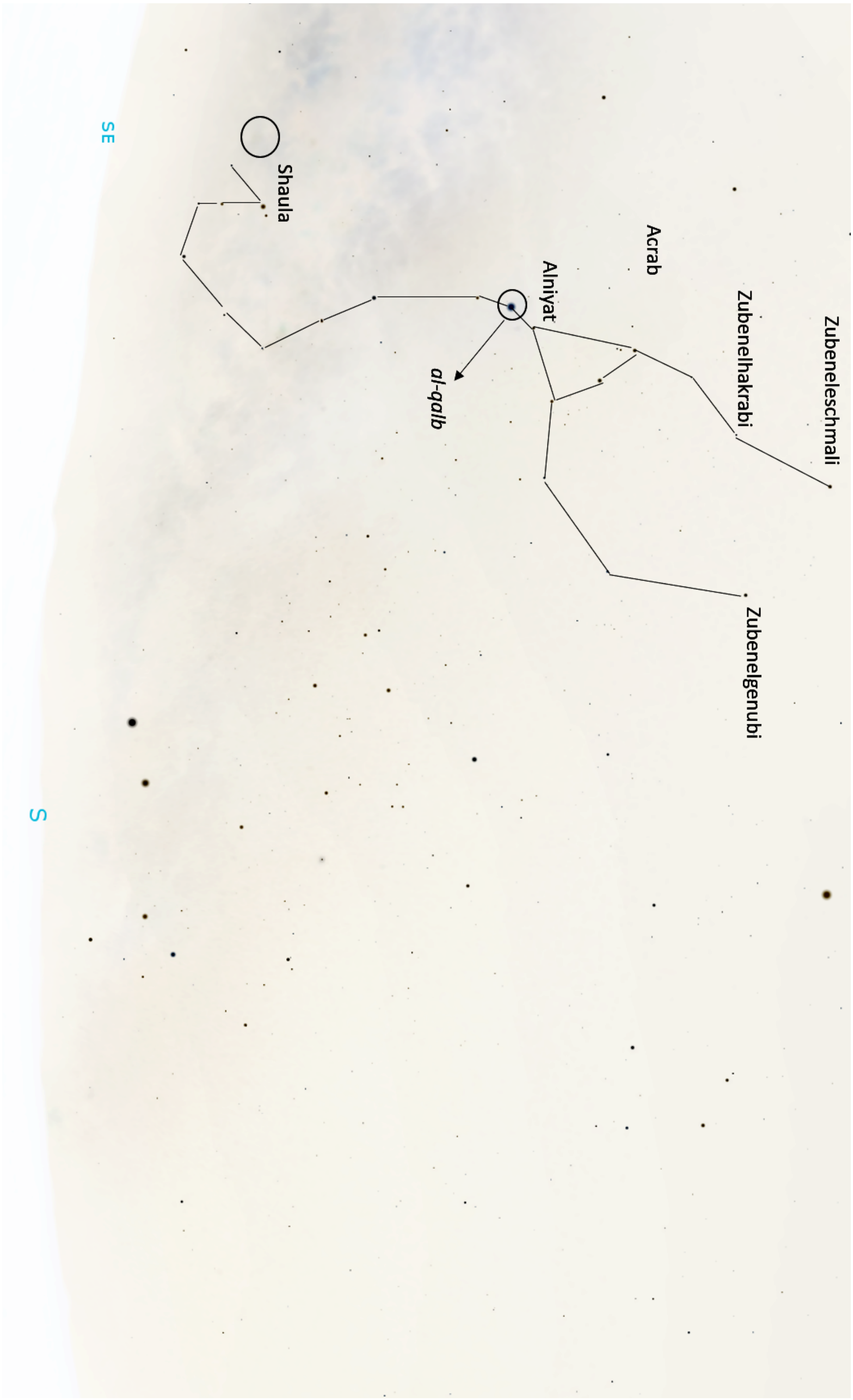
Question 5:

A key reference used in the setting of this question is https://onesky.arizona.edu/, which contains details about how these stars were used to keep track of time.


SE
S
Shaula
Acrab
Alniyat
al-qalb
Zubenelhakrabi
Zubeneleschmali
Zubenelgenubi

[End of paper]