AstroChallenge 2017
Data Response Questions

(JUNIOR)

PLEASE READ THESE INSTRUCTIONS CAREFULLY

1. This paper consists of **12** printed pages, excluding this cover page.
2. Do **NOT** turn over this page until instructed to do so.
3. You have 2 hours to attempt all questions in this paper.
4. At the end of the paper, submit this booklet together with your answer script.
5. Your answer script should clearly indicate your school (and team number) on **EVERY** page, as well as the individuals in the said team on the first page.
6. It is your team's responsibility to ensure that all pages of your answer script have been submitted, including pages to be detached from this booklet.

DRQ 1: Cave in the Tundra [20 marks]

One of the best blackbodies known in physics is a black hole, emitting a form of blackbody radiation known as Hawking radiation. Over time, if no incident matter or energy is absorbed by the black hole, this radiative emission results in the black hole losing mass in a process known as evaporation.

Therefore, a black hole can be treated as a blackbody of temperature T, where T is given by

$$T = \frac{\hbar c^3}{8\pi \mathrm{kGM}}$$

Where
ħ is the reduced Planck constant ħ = h/2π
k is the Boltzmann constant
G is the Universal Gravitational Constant
and M is the mass of the black hole

i) Find the blackbody temperature of Sagittarius A*, the supermassive black hole at the centre of the Milky Way Galaxy and comment on your result. Sagittarius A* has a mass of $4 \times 10^6\ M_\odot$. [2 marks]

The escape velocity from an object of mass M is given by

$$v_{escape} = (2GM/R)^{0.5}$$

ii) Using the equation for escape velocity, obtain an equation relating the mass of a black hole to the radius of its event horizon. [2 marks]

iii) Using the Stefan-Boltzmann law, the result from (ii) and the equation for black hole temperature above, derive an equation relating the total power emitted from Hawking radiation to the radius of a black hole. Hence, comment on the relationship between the total emissive power from Hawking radiation and the radius of the black hole event horizon. [3 marks]

iv) Based on your result in question (iii), explain why black holes are thought to radiate in a "flash" at the end of the evaporation process. [1 mark]

v) Find the solar radiative flux at a distance of 1AU [1 mark]

Given that at 1 AU, there is a solar radiative flux of 1.4 kW/m^2.

vi) Estimate the total amount of solar radiation absorbed per second by a black hole with an event horizon of radius R, treating it as a solid blackbody sphere of **1.5R (photon sphere radius)** located at a distance of 1AU from the sun. [1 mark]

In equilibrium, the incident radiative energy will be equal to the radiant energy from the black hole.

vii) Consider a black hole located at a distance of 1AU from the Sun. Find the equilibrium radius for such a black hole. [3 marks]

viii) Calculate the equivalent blackbody temperature of this black hole [1 mark]

ix) Comment about the significance of this equilibrium radius in terms of how the mass of a black hole changes over time [1 mark].

Consider instead a black hole located deep in intergalactic space, with no significant sources of radiation aside from the Cosmic Microwave Background Radiation (CMBR).

x) Given that the temperature of the CMBR is 2.725K, find the equilibrium radius and mass for such a black hole. [3 marks]

xi) Explain the significance of this result on any black hole with a radius larger than a black hole with Hawking radiation in equilibrium with the CMBR. [2 marks]

Life Around a Chubby Sun [20+1 marks]

Suppose that a group of amateur astronomers has discovered a new star system that is similar to our Solar System (consists of a main-sequence star and several planets). They used a telescope that has a limiting magnitude of $+10$. From careful analysis of the data obtained, it is known that the star has a mass of $20\ M_{\odot}$.

i) The distance of the star from Earth is roughly 1000 pc. Determine the visual magnitude, m_V, of the newly discovered star, if the luminosity of the star is $100L_{\odot}$. [1 mark]

ii) This star, like all other main-sequence star, produces energy in its core via hydrogen-burning process or also known as the proton-proton fusion chain reaction. In this reaction, 4 protons are fused to produce one He atom. If only 10% of the mass is available for burning, determine the mainf sequence lifetime of this star **in years**. You may assume that the star is made up of hydrogen only where applicable. [3 marks]

iii) A boy who is eager to learn astronomy knows about the discovery of this star. He wants to observe it but using a weaker telescope. The diameter of his telescope is one-half of that of the telescope used by the astronomers. Is he able to observe it? Hint: calculate and compare the limiting magnitudes of the two telescopes. [2 marks]

iv) A fast-rotating planet of radius $R_{\oplus}$ is discovered to be revolving around this star with a circular orbit at distance d = 1.523 AU. The surface albedo of this planet is 0.25. Determine the average blackbody temperature at the surface of the planet. [5 marks]

v) Judging from the surface temperature only, is it possible for humans to live on that planet? If not, propose the best temperature range and the suitable distance between the star and planet so that humans are able to live on it. [5 marks]

vi) This star, after ending its lifetime in the main-sequence stage, will eventually move up along the giant branch, during which its temperature drops by a factor of 3 and its radius increases 100-fold. Determine its new visual magnitude. [4 marks]

Bonus: State what's unrealistic with the scenario presented in this question. [1 mark]

DRQ 3: Space Telescope [20 marks]

The Hubble Space Telescope, which has been in service since 1990, is going to be succeeded by a new telescope, the James Webb Space Telescope (JWST). We will investigate the properties of the space telescope.

Part I: The Cosmological redshift [5 marks]

Due to the nature of the universe, very distant objects are subject to redshift. As a result, most features that would usually be observed in the visible spectrum will become redshifted. For a homogeneous and isotropic universe,

$$1+z = a_{now}/a_{then}$$

Where a is the cosmic scale factor, and z is redshift. A_{now} is assumed to be 1.

The James Webb Space Telescope aims to observe distant objects, and as such, will be able to analyze the redshifted spectra more readily. It has a detection range from 600 to 28,500 nm. Notice that this only covers a small portion of the visible spectrum.

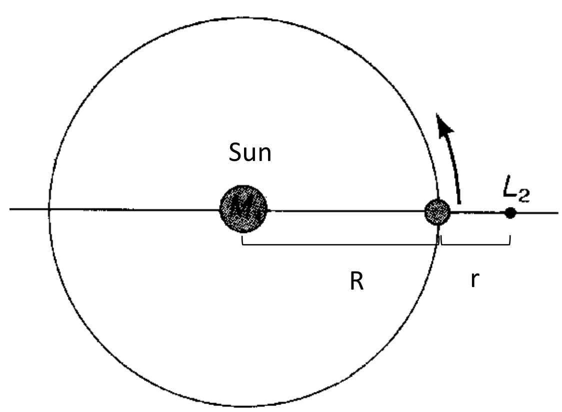
i) For the following lines, which ones are visible for a scale factor of 0.12? [3 marks]

Line	Hα	Hβ	Lα	Lβ	Ne II	He II	H_2O
Wavelength (nm)	656.3	486.1	121.6	102.6	44.8	30.4	19230

ii) The most prominent feature of extremely distant galaxies is their Lyman lines. What is the maximum redshift that can be analyzed? [2 marks]

Part II: Orbit [5 marks]
[Assume a spherical orbit, and disregard the moon in the following questions]

Due to the combined gravitational pull of both the sun and the earth, the James Webb Space Telescope orbits the sun under what is known as a "halo orbit" in the L2 Lagrange point. In this orbit, the relative position of the sun, earth and satellite is always fixed (refer to figure below):



iii) Derive an expression involving only m_{earth} , M_{sun} , R , and r. Confirm this for $r = 1.5\times10^9\ m$.
[4 marks]

iv) Compute the ratio of r to the earth-moon distance and comment on the feasibility of conducting repairs on the JWST. [1 mark]

Part III: Orbital Redshift [5 marks]

In addition to the galaxy's expansion, the JWST's orbital motion around the sun leads to received electromagnetic waves undergoing a further Doppler shift as given by

$$z = \sqrt{\frac{c+v}{c-v}} - 1$$

v) Calculate the JWST's orbital velocity in the L2 point. [1 mark]

vi) Calculate the resulting Doppler shift, z. By how much, in nm, is the H-alpha line shifted due to this? [2 marks]

vii) Compare your value of z with that obtained in part I, and comment on your result.
[2 marks]

Part IV: Optics [5 marks]

Now, we analyze the mirror of the JWST. It is larger than the one in HST. Once again, the nature of the mirror necessitates the use of multiple segments as opposed to one large segment. As shown in the diagram, the telescope consists of 18 hexagonal segments, each 1.32m in diameter (flat edge to flat edge).

viii) Derive the total area of the mirror. What would be the equivalent diameter for a standard circular mirror? [2 marks]

ix) Compared to the Hubble Space Telescope (D=2.4m), how much more collecting power does the telescope have? [1 mark]

x) At what wavelength will the resolving power of the JWST match the one of the HST at visible light? Assume that is the JWST mirror is approximately circular. [2 marks]

DRQ 4: Earth in another Turf [20 marks]

Part I: Binary Star Systems [10 marks]

According to stellar surveys, more than half of all Sun-like stars are part of multiple star systems. This means that in considering the search for extra-terrestrial life, there is a significant chance that life may evolve on planetary systems in such multiple star-systems. In the first part of this question, we analyse a binary star system to understand how a planet orbiting in such a configuration will experience temperature variations.

	Star A	Star B
Mass (M)	0.9 $M_{\odot}$	1.1 $M_{\odot}$
Radius (Solar Radii)	0.83 $R_{\odot}$	2.1 $R_{\odot}$

Distance between star A and star B = 12 AU

i) Estimate the star surface temperatures [3 marks]

The stars A and B are separated by a distance of 12 AU. A planet with characteristics identical to the Earth orbits a point X in a circular orbit with a radius of 2 AU (see figure 1).

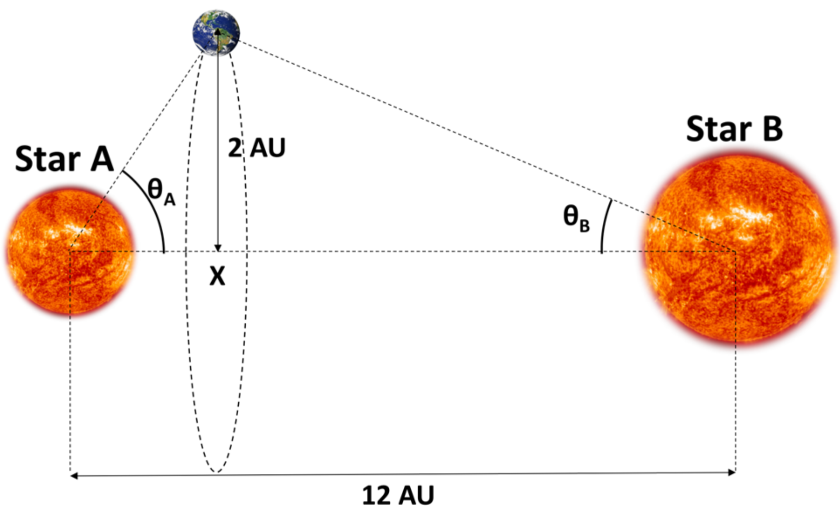


Figure 1 *Diagram of the A-B binary system with the Earthlike planet orbiting Star A. This diagram is not drawn to scale.*

The gravitational field strength around by a star is given as

$$g_{star} = GM_{star}/r^2$$

Where

G is Newton's gravitational constant (see formula booklet)

M_{star} is the mass of the star

And r is the distance from the centre of the star.

At point X, which lies between stars A and B, the gravitational fields of both stars cancel each other out completely (i.e. the magnitude of the gravitational field strength of star A and Star B are equal).

ii) Using the equation above or otherwise, calculate the distances from the centres of stars A and B to point X respectively [2 marks].

iii) Is such an orbit stable? Explain. [2 marks]

The average surface temperature of a planet orbiting a single star can be estimated to be

$$[I_0/(16\pi d^2)](1-\alpha) = \sigma T_p^4$$

Where I_0 is the star's luminosity constant (3.827×10^{26} J/s for the Sun);
T_p is the average temperature of the planet;
α is the planetary albedo, which has a value of 0.3 for the Earth;
d is the distance from the star;
and σ is the Stefan-Boltzmann constant, which has a value of $5.67 \times 10^{-8} W/(m^2 \cdot K^4)$.

iv) Modify the above equation for the case of a binary star system. [1 mark]
(Hint: Linear addition can be performed on the left hand side of the equation)

v) Find the steady state temperature of the planet as described in figure 1. [2 marks]

Part II: Tidal Locks and Love Numbers [10 marks]

When searching for potentially habitable worlds, there is generally both a lower and upper limit for stellar masses for parent stars. The upper limit of stellar masses exists due to the fact that stars of higher mass tend to exhaust their nuclear fuel at much greater rates, dying at timescales too short for life to evolve.

On the other hand, there is also a lower limit of stellar masses which arises from a less intuitive phenomenon. As the mass of a star decreases, its luminosity similarly decreases more than proportionately. This means that planets orbiting lower mass stars need to orbit at radii much closer to their host stars. At a certain point, the planet would become tidally locked to the star, with one side being plunged in constant daylight and another that never sees the light.

Such planets with incredibly asymmetric temperature profiles could therefore impact the habitability of life even in supposed habitable zones. Therefore, it would be of particular interest to get a ballpark estimate of the lower limit for such stars. This can be done by making several reasonable assumptions, and applying the equation for tidal locking and stellar luminosity.

The time for a planet to become tidally locked to its parent star t_{lock} can be estimated using the equation (in SI units) as follows:

$$t_{lock} = (\omega a^6 IQ)/(3GM_{star}^2 k_2 R^5)$$

where ω is the initial spin rate in radians per second
a is the semi-major axis of the motion of the planet around the star
R is the radius of the planet
$I = 0.4M_{planet}R^2$ is the moment of inertia of the planet
G is the gravitational constant
M_{star} is the mass of the star
k_2 is the Love number of the satellite, which measures a body's rigidity
Q is the dissipation function of the satellite

In practice, Q and k_2 are not easily determined values. In this case, for our estimation we consider the case where $k_2/Q = 0.0011$ which is equal to the known value for the Moon

Assuming an Earth-like planet in a circular orbit, we will calculate the smallest mass of star in which the planet can orbit in the habitable zone without becoming tidally locked within 1 billion years.

vi) Show that for this planet, $a^3/M_{star} = 101.52$ [3 marks]

The inner and outer radii of the habitable zone of a star can be approximated by the following equations:

$$r_{inner} = (L_{star}/1.1L_{sun})^{0.5}$$
$$r_{outer} = (L_{star}/0.53L_{sun})^{0.5}$$

Where r is measured in Astronomical Units

vii) Using the above information, provide an expression for a (in appropriate units), the semis major axis of the planet for our calculations, in terms of L_{star} and L_{sun}. Briefly explain your answer in qualitative terms [2 marks].

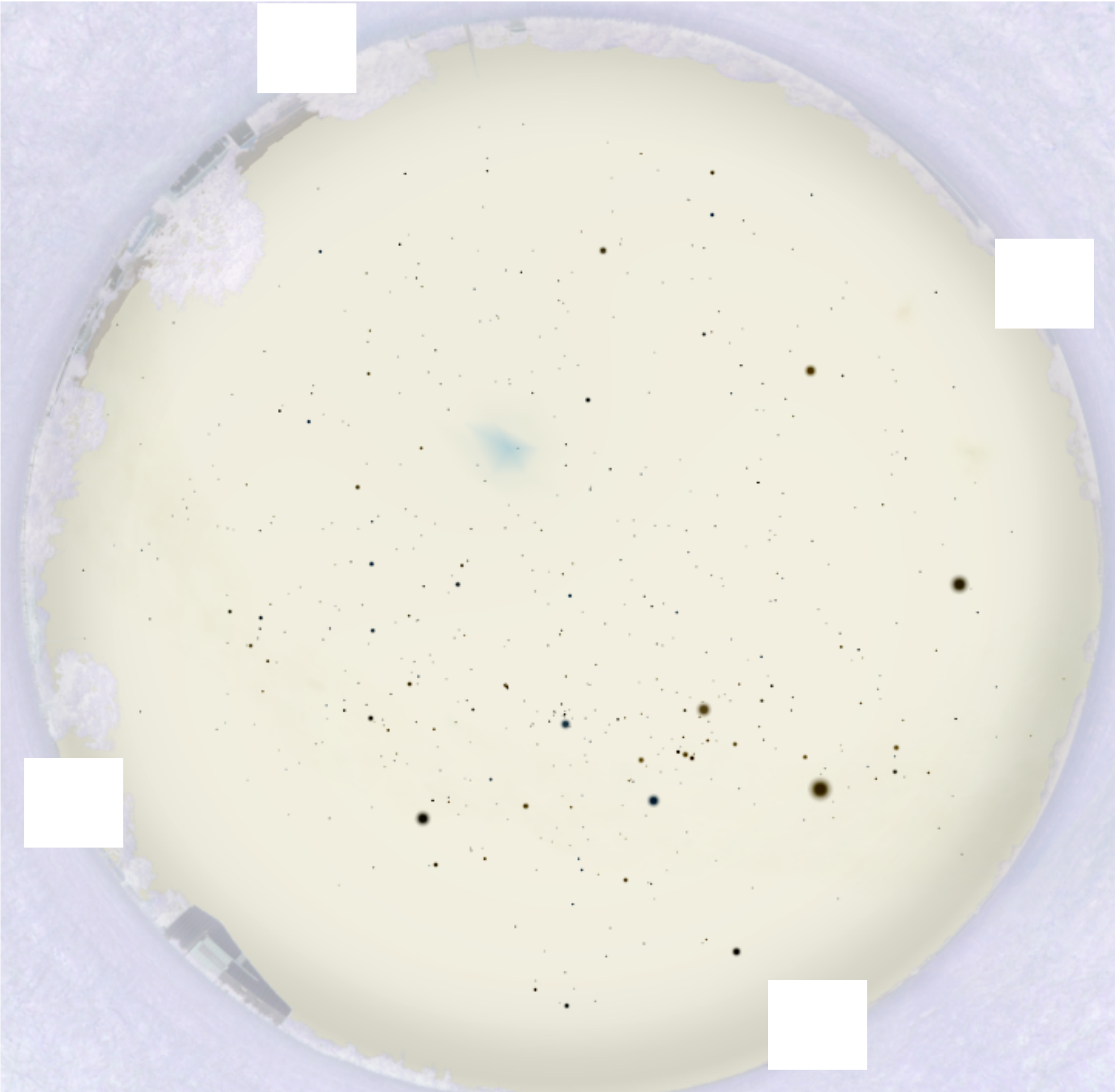
viii) Using the answers obtained in Questions (vi) and (vii), find the minimum stellar mass for an Earthlike planet to orbit around it without being tidally locked for at least 1 billion years. [3 marks]

TRAPPIST-1 is an ultra-cool dwarf star with 7 planets orbiting around it, of which five are approximately Earth-sized and 3 of which lie within the habitable zone. TRAPPIST-1 has a mass of 0.0802±0.0073 solar masses.

ix) Based on your result in question (viii), explain qualitatively if there is any insight it provides on the habitability of these planets. [2 marks]

DRQ 5: Practical Astronomy [20 marks]

Part I: The Night Sky [10 marks]



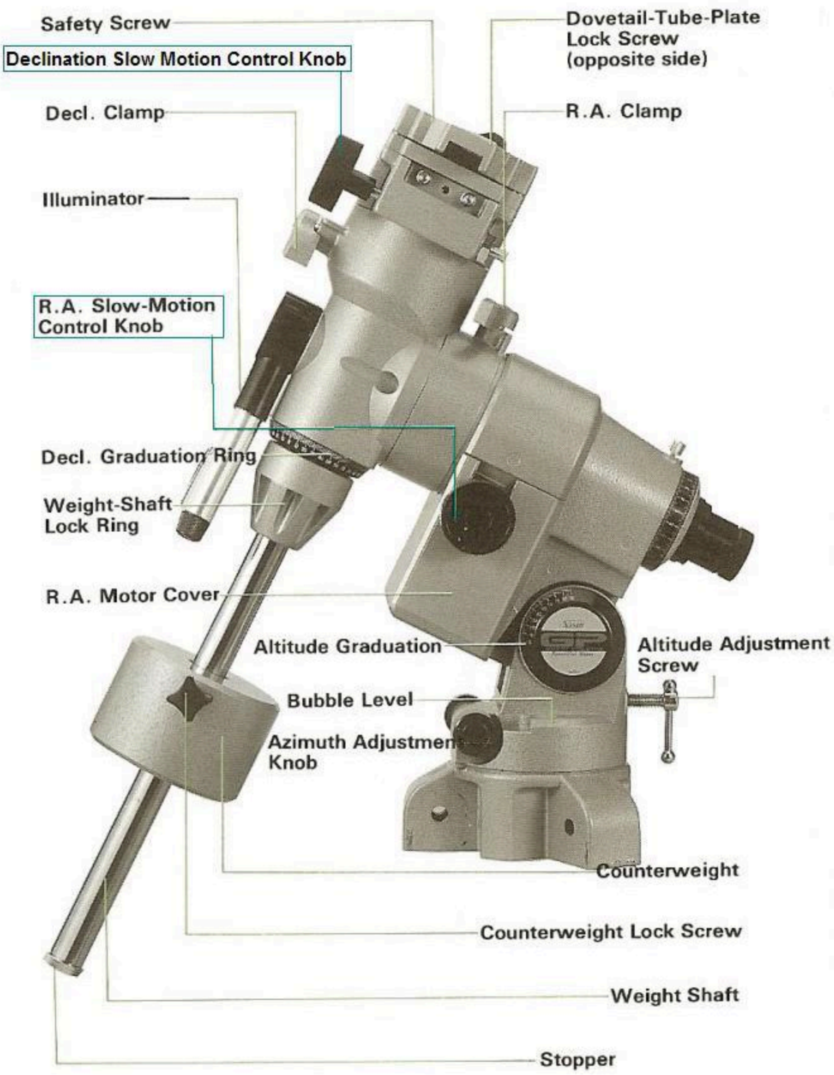
Identify the following on the diagram given:

i) Circle Bellatrix (γ Ori). [1 mark]
ii) Circle Achernar (α Eri). [1 mark]
iii) Trace out the 'Winter Triangle' and label it. [1 mark]
iv) Trace out the 'Great Square of Pegasus'. [1 mark]
v) Trace out the constellation lines of 'Auriga'. [1 mark]
vi) List down 2 prominent galaxies and mark their approximate positions on the diagram. [2 marks]
vii) List down 1 prominent nebula aside from M42 and mark their positions on the diagram. [1 marks]
viii) List down 2 prominent open star clusters and mark/circle their approximate positions on the diagram. [2 marks]

Part II: Setting up a German Equatorial Mount [10 marks]

You want to set up an equatorial mount to observe the night sky in Singapore. A detailed procedure of the setup process is written on the following page. Identify as many errors as possible, and explain how it should be corrected. Write your corrections on the blank space provided on the right side of the page.

A photograph of a standard equatorial mount is shown below for reference, with the parts labelled.
(Note: There are no errors in the photograph. It is for your reference only)



Sample format of answers

Procedure	Your answers here
1. Install the tripod on top of the mount head and secure it using the locking screws	*The mount head should be installed on the top of the tripod, not the other way round*

Note: You will be awarded 0.5 marks for every error found, and 1 mark for a correct explanation for a total of 1.5 marks per error

Procedure to Set up a German Equatorial Mount

Initial Steps **Your answers here**

1. Transport the mount to the observation site. Ensure that both the RA and DEC clamps are locked and engaged so that the axes do not freely rotate when transported.

2. To begin the setup process, spread the tripod legs and place the tripod on flat ground. If the ground is soft or muddy, drive the tripod legs as deep into the ground as possible to ensure stability. Orient the tripod roughly in such a position that when the mount head is installed, the front of the mount (where the counterweight bar protrudes) will point due south.

3. Install the equatorial mount head onto the top of the tripod, and tighten the main central screw, leaving a small amount of slack so that the azimuth axis can be adjusted for more precise polar alignment.

4. Using a compass, ensure that the mount head is roughly polar aligned with the mount head pointing due south. Level the mount using a bubble level, and use the altitude adjustment screw to ensure that the latitude reading corresponds correctly to your geographic location.

Installing the telescope and counterweights **Your answers here**

5. In steps 6 and 7, ensure that the mount's axes are unlocked to ensure smooth movement.

6. Install the telescope tube by sliding the telescope's dovetail into the mount's dovetail saddle. Tighten the locking screws on the saddle to hold the telescope in place.

7. Install the counterweight shaft onto the equatorial mount. Slide the counterweights into the counterweight shaft and tighten the counterweight's locking screw to hold it in place. Install the stopper to prevent the counterweights from falling off the shaft.

Balancing the setup **Your answers here**

8. Balance the Right ascension axis: Rotate the RA axis such that the counterweight shaft is parallel to the ground. Lock the RA axis and leave the DEC axis unlocked to determine the balance point. Shift the counterweights along the counterweight shaft until the RA axis is balanced (i.e. no tendency to rotate in any direction around the RA axis).

9. Balancing the Declination axis: With the counterweight shaft still parallel to the ground, Lock the RA axis and leave the DEC axis unlocked to determine the balance point. Shift the telescope tube forwards or backwards to balance the DEC axis. When there is no tendency for the DEC axis to rotate, the mount is sufficiently balanced.

10. As far as possible, do not install all the accessories (diagonals, eyepieces and finderscopes) until the balancing is complete to ensure that the balance of the system is not disrupted.