ASTROCHALLENGE 2025
SENIOR OBSERVATION ROUND

SOLUTIONS

Wednesday 4th June 2025

School and Team Number	
Team Member 1	
Team Member 2	
Team Member 3	
Team Member 4	
Team Member 5	

References

[1] Fiona Vincent. *Positional Astronomy Fiona Vincent APM Rev 3.2*. Copyright Fiona Vincent 1998. Revised in 2003, PDF by Alfonso Pastor (2005), revised 2015. Cited page: p. 29. 1998. URL: http://archive.org/details/PositionalAstronomyFionaVincentAPMRev32 (visited on 12/05/2025).

[2] Harmonic Drive SE. *Harmonic Drive® Triangle Technology*. Product brochure. 2024. URL: https://harmonicdrive.de/fileadmin/Downloads/Produkte/Produktbroschueren/Harmonic_Drive_Triangle_Technology_EN_1064631.pdf.

[3] Jahobr. *Harmonic-Drive Animation (strain wave gear (SWG))*. Created with MATLAB. Licensed under Creative Commons CC0 1.0 Universal Public Domain Dedication. Accessed May 12, 2025. 5th Dec. 2016. URL: https://commons.wikimedia.org/wiki/File:HarmonicDriveAni.gif.

[4] Bartłomiej Okone. *Satellite image captured using ZWO ASI178MM and Celestron C9.25*. Single frame with curve adjustment and contrast. Image taken in Wrocław, Poland. Accessed May 12, 2025. 2022. URL: https://www.astrofoto.pl/satellites/.

Question 1 Equipment Mayhem

Part I Camera

(a) Given that the camera at the end of the image train is a (hypothetical) sensor with size of 5.5 mm by 3.7 mm (width × height), what focal length would allow the International Space Station (ISS) to fill approximately one-ninth of the frame in the orientation shown in Figure 1? Assume that imaging will take place at sea level with the ISS directly overhead. **[4]**

It is known that the ISS is roughly 109m long, 73m wide, and orbits at an altitude of 400km.

Hint: For a given sensor size of H, the apparent field of view θ for a lens of fixed focal length f is given by

$$\theta = 2\arctan\left(\frac{H}{2f}\right)$$

Solution:

Using small angle approximation, we get

$$\theta \approx \frac{H}{f}$$

Equivalent size of the image on the sensor: $1.84 \times 1.23\,\text{mm}$

Focal length by width: $\frac{400000}{109} \times 1.84 = 6752\,\text{mm}$

Focal length by height: $\frac{400000}{73} \times 1.23 = 6739\,\text{mm}$

Acceptable range: (6700 ± 100)mm

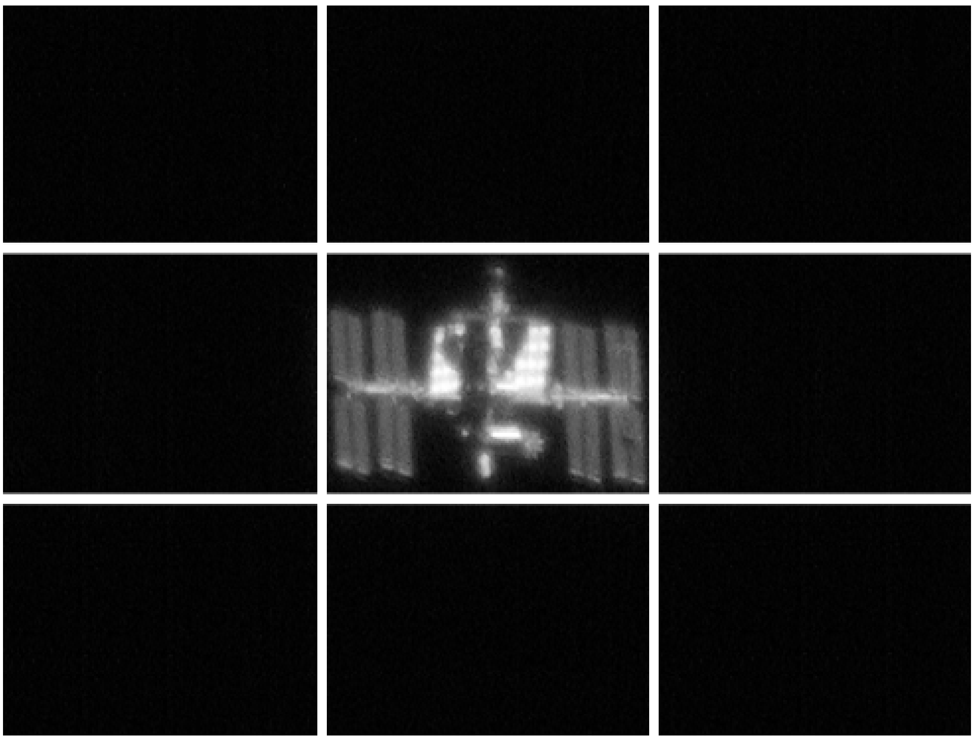


Figure 1: The ISS as seen in the camera frame [4]

Part II Computerised mounts

Computerised mounts require high resolution to be able to slew and track objects accurately. A rotary encoder is used to detect the position, speed and direction of the motor shaft, in which the resolution is dependent on the encoder bits. A 4-bit encoder can measure $2^4 = 16$ positions per revolution, while an 8-bit encoder can measure $2^8 = 256$ positions per revolution.

Equatorial mounts are often physically incapable of rotating around the right ascension axis past 90° in either direction, hence allowing us to divide the limited number of positions around just half of a rotation. Even then, its resolution is nowhere near enough to accurately track the night sky.

We wish to have a resolution of $0.1''$.

(b) Find the number of positions that the bit encoder needs to account for, if we wish the motor is coupled directly to the right ascension (RA) axis. **[2]**

Solution:

$$0.1'' = \frac{1}{3600} \times 0.1 = 2.78 \times 10^{-5\circ}$$

$$\frac{360}{2.78 \times 10^{-5}} = 1.295 \times 10^{7} \text{positions}$$

This is impractical in real life. We would need a 24-bits mechanical encoder, which is extremely costly, if not impossible, to manufacture.

Instead of a direct drive, where the rotation of the motor is coupled to the RA axis in a 1:1 ratio, we can use a high gear ratio, such as 250:1, meaning that the input gear must complete 250 revolutions to turn the output gear for one complete revolution.

We will also adopt bit overflow. This means that we will have an electronic memory to store how many revolutions the gear has completed, while our mechanical encoder tracks the position of the gear in each rotation.

(c) Suppose that we have a maximum possible gear ratio of 1000:1, to achieve the (output) resolution as specified in (b), how many positions, minimally, does our mechanical encoder need to track? **[1]**

Solution:

We want to use the maximum possible gear ratio to track the least number of positions possible.

$$\frac{12.95 \times 10^6}{1000} = 1.295 \times 10^4 \text{ positions}$$

(d) Determine the minimum number of bits the mechanical encoder needs in tracking the input gear, to achieve the (output) resolution as specified in (b) with the maximum gear ratio as specified in (c). **[1]**

Solution:

$$\log_2(1.295 \times 10^4) = 14 \text{ bits}$$

Note that this answer is rounded up since 13 bits would only be able to track $2^{13} = 8192$ positions, which is less than that needed from part (c).

Part III Imaging the ISS

(e) A friend sees your photos of the ISS on Instagram and is impressed. He wants to capture his own image of the ISS but has no experience in the hobby, apart from some basic understanding of photography. Suggest an alternative method for him to capture the ISS that are more approachable if he does not have a telescope. **[1]**

Solution:

- Wide-field long exposure nightscape to capture a streak of light as the ISS travels across the sky.
- Piggyback his camera on your telescope and mount.
- Lunar transit. Plan a moment when the ISS will transit the moon to take a silhouette of the ISS.
- Hand-held tracking with a telephoto lens.

Part IV Harmonic reducer

The key components of harmonic mounts are harmonic reducers. A harmonic reducer consists of three main components, the circular spline, the flex spline and the wave generator, as shown in Figure 2.

The wave generator moves two (or more) equidistant points of the flexible spline, which in turn engages the teeth at these points with the circular spline. The flex spline changes its shape as the wave generator rotates. The wave generator is coupled to the motor shaft (driver), and the circular spline is the output (driven).

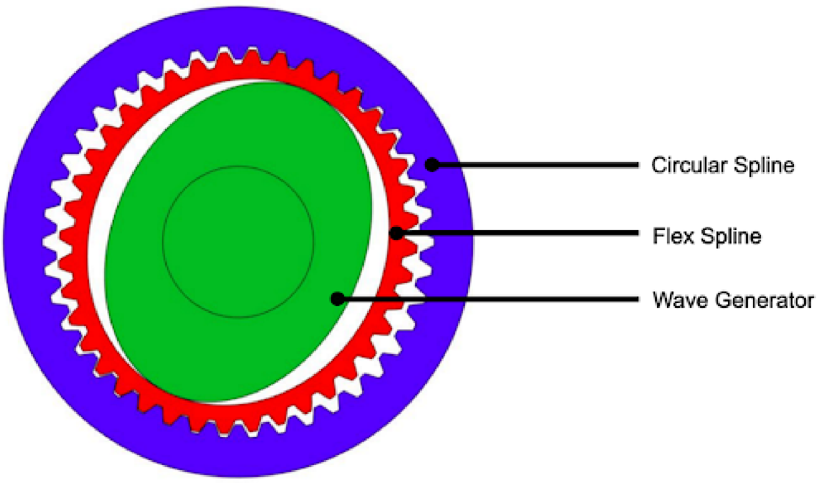


Figure 2: Overall diagram of a harmonic reducer [3]

Figure 3 shows how the flex spline advances by 1 tooth for every 1/3 of a rotation of the wave generator.

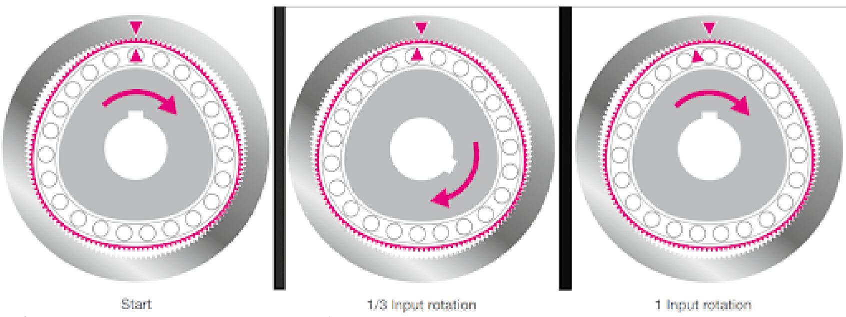


Figure 3: Mechanism of the flex spline [2]

Please note that although the relative position of the teeths in the above diagram are labelled correctly, in reality, the flex spline only deforms and not rotates. That is, the circular spline would have rotated clockwise by some angle after one input rotation.

(f) To achieve the gear ratio of 250:1, how many teeth are required on the circular spline and flex spline respectively? Is there a detail that needs to be specified to come up with your answer?

Hint: Notice how many contact points between the circular and flex splines in Figures 2 and 3. **[3]**

Solution:

Decide the number of engagement points, 2, 3 or 4 (no more than 4). The number of teeth on the flex spline, n_F must have 2, 3 or 4 fewer teeth than on the circular spline, n_C. For example, if we decide that the number of engagement points is 3, then we have $n_C = 750$ and $n_F = 750 - 3 = 747$ such that

$$\text{Gear ratio} = \frac{n_C}{n_C - n_F}$$

$$250 = \frac{750}{3}$$

$$250 = \frac{750}{750 - 747}$$

Part V GEM vs CEM

Aiden purchased a German Equatorial Mount (GEM) in 2022, just a few months before a reputable company released a reliable harmonic mount that is lauded to be better than any GEMs or Center-mounted Equatorial Mounts (CEM). Buyer's remorse has been tempting him to replace his current GEM to a harmonic mount. Help him compare and understand the differences between the two types of mounts.

(g) Which type has a higher degree of backlash due to the mechanical play between the gears? Explain why this matters in astrophotography. **[2]**

Solution:

GEMs. Backlash refers to the slack between gears. When changing directions, this leads to inaccuracies in tracking, especially when small corrections are made when autoguiding.

(h) Assuming a perfect polar alignment, which type of mount would benefit more from a guiding scope for long exposure astrophotography? Why is this the case? **[2]**

Solution:

Harmonic mounts. They have larger periodic errors with shorter intervals due to its reliance on several layers of gears between the motor and the mount itself.

(j) Which type of mount tend to have a better payload capacity, assuming the mount themselves have the same weight? Why? **[2]**

Solution:

Harmonic mounts have a higher payload to mount weight ratio. The worm gear of a typical GEM has less torque capabilities than the strain-wave gear of harmonic mounts.

(k) Which of the two designs have more affordable options? Suggest why that might be the case apart from the fact that harmonic mounts are developed more recently. **[2]**

Solution:

GEMs have less complex designs and tend to have a lesser number of components that make it simpler to manufacture.

Question 2 Geo Guesser

Part I Sextants

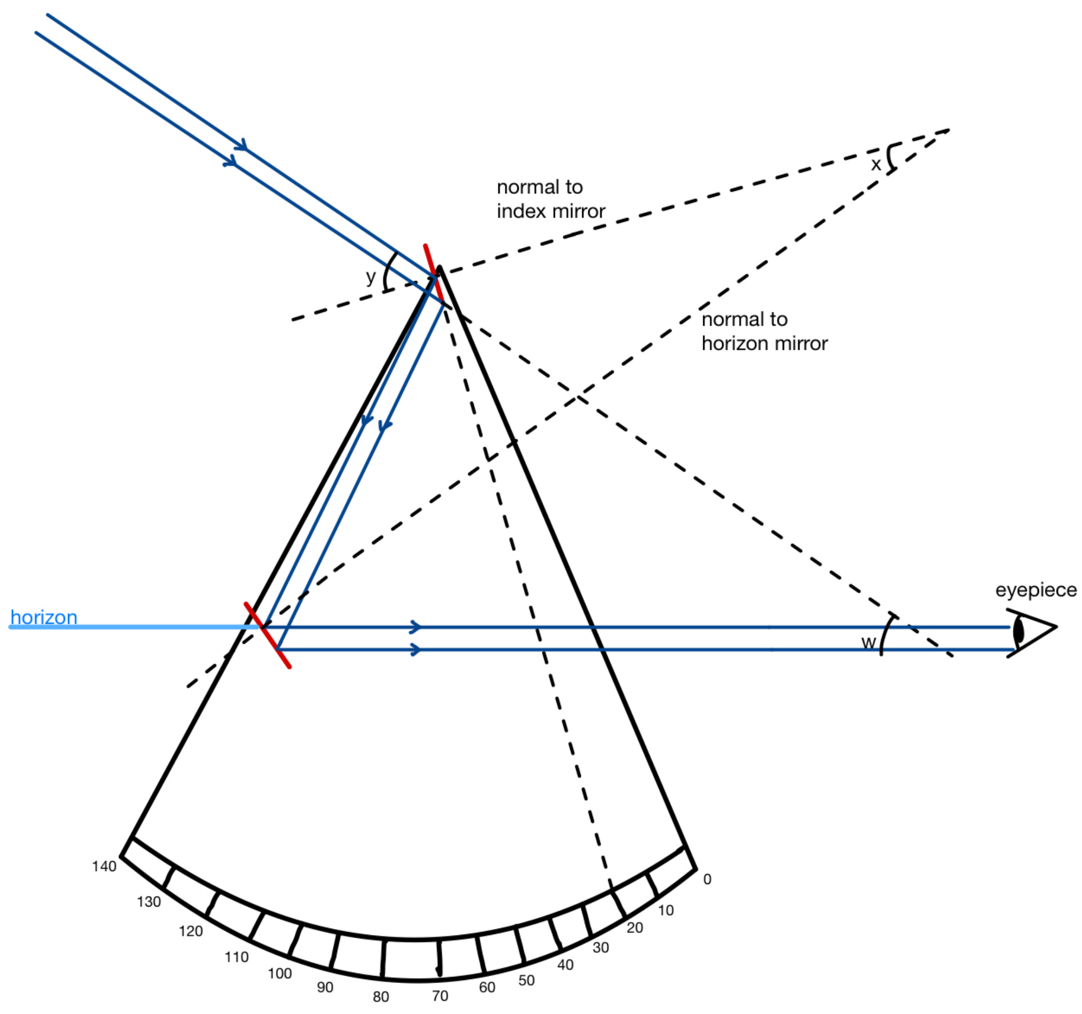
A sextant is a doubly reflecting navigation instrument that measures the angular distance between two visible objects. In celestial navigation, it is used to measure the angle between an astronomical object and the horizon. Figure 4 shows an incomplete ray diagram of a sextant.

The index mirror is a fully reflective mirror that is used to reflect an image of the body you are measuring towards the horizon mirror. The horizon mirror is a partial mirror that enables the user to overlay the reflected image from the index mirror, with the horizon viewed right ahead. When the sextant is lined up, the rays from the object appear to be parallel to the horizon. In this question, the horizon will be considered to have an altitude of 0 degrees.

A sextant measures angles between sightlines by viewing those sightlines through a series of mirrors. It then adjusts the angle of the sightline by changing the angles of the mirrors. The horizon mirror is in front of only half your view. So the view to the left is straight. The view to the right is towards the index mirror. What you see in the index mirror depends upon the angle of that mirror. One of the curious features of a sextant is that if you change the angle of the index mirror by 10-degrees, you change your view by 20-degrees. This is because changing the angle of the mirror changes both the angle of incident and reflection, giving it a doubling effect. Therefore the angle between the index and horizon mirrors is equal to half the altitude angle of the object.

(a) Complete the light ray diagram (Figure 4). **[2]**

Solution:



Step 1: Show understanding for what 'lined up' means from reading the preamble. Draw 2 rays going into the eyepiece that are both parallel to the horizon. [1]

Step 2: Show understanding for the laws of reflection, especially angle of incidence = angle of reflection. The rays drawn in step 1 are the reflected rays at the horizon mirror. At the horizon mirror, draw the incident rays. [1]

Minus 0.5 marks if they forget to indicate the direction of rays drawn.

(b) On Figure 4, label an angle w that is equivalent to the altitude of the celestial object that it is lined up to. You may draw additional dotted line(s) if necessary. **[1]**

Solution:

Labelled in ans scheme for (a)

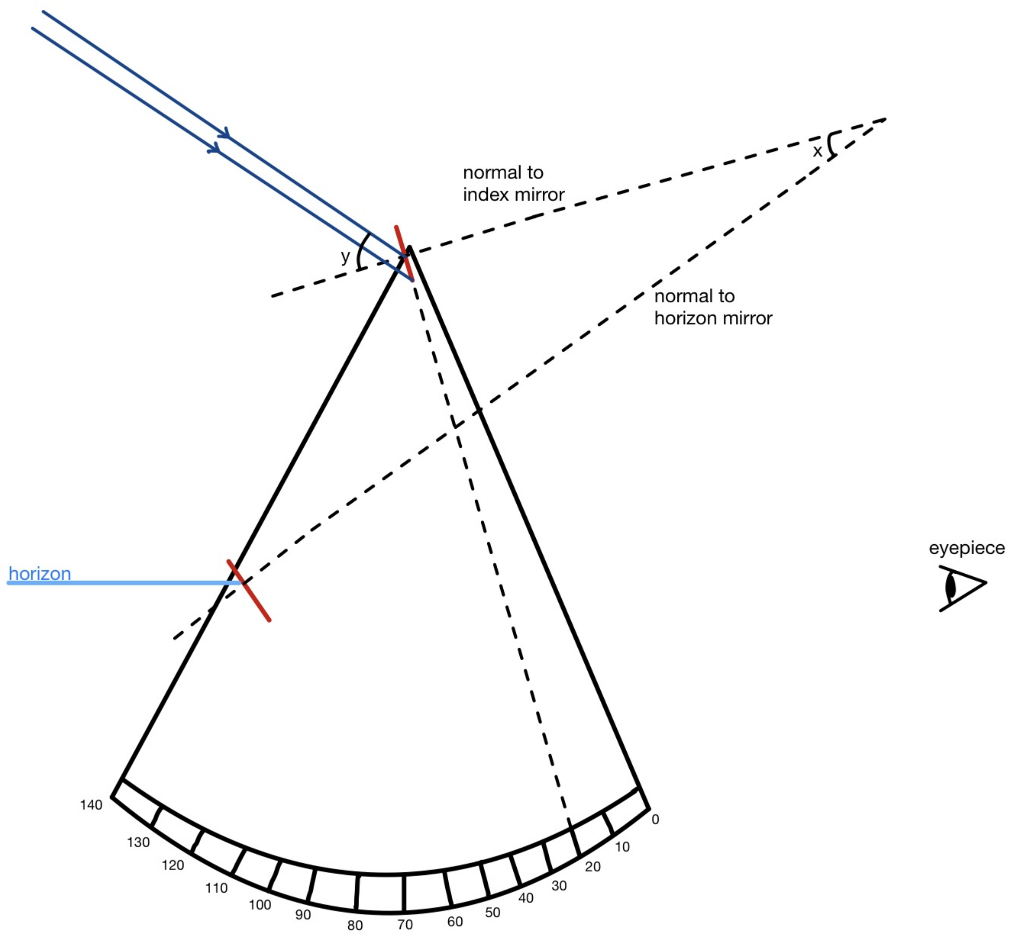


Figure 4: Ray diagram of a sextant

Part II Look up!

You come ashore to an island with a very clear horizon on all sides as it is a very flat island with no mountains or hills, just beautiful beaches.

At night, since you are stranded on the island, you look up to the sky and watch the stars appearing to rotate anti-clockwise about a single point, which is very close to a certain star. This star is decently bright and doesn't seem to move at all over the course of the night, and all other stars seem to rotate around it.

(c) Which hemisphere are you in and what is the name of this star? **[2]**

> **Solution:**
>
> Northern hemisphere [1], Polaris / α Ursae Minoris [1]

When you point your trusty dusty sextant at this star, the angle x labelled in Figure 4 is 10 degrees.

(d) Hence, what is the latitude of your current location? **[1]**

> **Solution:**
>
> x = angle between index and horizon mirrors
>
> current latitude = altitude of polaris = $w = 2x = 20\,^{\circ}$N

You wait for sunrise, a dawn of hope. At dawn, you see the locals preparing for a festival to be celebrated that day. They tell you that it is to celebrate the spring equinox (assume they have a perfect solar calendar and that this day is truly the day of the spring equinox).

(e) What is meant by the "spring equinox", taking into consideration which hemisphere this island is on? **[2]**

> **Solution:**
>
> It is the moment the path of the Sun crosses the celestial equator [1]
>
> From South of the celestial equator to North of the celestial equator [1] (error carried forward if they thought the island was in the Southern hemisphere from part (c))

(f) What is the special name of the point at which the Sun is at on this day? Explain the significance of this point in the celestial coordinate system. **[2]**

> **Solution:**
>
> Spring equinox / vernal equinox / March equinox / First point of Aries / cusp of Aries. [1]
> It is the origin ((0,0) point) for the RA/DEC coordinate system. / It is the point where the celestial equator and ecliptic intersect. [1]

The Sun on this day currently lies in the constellation of Pisces, and not in Aries.

(g) Why is this the case? **[1]**

> **Solution:**
>
> Axial precession / precession of the equinoxes / precession of the equator / lunisolar precession [1]
>
> Precession [0.5]

Part III Sunrise Equation

You happen to have brought a digital pocket watch from home, which shows the exact UTC time. You observe sunrise at 4:34pm UTC. Ignore atmospheric refraction

The sunrise equation states that:

$$\cos(H) = -\tan(\phi)\tan(\delta) \tag{1}$$

where H is the hour angle of the sun, δ is the declination of the sun and ϕ is your current latitude.

(h) Calculate the hour angle of the Sun at sunrise, using the sunrise equation or otherwise. **[1]**

Solution:

-90 degrees / -6 hours / 270 degrees / 18 hours

Substitute in current latitude as 20 °N from part (d) and declination of the Sun as 0° from part (f).

Hour angle of the Sun is defined as 0 at noon, and is counted in the negative direction before noon in the morning.

(j) What is your current longitude, rounded down to the nearest whole number? **[4]**

Solution:

Observed UTC time for sunrise = 4.34pm = 16:34hrs UTC [0.5]

Local solar time of sunrise = 6.00am since it is spring equinox [0.5]

Time difference between UTC and local time = 16:34hrs - 06:00hrs = 10h 34min [1] = 634 minutes (behind UTC)

Using the fact that 24 hours corresponds to 360 degrees, by proportion, 1 minute corresponds to 0.25 degrees. [1]

Longitude = (634 min)(0.25 degrees per minute) = 158.5 degrees = 158 degrees West [1] (rounded down to nearest whole number).

Note that longitude is measured from Greenwich and measured in the positive direction westwards.

If any participants used 361 degrees (sidereal time) they should get the same answer since it is rounded down.

The sunrise equation can actually be proven using our knowledge about the celestial coordinate system. The 2 commonly used celestial coordinate systems are the right ascension-declination (RADEC) system and the altitude-azimuth (ALTAZ) system, as shown in Figure 5.

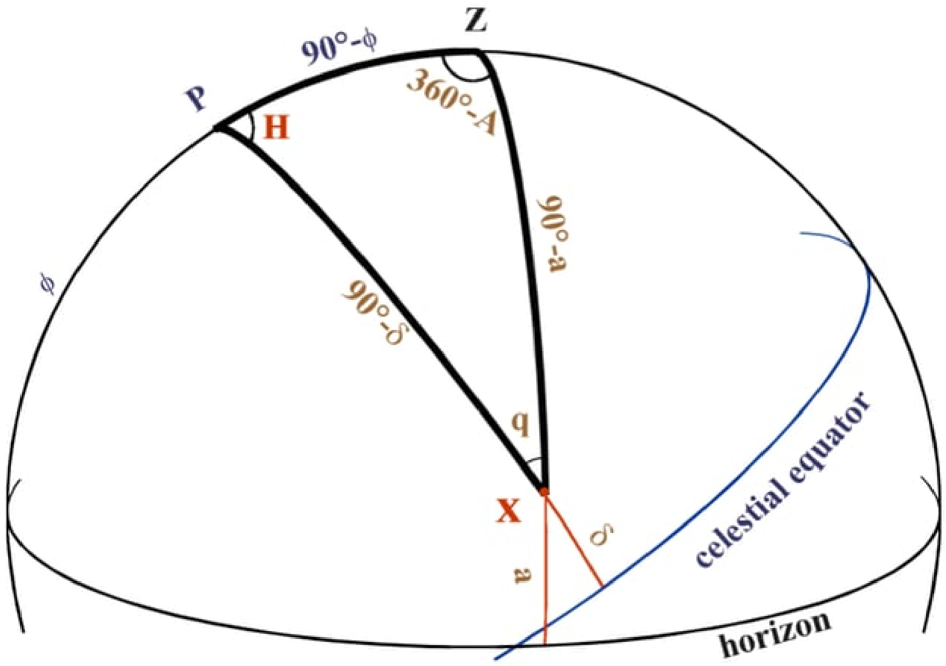


Figure 5: The Celestial Coordinate System [1]

In Figure 5, the definitions of notations are as follows:

- Right ascension: α
- Declination: δ
- Altitude: a
- Azimuth: A
- Latitude: ϕ
- Local hour angle (of X): H

Here, the observer (you) is standing at the centre of the sphere. As you are standing on flat ground, you can see everything that is above the ground and cannot see anything below the ground. The line marking this separation is the horizon. On the celestial sphere, the point of interest, which is the Sun, is marked at X. The point marked Z is the zenith, or the point directly overhead you. The point marked P is the north celestial pole.

Distances on a spherical coordinate system can be expressed in terms of angles. This is because we do not care about the actual physical distances between two celestial objects, but rather we care about how far they appear from one another in the sky from our point of view. For example, by definition, the angle between the zenith and any point on the horizon is 90°.

Using the definitions above, we can derive some other interesting angles that will be useful later.

Take a point X', with the same azimuth as X but on the horizon. The angle XX' is a, the altitude of X. As stated before, the angle ZX' is 90°. Therefore the angle XZ is $90° - a$.

Similarly, take a point R, with the same right ascension as X but on the celestial equator. The angle XR is δ, the declination of X. The north celestial pole has a declination of 90°, so the angle PR is 90°. Therefore the angle PX is $90° - \delta$.

Similarly, the angle PZ is $90° - \phi$.

(k) Prove the sunrise equation (Equation 1). **[5]**

Reminder: please use the appropriate trigonometry laws given in the formula booklet.

Hint: Consider the trigonometric properties of the triangle PZX on the celestial sphere.

Solution:

Let us remind ourselves that the triangle PZX lies on the surface of a sphere. Hence, the trigonometric laws on a sphere apply.

Using the cosine law for a sphere, given in the formula booklet:

$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

[1 mark for correct choice of trigonometry formula]

substituting

$$a = PX = 90^\circ - \delta$$
$$b = PZ = 90^\circ - \phi$$
$$c = XZ = 90^\circ - a$$
$$C = H$$

we get

$$\cos(90^\circ - a) = \cos(90^\circ - \delta)\cos(90^\circ - \phi) + \sin(90^\circ - \delta)\sin(90^\circ - \phi)\cos(H)$$

[1 mark for correct substitution]

Using the identity $cos(90^\circ - \theta) = \sin(\theta)$,

$$\sin a = \sin\delta \sin\phi + \cos\delta \cos\phi \cos H$$

[1]

At sunrise, the altitude of the Sun should be 0, since it appears at the horizon. Therefore, $\text{LHS} = \sin 0 = 0$ so

$$0 = \sin\delta \sin\phi + \cos\delta \cos\phi \cos H$$

[1]

$$-\sin\delta \sin\phi = \cos\delta \cos\phi \cos H$$

Using the identity $\frac{\sin\theta}{\cos\theta} = \tan\theta$,

$$\cos H = -\frac{\sin\delta \sin\phi}{\cos\delta \cos\phi} = -\tan\theta \tan\phi$$

[1]

(m) Just for fun, suggest what island you are on. **[0]**

Solution:

Hawaii.