AstroChallenge 2023
Senior Data Analysis Answers

Monday 13th March 2023

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

X

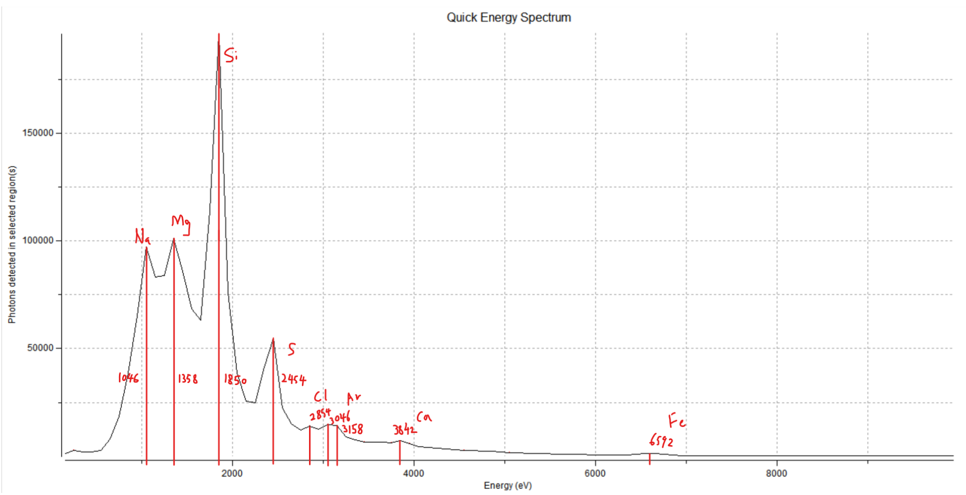
1. **(a) What do you observe about the image? Explain how the change in scale caused this change.** [2]

> Fainter features in the image are better highlighted upon switching the scaling from linear to sqrt. [**0.5**]
>
> **0.25 if the answer only mentions the image became brighter without stating the increase in contrast in the dimmer spectrum of the image.**
>
> For the same absolute difference in brightness, brighter pixels are less different compared to dimmer pixels when square-rooted [**0.5**].
>
> Hence, a larger range of the brightness scale is allocated to dimmer pixels [**0.5**] and the differences among dimmer pixels become more apparent [**0.5**].
>
> Any other reasonable explanation [**1.5**].

(b) The `Quick Energy Spectrum Plot` plots the number of photons detected within regular intervals of energies over the entire range of energies of the photons collected (i.e., a histogram of photon energies). How does this allow us to capture the emission and absorption lines of elements from Cassiopeia A? [1]

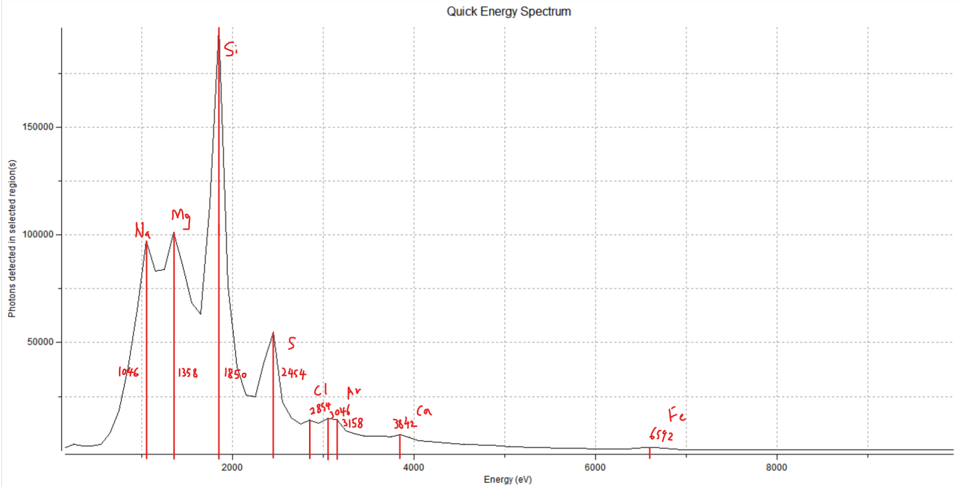
> An emission line can be identified by a sharp peak in photons detected within a narrow range of photon energies [**0.5**] and an absorption line is identified by a sharp drop in photons detected in a narrow range of photon energies [**0.5**].

(c) Based on your answer in part b, identify two emission lines and their energies in the `Quick Energy Spectrum Plot`. Show your working clearly. [1]



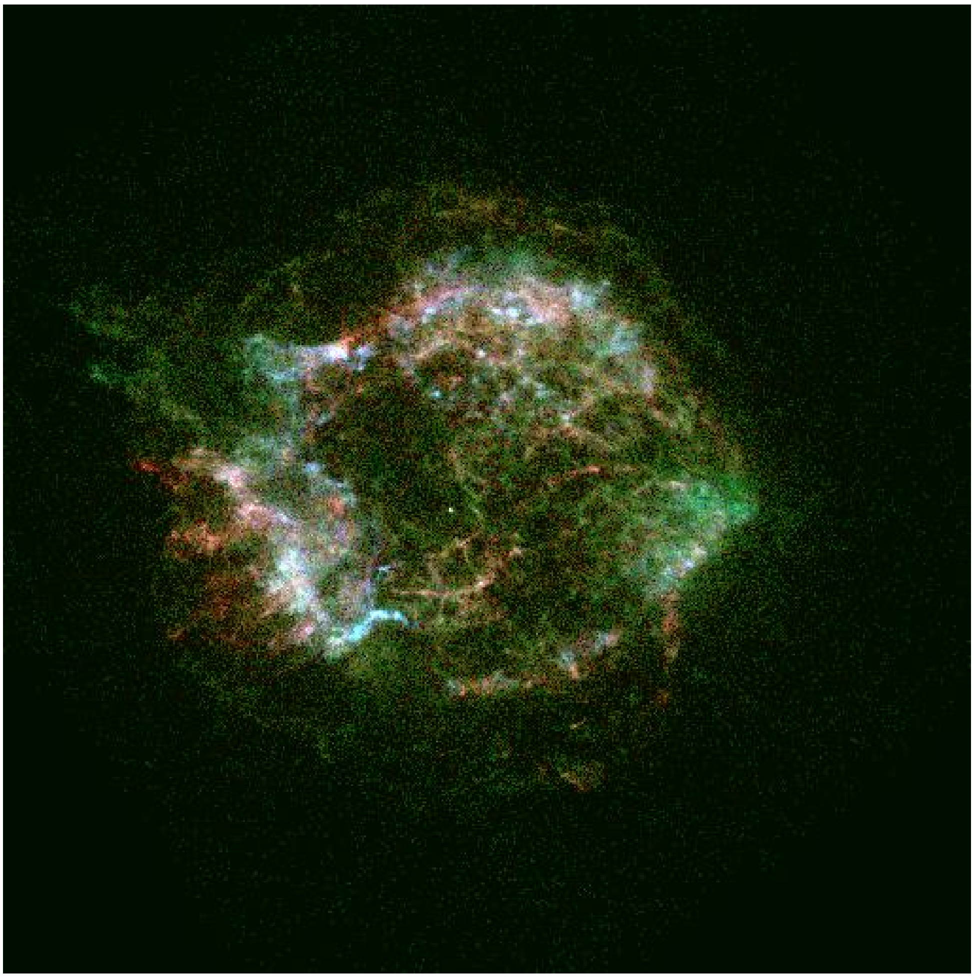
> Correct graph generated [**0.5**]
>
> Any two emission lines identified [**0.5**]

(d) Identify three elements from the `Quick Energy Spectrum Plot` of Cassiopeia A and create an RGB composite image of Cassiopeia A with each colour representing an element you have chosen. Mark out the elements you have identified on the energy spectrum plot and present it as working to this part of the question. Save your created image in SAO as an `RGB Image` (.fits) under `File` → `Save As` in the menu bar. Also, save the colour image as PNG and attach it to your report (`File` → `Save Image` → `PNG...`). [2]



[0.5]

Possible answers[1] (any three) : Sodium, Magnesium, Silicon, Sulphur, Chlorine, Argon, Calcium, Iron **[0.5] (-0.25 for each wrong element, capped at -0.5)**



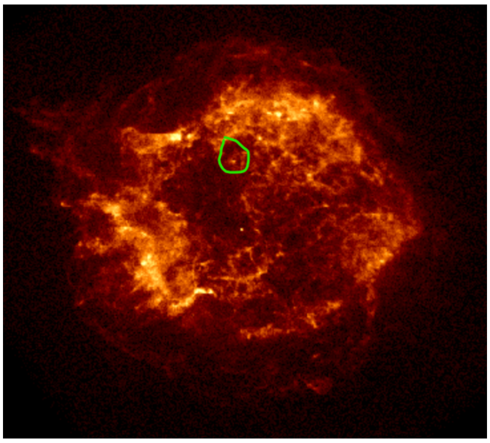
[0.5]

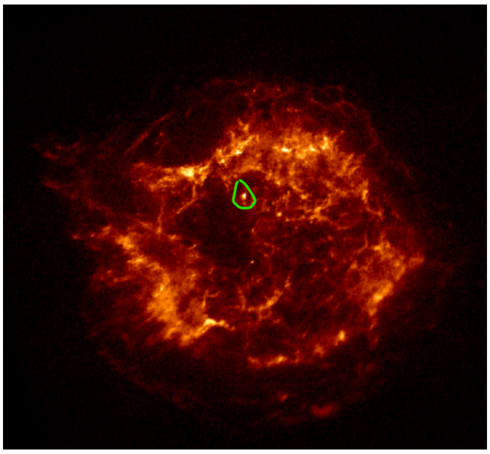
Magnesium – Red, Silicon – Green, Sulphur – Blue. Emphasis of the colours were given in accordance to the abundance of elements in the supernova Remnant, in the order of the most abundant being Silicon, followed by Magnesium, then Sulphur. **[0.5 for brief explanation of the image and/or what its colours represent]**

[1] https://www.chess.cornell.edu/users/calculators/characteristic-emission-lines-elements

(e) **Take a screenshot of the feature you have identified in each image, label them, and include your screenshots in the report. Take the angular distance of these features from the centre star of the Cassiopeia A (circled in Fig. 3) by drawing a `Ruler` region[2] (take screenshots of your working). What is the difference in angular distance from the neutron star to the feature in each image? What does this tell you about Cassiopeia A?** [2]

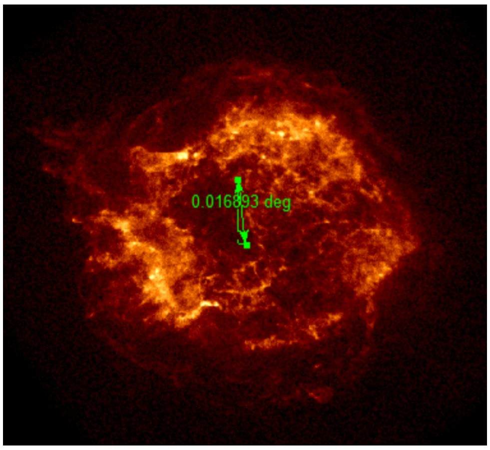
Identify same feature (needed for angular measurement marks to be awarded)



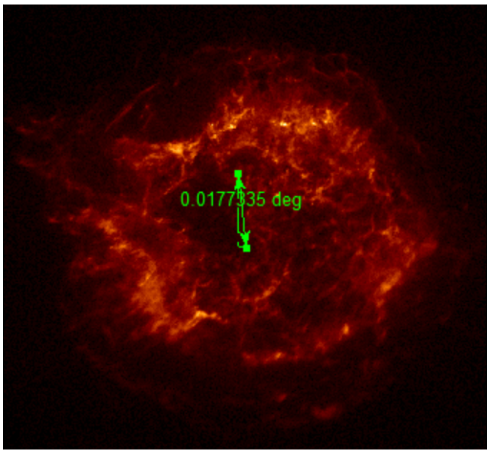


2000-01-30T10:41:45 -

2019-05-13T12:25:32 -



[0.5]



[0.5]

Difference in angular distance = (0.0177335 – 0.016893) degree = 0.0008405 degree [**0.5**]

CAS A is expanding with time [**0.5**]

Poor choice of feature -0.5 Measurement inaccuracy -0.25

[2] Change the shape of the region to `Ruler` in the menu bar and user a region tool under `edit` in the buttons bar.

(f) Given that the shell of Cassiopeia A is expanding at around 5000km s^{-1}. Approximate the size of Cassiopeia A in terms of light years across. **[2]**

Time difference between 2000-01-30T10:41:45 and 2019-05-13T12:25:32: 608521427 s [**0.5**]

Angular speed of moving feature: $\frac{0.0008405^\circ}{608521427s} = 1.38122 \times 10^{-12\circ} s^{-1}$ [**0.5**]

Hence one degree corresponds to $\frac{5000kms^{-1}}{1.38122\times10^{-12\circ}} s^{-1} = 3.62 \times 10^{15} km/^\circ$ [**0.5**]

Diameter of CAS A: $0.107^\circ \times 3.62 \times 10^{15} km/^\circ = 3.8734 \times 10^1 4m \rightarrow 40.9ly$ [**0.5**]

Better estimates has CAS A at a diameter of 10 ly.

1 for using age of CAS A to determine its size.

Cosmic Rolex

2. (a) **Calculate the average energy of a photon that the detector received.** **[0.5]**

Total energy = 1.61261E-09J

Total photon count = 2945841

Average photon energy = $\frac{1.61261\times10^{-9}\text{J}}{2945841} = 5.47480 \times 10^{-16}$J, 3417eV **[0.5]**

(b) Given that the average photon count when the X-ray beam from Cen X-3 is facing us is 78.8596 per second and the effective aperture of the Chandra X-ray Observatory has an area of 353 cm^2 , calculate the apparent bolometric magnitude of Cen X-3 when it is the brightest, assuming Cen X-3 emits most of its energy in X-ray. You may reference the intensity of radiation from a celestial source with a konwn apparent magnitude. **[1.5]**

Intensity of x-ray received by Chandra Telescope when Cen X-3's x-ray beam is facing us:
$\frac{78.8596\text{s}^{-1}(5.47480\times10^{-16}\text{J})}{353\text{cm}^2} = 1.22306 \times 10^{-16}\text{Wcm}^{-2}$ **[0.5]**

$$\frac{I_1}{I_2} = 10^{\frac{m_2-m_1}{2.5}}$$ **[0.5]**

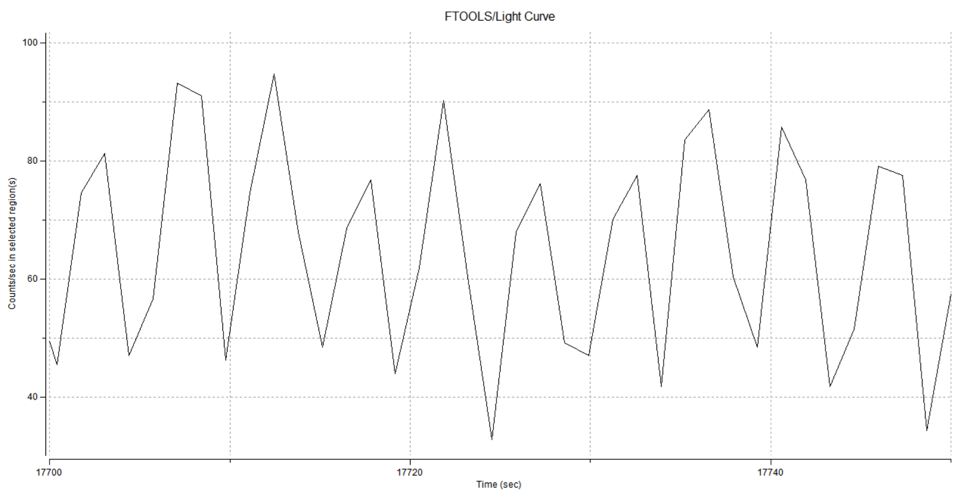
Intensity of solar radiation at Earth's surface = $\frac{3.846\times10^{26}\text{W}}{4\pi(1.49597870700\times10^{13}\text{cm})^2} = 0.136757\text{Wcm}^{-2}$

Apparent Magnitude of the Sun = -26.74

Lowest apparent Magnitude of Cen X-3 = $2.5\log\frac{0.136757}{1.22306\times10^{-16}} - 26.74 = 10.88$ **[0.5]**

Other stars/standards may be used for magnitude reference (other than the Sun).

(c) **Take a screenshot of your plot and measure the period of brightness oscillation. Show your working on your screenshot.** **[1]**



10 oscillations → 17700.5s to 17743s **[0.5]**

Period of one oscillation = 4.72s **[0.5]**

(d) Based on information from the light curve only, do you expect the radiation received to be from a small, concentrated source (e.g., a small star), or a large extended one (e.g., a nebula/gas cloud)? Explain. [1]

Given the short period of oscillation in brightness, the source is like to be a small concentrated one **[0.5]** since light from further parts of a extended source will take longer time to reach us and hence its brightness cannot change so rapidly like what we observe with Cen-X3 **[0.5]**.

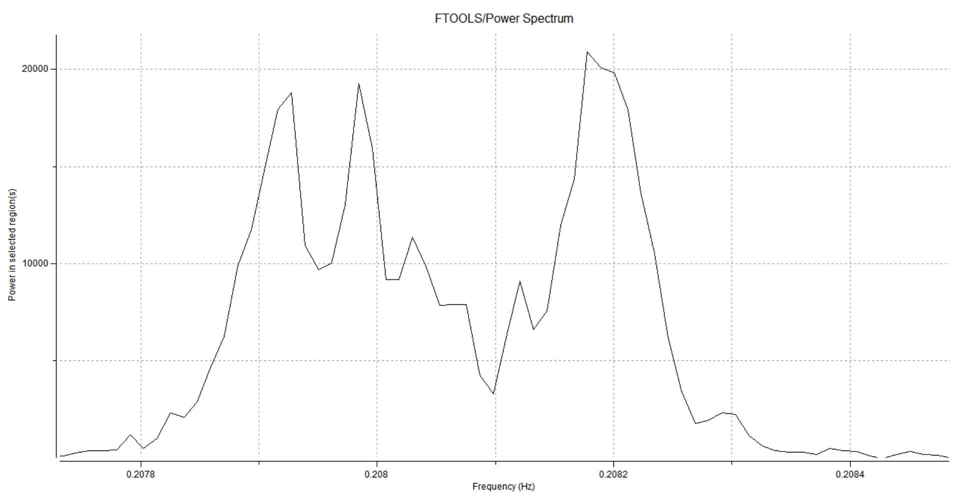
Accept other reasonable answers + explanation **[1]**.

(e) Explain how doppler shift results in this distribution of the observed frequencies of the pulsar's rotation. [1]

Like other types of waves or oscillations, the rotation of the pulsar produces periodic oscillations in its x-ray brightness. When the pulsar is moving towards us, the period between each x-ray beam aligned towards us is shorter since the pulsar would have travelled closer to us by the next time its beam faces us **[0.5]** and hence the radiation requires less time to travel to us **[0.5]**. This results in the duration between two brightness peaks of the pulsar being shorten. The opposite is true when the pulsar is moving away from us.

0 if answer discusses doppler shift in the context of x-rays emitted by the pulsar rather than its period of rotation.

(f) With information from the power spectrum plot, find the maximum and minimum doppler-shifted frequency of the rotation of the Cen-X3. Hence, find the maximum radial velocity of Cen-X3. Include your working on a screenshot of the power spectrum plot. [2]



Min. Frequency: 0.2078 Hz **[0.5]**
Max. Frequency: 0.20835 Hz **[0.5]**
Mean. Frequency: 0.208075 Hz

$$\frac{\Delta f}{f_0} = \frac{v}{c} \quad \textbf{[0.5]}$$

$$v = \frac{(0.20835 - 0.2078)/2}{0.208075} c = 396492 \text{ms}^{-1} \quad \textbf{[0.5]}$$

Hence, the maximum/minimum radial velocity of Cen-X3 is 396492ms^{-1}.

(g) Hence, calculate the mass of the binary system of Cen-X3, give your answer in solar masses.
HINT: Use Kepler's third law for binary systems. **[2]**

Assuming a circular orbit of the pulsar around its companion star.

$$a = \frac{P\left(v_1 + v_2\right)}{2\pi} = \frac{2.1 \times 24 \times 3600 \times (396492 + 26000)\ \mathrm{m}}{2\pi\left(\sin 70^\circ\right)} = 1.29833 \times 10^{10}\mathrm{m} \quad \mathbf{[0.5]}$$

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3 \quad \mathbf{[0.5]}$$

$$m_{\text{total}} = \frac{4\pi^2}{6.674 \times 10^{-11}\mathrm{Nm^2kg^{-2}}\,(2.1 \times 24 \times 3600\mathrm{s})^2}\left(1.29833 \times 10^{10}\mathrm{m}\right)^3$$

$$= 3.93 \times 10^{31}\mathrm{kg} \rightarrow 19.8 M_{\text{sun}} \quad \textbf{[0.5 for working, 0.5 for correct answer]}$$

(h) What is the mass of the Cen-X3 and its companion star respectively? **[1]**

Since both members of the binary system have the same orbital period.

$$v_1 r_1 = v_2 r_2 \quad \mathbf{[0.25]}$$

Considering the distance of the barycentre

$$m_1 r_1 = m_2 r_2$$

$$\frac{m_1}{m_2} = \frac{v_1}{v_2} \quad \mathbf{[0.25]}$$

$$m_{\text{star}} = \frac{19.8 M_{\text{sun}}}{1 + \frac{26}{396.492}} = 18.6 M_{\text{sun}} \quad \mathbf{[0.25]}$$

$$m_{\text{pulsar}} = 19.8 M_{\text{sun}} - 18.6 M_{\text{sun}} = 1.2 M_{\text{sun}} \quad \mathbf{[0.25]}$$

1 for any other valid methods