AstroChallenge 2018
Data Response Questions
(Junior)

PLEASE READ THESE INSTRUCTIONS CAREFULLY

1. This paper consists of **22** printed pages and **6** blank pages, including this cover page.
2. Do **NOT** turn over this page until instructed to do so.
3. You have **2 hours** to attempt all questions in this paper.
4. At the end of the paper, submit this booklet together with your answer script.
5. Your answer script should clearly indicate your school (and team number) on **EVERY** page, as well as the individuals in the said team on the first page.
6. It is your team's responsibility to ensure that all pages of your answer script have been submitted, **including pages to be detached from this booklet**.

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Question 1
Do You See What I See?
(Total: 20 points)

Part 1: The Many Eyes of Astronomy (Sub-total: 11 points)

The layman is familiar with using binoculars and telescopes in the backyard to look at stars. This is an example of *optical astronomy*, i.e. astronomy in the visible light spectrum. We also call this *visible light astronomy.* Astronomy, however, comes in many different forms, across the entire electromagnetic spectrum, and more.

The electromagnetic spectrum describes the range of electromagnetic waves in terms of their wavelength. Broadly speaking, there are seven distinct classes of electromagnetic waves.

Class	Average wavelength
Gamma	1 pm
X-ray	500 pm
Ultraviolet	100 nm
Visible	500 nm
Infrared	10 μm
Microwave	1 mm to 10 cm
Radio	10 m

Of course, these classes are really "bands", so each class really establishes a range on the electromagnetic spectrum.

Astronomy takes place across all these bands. Many objects in the universe (e.g. gas clouds, black holes) are not visible to the naked eye. In other words, they emit very little, if at all, radiation in the visible light spectrum. However, any object with heat does emit radiation. Therefore, to obtain a good picture of the universe, we need to consider the entire electromagnetic spectrum. In fact, most images one might see of beautiful objects in the universe are false colour images. If one were to look at these objects in true colour, one should expect to literally see space, because our eyes cannot detect the radiation these objects emit.

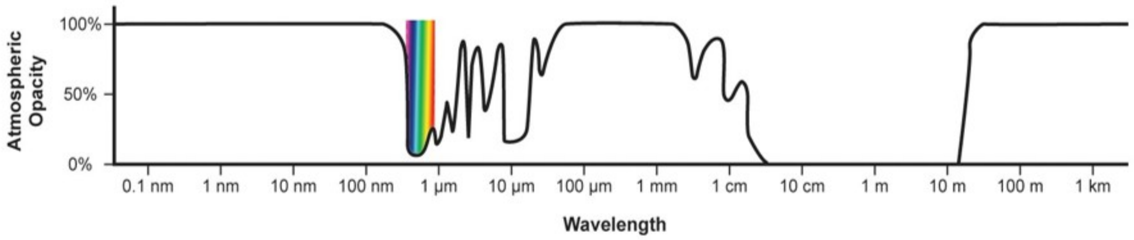
(i) [2 points] Provide, with clear justification, a use for microwave astronomy and a use for X-ray astronomy.

Due to the varying wavelengths of each band, different types of telescopes are needed to perform different types of astronomy. For example, huge satellite dishes are used to perform radio astronomy, standard optical telescopes are used to perform optical astronomy, and gamma ray detectors used to perform gamma ray astronomy.

(ii) [1 point] Explain why radio telescopes need extremely large receivers (satellite dishes).

(iii) [2 points] State and explain two difficulties of gamma ray astronomy if we were to try to use the conventional method of focusing light to detect gamma rays.

One additional difficulty with performing astronomy in each of these ranges is due to *atmospheric opacity.* Gases in Earth's atmosphere prevent certain wavelengths of electromagnetic radiation from permeating the atmosphere. Because of this, certain telescopes and detectors cannot be built on the ground, and must instead be put in space.



(iv) [3 points] The atmosphere is completely opaque to gamma radiation. Yet there are ground-based gamma ray detectors. How do they work, and why are they in fact superior to space-based gamma ray telescopes?

(v) [1 point] What is an advantage to using space-based telescopes as compared to ground-based telescopes for optical astronomy? Explain.

Finally, we may of course study the same object in space using different ranges of the electromagnetic spectrum to gain for information about the object. Additionally, even when we expect objects to emit radiation in some range of the electromagnetic spectrum, this may not be the case. Such is the case with *embedded star clusters.* A typical example of an embedded star cluster is shown below.

Figure 1: Orion Trapezium Cluster, in optical (left) view and in infrared (right) view.

(vi) [1 point] What are embedded star clusters?

(vii) [1 point] Explain why infrared astronomy has been very useful in detecting such star clusters.

Part 2: Another Spectrum (Sub-total: 9 points)

Basics of Spectra

Another very important tool used when studying astronomical objects is the *spectrum* of the object. As an example, here is a sample absorption spectrum of a star HD 145482.

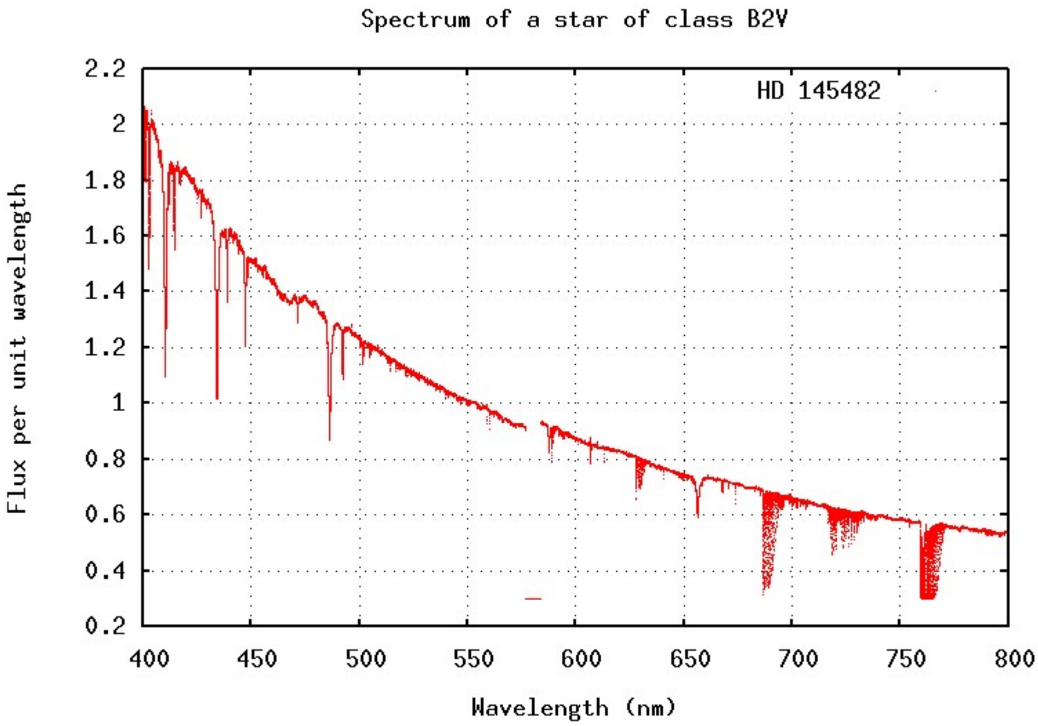
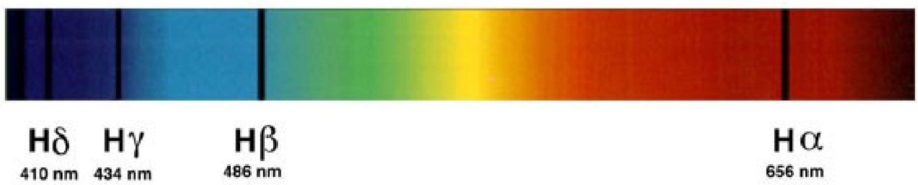
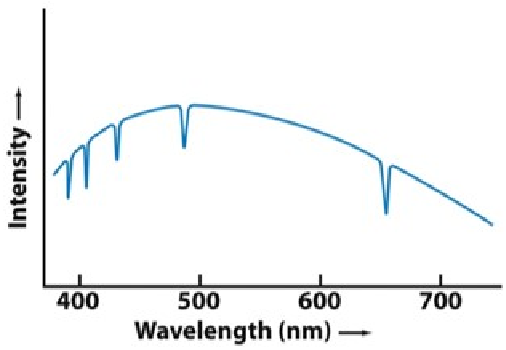


Figure 2: Absorption spectrum of HD 145482.

Each of these dips is called an *absorption line*. These absorption lines yield a wealth of information about the star. Each element and/or molecule is associated with a set of wavelengths. For example, the absorption lines of hydrogen (and the associated graph) are as follows.





By studying the absorption lines of a stars spectrum, one can obtain information about the composition of the star by checking for the characteristic lines of the spectra of the element or molecule in question. We can furthermore deduce properties based on the elements and molecules present, for example ongoing fusion processes. Rotational properties can be deduced from the broadening of the spectral lines, although these can be caused by collisions (known as collision damping) and thermal movement as well. Horizontal scaling of spectral lines suggests radial movement of the star.

(viii) [1 point] A copy of the absorption spectrum graph of HD 145482 is attached to the back of this question. Identify the hydrogen lines $H\alpha, H\beta, H\gamma, H\delta$ in the spectrum of the star HD 145482, and indicate them on this graph.

(*Note: You should detach the graph provided and attach it to your answer script.*)

(ix) [2 points] There are in fact two types of spectra – absorption spectra and emission spectra. Since absorption spectra involves light passing through a gas, and an emission spectra looks at diffracting emitted light, one might expect us to use emission spectra to study stars instead of absorption spectra. Why do we use absorption spectra instead of emission spectra?

(x) [1 point] HD 145482 is in fact a spectroscopic binary. What is a spectroscopic binary, and how would you determine that HD 145482 is a spectroscopic binary?

Doppler Line Broadening

Doppler broadening due to thermal effects may be modelled by the Maxwell-Boltzmann distribution. Informally speaking, the random motions of a gas depend on temperature, and the distribution of movement of molecules thus also depends on temperature. By the Doppler effect on molecules in a gas, therefore there will be a range of wavelengths (and hence frequencies) detected by spectroscopic methods. The profile function for the distribution of frequencies due to thermal effects is given by

$$\phi(f) = \frac{1}{\Delta f_D \sqrt{\pi}} e^{-\frac{(f-f_0)^2}{(\Delta f_D)^2}},$$

where $\Delta f_D = \frac{f_0}{c}\sqrt{\frac{2kT}{m}}$ is the *Doppler width*. Here, f_0 is the average (mean) frequency, c is the speed of light, k is the Boltzmann constant, T is the temperature of the gas, m is the particle mass, and $e = 2.71828\ldots$ is Euler's number, a constant.

With the formula $\phi(f)$ above, the *full width at half maximum* (FWHM) is the width of $\phi(f)$ at half its maximum value. In layman's terms, the FWHM describes the average spread of the function. Graphically, the FWHM is seen as follows.

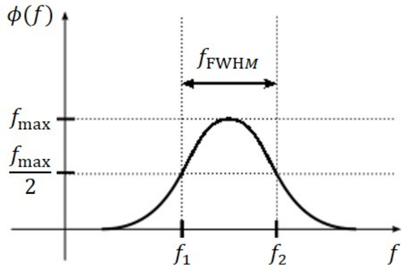


Figure 3: FWHM of $\phi(f)$.

Where there is a spectral line at f_0, the function $\phi(f)$ will describe the broadening of the line. The FWHM translates to the width of the line, i.e. at half-maximum the line will range from $f_1 = f_0 - \frac{f_{\mathrm{FWHM}}}{2}$ to $f_2 = f_0 + \frac{f_{\mathrm{FWHM}}}{2}$, where f_{FWHM} is the FWHM of $\phi(f)$.

The Doppler width is extremely important, as it determines the influence of the considered effect (in this case, thermal) on the width of the broadened spectral line. In the questions to follow, you will see how this is true.

(xi) [2 points] The maximum of $\phi(f)$ is $\phi(f_0)$. Find the FWHM of $\phi(f)$. Express your answer in terms of Δf_D.

(xii) [1 point] Determine Δf_D in terms of wavelength λ.

(xiii) [2 points] Assume the Sun has temperature 6000 K. The effect of collision damping on the width of the Balmer lines of hydrogen is approximately 0.1 pm, i.e. the Balmer lines are broadened about the mean by approximately 0.1 pm due to collision damping.

Hence, or otherwise, prove that the broadening of the Balmer lines due to thermal effects dominates the broadening due to collision damping. You may assume the mass of hydrogen is $1.0078u$. Here, u is the atomic mass unit.

(Note: The function ϕ has the property that the area under the curve from $\phi(f_1)$ to $\phi(f_2)$ is the probability that a random particle selected in the gas will have a Doppler-shifted frequency between f_1 and f_2. More advanced students will recognise ϕ as a probability distribution function known as a *Gaussian distribution*, although this fact is not needed in this question.)

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Detach this page and attach it to your answer script.

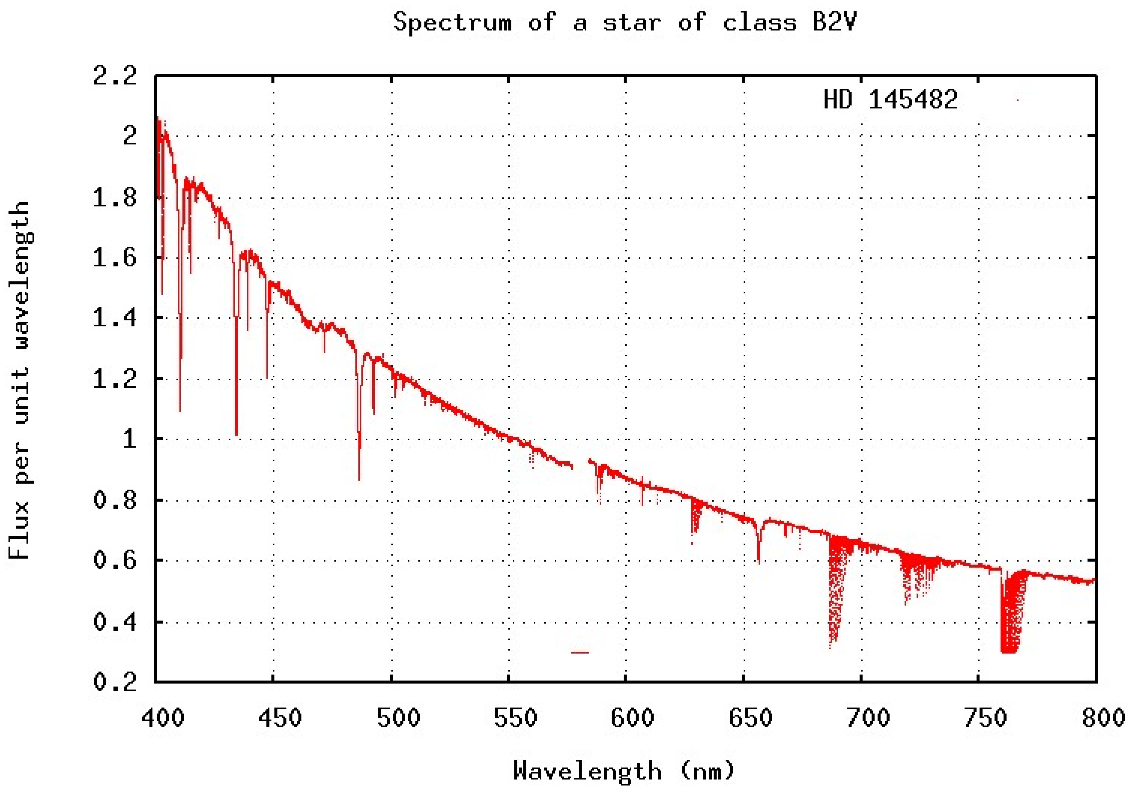

Figure 4: Copy of absorption spectrum of HD 145482.

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Question 2
Planetary Analysis
(21 points)

Introduction

On 14 December 2017, NASA and Google announced the discovery of an eighth planet, Kepler-90i, in the Kepler-90 system. The discovery was made using a new machine learning method developed by Google. However, how much do we truly know about the exoplanet Kepler-90i, or in fact about any of the exoplanets that are currently being discovered in general? Let us find out more by investigating the underlying geophysical concepts and the astronomy concepts revolving around the structure of a planet itself, as well as its orbital characteristics. Furthermore, the chances of life on Kepler-90i will be discussed at the end of this question.

Part 1: Radiogenic Heat and Conceptual Preliminaries (Sub-total: 4 points)

Kepler-90 system is a star system formed 2 billion years ago. Hence, we would thus expect the primary source of internal heat to be from radioactive decay. For this part of the question, we will investigate the physics behind radioactive decay and how it becomes the primary heat source a few billion years after the formation of the star system.

(i) [1 point] We tend to assume that one primary heat source for a planet a few billion years after the formation of the star system is via radioactive decay. Briefly explain why this assumption is valid.

In any radioactive decay reaction, they tend to follow the universal law of radioactive decay. This law, in summary, states that the decay of an unstable nucleus is entirely random, and it is impossible to predict when a particular atom will decay. Hence, we can expect to model this as a rate equation

$$\frac{\Delta N}{\Delta t} = -\lambda N,$$

where N refers to the number of unstable nuclides, λ is a positive constant (also known as the decay constant), and $\frac{\Delta N}{\Delta t}$ is the rate at which the number of unstable nuclides N changes. Since $\lambda > 0$, we would expect that N decreases with time.

The solution to this rate equation is given by

$$N = N_0 e^{-\lambda t},$$

where N_0 denotes the initial number (at $t = 0$) of unstable nuclides.

The half-life of an unstable nuclide X (e.g. the half-life of uranium-235, or plutonium-238, etc.) is defined as follows. Given a sample of X, the half-life $t_{\frac{1}{2}}$ of X is the time taken for the number of nuclides of X in the sample to decrease to half the original amount.

(ii) [1 point] In a sample of nuclide X, there are N_0 initial nuclides of X. Suppose X has decay constant λ. Prove that $t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$.

There are a few types of radioactive decay. Two of them are given below.

- α decay refers to the emission of a helium-4 nuclide (we call the helium-4 nuclide an *alpha particle*) and a daughter nuclide (the main product nuclide of the decay).
- β^- decay refers to the emission of an electron ${}^{0}_{-1}\beta$ and a daughter nuclide.

For instance, a uranium-238 nucleus decays via α decay to form a thorium-234 nucleus and an α particle by the following decay chain:

$$ {}^{238}_{92}\mathrm{U} \to {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}. $$

The energy released from this nuclear reaction is calculated using the Einstein's mass-energy equivalence equation

$$ E = (\Delta m)c^2, $$

where c is the speed of light and Δm is the net decrease in the total mass of the nuclei, calculated by taking the difference between total mass of the reactants and the products.

A *radioactive chain decay* is a process whereby the daughter nuclide is also unstable and thus decays further. This process usually generates an additional amount of heat. Since the half-life of the unstable daughter nuclide of this reaction is extremely fast relative to the half-life of the parent nuclide, it can be assumed that each radioactive decay of the parent nuclide releases energy from the radioactive decay of the nuclide itself plus the radioactive decay of the daughter nuclide(s).

Refer to Appendix A for the radioactive decay chain of uranium-238. The nuclide ${}^{206}_{82}\mathrm{Pb}$ is stable in this chain reaction. The relevant data required for the remaining part of this section is also given in the following table.

Nuclide	**Mass (in u, the atomic mass unit)**
Helium-4	4.002602
Uranium-238	238.051
Thorium-234	234.044
Protactinium-234m	234.043
Uranium-234	234.041
Lead-206	205.974

Note that the mass of an electron can be found in the Formula Booklet.

(iii) [2 points] Determine the total energy released from the entire chain of radioactive decay of ONE uranium-238 nuclide, and show that it is approximately 7.9×10^{-12} J.

Part 2: Planetary Thermodynamics **(Sub-total: 7 points)**

To analyse the thermodynamics of a planet, we will try to illustrate the concepts by using Earth as an example. Beneath the surface of the Earth, radiogenic heat is being generated mainly by the radioactive decay of unstable nuclides such as ${}^{40}_{20}\mathrm{K}$, ${}^{238}_{92}\mathrm{U}$, and ${}^{232}_{90}\mathrm{Th}$. The power generated by ${}^{232}_{90}\mathrm{Th}$ is approximately equal to that of ${}^{238}_{92}\mathrm{U}$, and is twice of that of ${}^{40}_{20}\mathrm{K}$ today. Geoneutrino detectors can detect the decay of ${}^{238}_{92}\mathrm{U}$ and ${}^{232}_{90}\mathrm{Th}$, but are unable to detect the decay of ${}^{40}_{20}\mathrm{K}$.

Hence, the power generated by the decay of ${}^{40}_{20}\mathrm{K}$ is being detected by other similar means. It has been determined that the current total heat flux from Earth to space is 44.2 TW. Note that the prefix T here is Tera and represents 10^{12}.

The half-life of uranium-238 is 4.468 billion years. Radiometric dating has determined that uranium made up $3.4 \times 10^{-6}\%$ of the mass of the Earth when it was first fully formed approximately 4.5 billion years ago.

(iv) [3 points] Show that the power generated by the radioactive decay of ${}^{238}_{92}\mathrm{U}$ is approximately 10 TW.

(Hint: Don't be scared! You can split this question into solving multiple mini questions.

- Uranium made up $3.4 \times 10^{-6}\%$ of the mass of the Earth when it was first fully formed approximately 4.5 billion years ago. What is the mass of uranium now? Consider Part 1 of this question.
- With the above, how many uranium nuclei, N_0, are there now?
- How can you then proceed to calculate the power generated from radioactive decay? (Hint for a Hint: State how P, $\frac{\Delta N}{\Delta t}$, and E for the decay of a nuclide all relate to each another.)

Points will be awarded for solving the mini-questions!)

(v) [1 point] Hence, show that the total power generated by the radioactive decay of ${}^{40}_{20}\mathrm{K}$, ${}^{238}_{92}\mathrm{U}$, and ${}^{232}_{90}\mathrm{Th}$ is approximately 25 TW.

(vi) [1 point] Hence, comment on the assumption made in Part (i).

(vii) [1 point] Explain briefly what equation below means to the layman.

$$P_{\text{Earth}} = \frac{(1-\varepsilon)(\pi R_E^2)}{4\pi d_{S-E}^2} L_S,$$

where

- P_{Earth} is the rate at which Earth receives energy from the Sun,
- ε represents the albedo of Earth,
- R_E represents the radius of Earth,
- d_{S-E} represents the Sun-Earth distance, and
- L_S represents luminosity of the Sun.

(viii) [1 point] Determine, with suitable calculations, which source of heat is more important in determining the surface temperature of a planet that is relatively not very far from its own star, such as Earth itself. Earth has an albedo of approximately 0.30.

Part 3: Kepler-90i Orbital Characteristics and Chance of Life (Sub-total: 10 points)

The following table describes some characteristics of the Kepler-90 star system.

Characteristic of Kepler-90	**Value**
Spectral type	G0V
Distance from Earth	2545 ly
Apparent magnitude V	+14
Absolute magnitude (M_V)	−0.5
Mass	$1.2M_{\odot}$
Surface temperature	6080 K
Age	Approx. 2 billion years

The following table describes some characteristics of the planet Kepler-90i, discovered recently.

Characteristic of Kepler-90i	**Value**
Radius	$1.32R_{\oplus}$
Semi-major axis	See Part (ix)
Mass	$2.5M_{\oplus}$
Orbital eccentricity	Approx. 0
Orbital period	14.44912 Earth days
Age	Approx. 2 billion years
Albedo	Approx. 0

(ix) [1 point] Determine the semi-major axis of Kepler-90i about its star Kepler-90.

(x) [1 point] Show that the luminosity of Kepler-90 is approximately 7.28×10^{26} W.

(xi) [2 points] Show that the surface temperature of Kepler-90i is 768 K.

(xii) [6 points] With reference to any of the concepts mentioned in the previous parts of this question, the characteristic data of Kepler-90, and its orbiting planet Kepler-90i, discuss the probability of life on Kepler-90i.

In your answer, you should support your argument with reference to relevant calculation(s) and/or theory. You should include at least 2-3 points/arguments (with hopes of scoring more than half of the marks allocated for this question).

Question 3
The Story of a Space Strife Engineer
(19 points)

Disclaimer: The storyline in this question is a work of fiction. Names, characters, businesses, places, events, locales, and incidents are either the products of the author's imagination or used in a fictitious manner. Any resemblance to actual or fictitious persons, living or dead, actual or fictitious events is purely coincidental.

Introduction

A long time ago, in a galaxy far far away, the Galactic Kingdom was on the retreat. The Rebel Army had assassinated Emperor Palpable, and the sympathisers of the Galactic Kingdom were escaping into the Unknown Space.

Among the remnants of the Galactic Kingdom, a new leader emerged – Supreme Leader Snake. As his first edict, a mobile planet that housed a super-weapon capable of destroying star systems was to be constructed. This new military installation was codenamed Starslayer Base.

Eventually a suitable planet was found. Surprisingly, this planet was similar to Earth. During the construction of Starslayer Base, a large hole was drilled from an end of the planet to the other end of the planet along its equator and through the core of the planet. Once drilling was completed, the Weapons Division team was deployed to install the machinery.

Part 1: Not Just LOST, but Very LOST! (Sub-total: 7 points)

You are an engineer of the Weapons Division team. In a moment of folly, you dropped your spanner down the hole that had been drilled through the planet. Knowing that losing your spanner is punishable by death, you immediately boarded the Very Low-Orbit Satellite Transport system (Very LOST) to the other side of the planet. Very LOST is a mass rapid transport system that uses low-orbit satellites instead of trains to transport people across the surface of the planet. This satellite transport system is only orbiting slightly above the surface of the planet, so slightly that it can be assumed that the radius of orbit is the radius of the planet. Surprisingly, this planet has a uniform density, unlike that of Earth.

(i) [1 point] Show that the time it would take for Very LOST to transport you from one side to the planet to the other side of the planet is given by $\tau = \pi\sqrt{\frac{R^3}{GM}}$.

(Note: This is also the time taken for the spanner to fall from the entrance of the hole to the exit of the hole. Thus the Very LOST is able to save your life!)

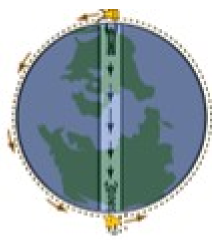

Figure 5: Pictorial representation of the journey of the spanner and Very LOST.

Being the careless engineer that you have always been, this time round, you dropped your spanner while riding the Low-Orbit Satellite Transport system (LOST) while surveying the planet. You gave the spanner enough speed to be out of your reach from LOST, while the spanner continues to orbit at nearly the same orbital radius, r, as LOST. Trying to save your life for the second time, you decide to take a space shuttle to reach an orbital radius of $r + \Delta r$, where Δr refers to a small difference in their orbital radii.

The period of the space shuttle is given by $T + \Delta T$, where T is the orbital period of LOST. The orbital speed of the space shuttle is given by $v - \Delta v$, where v represents the orbital speed of LOST at an orbital radius r.

(ii) [1 point] Show that the speed of LOST, v, at an orbital radius r, is given by

$$v = \sqrt{\frac{GM}{r}},$$

where M is the mass of the planet.

(iii) [3 points] Show that the difference in period of LOST and your space shuttle ΔT is given by

$$\Delta T = \frac{3\pi \Delta r}{v}.$$

(Hint: Try to obtain an expression that contains $\left(1 + \frac{\Delta r}{r}\right)^{\frac{3}{2}}$. Then, since $\Delta r \gg r$, hence $\frac{\Delta r}{r}$ is small. A binomial expansion from the Formula Booklet might help at this point.)

Note that using the binomial expansion, one can also show that the difference in speed of LOST and your space shuttle Δv is given by

$$\Delta v = \frac{\pi \Delta r}{T}.$$

On the space shuttle, the fear of not being able to catch the spanner with your Super Advanced Magnet (SAM) strikes you. Your plan is to wait for the spanner (which can be assumed to be at the same orbital radius as LOST), your space shuttle, and the planet to form a straight line to activate your SAM. However, being an engineer yourself, you start to worry and attempt to calculate the time at which such an alignment will next occur, if you by any chance happened to miss the upcoming perfect alignment.

(iv) [1 point] Explain why τ satisfies the equation

$$\frac{v\tau}{r} - \frac{(v - \Delta v)\tau}{r + \Delta r} = 2\pi.$$

(v) [1 point] The above expression simplifies to $\tau = \frac{2\pi r}{\Delta v + \frac{v\Delta r}{r}}$. By binomial expansion, this can be further simplified to

$$\tau = \frac{T^2}{\Delta T}.$$

What is the significance of this expression?

Part 2: Stellar Cannibalism **(Sub-total: 5 points)**

Starslayer Base was designed to consume stars as a power source via a method which drains off all radiation from the surface of the star. Seeking to exact revenge, Supreme Leader Snake ordered that Starslayer Base was to fire upon the Hasnian system where the Rebel Army had recently established the New Republic. Starslayer Base thus fired upon the Hasnian system and completely destroyed all 5 planets within the system. Note that these planets are Earth-like, i.e. we can assume that they each have the mass and radius of the Earth. Star-A1 was fully consumed to supply the energy to fire upon the Hasnian system.

You want to determine a lower bound on the mass of Star-A1 consumed to fire upon the Hasnian system to completely disintegrate the Hasnian system. You may assume Starslayer base has 100% efficiency of converting all the mass of Star-A1 into energy. You may assume that the planets are of uniform density.

(vi) [1 point] If Starslayer Base consumed a star and destroyed the Hasnian system, show that the minimum mass m of the star is

$$m = \frac{3GM^2}{R^2c^2},$$

where M and R refer to the mass and radius of the Earth respectively.

(Hint: Use Einstein's mass-energy equivalence.)

(vii) [1 point] Use the above equation to determine the minimum possible mass of Star-A1.

(Note: If you realise that the mass is too low, do not fret. Star Strife is parked under Science Fiction.)

The military advisers of Starslayer Base were discussing about habitability around Star-A2, as Star-A2 was the next best candidate star to be used to fire upon the Hasnian system. Habitability was discussed because the Starstrife Base must be positioned within the Goldilocks zone of Star-A2.

(viii) [1 point] Show that the surface temperature of a planet T, and the distance from the star to the planet r, is related by the proportionality relation $T \propto r^{-0.5}$.

(ix) [1 point] The Goldilocks zone of Star-A2 can be found by considering the zone to be bounded by the melting and freezing point of water at 1 atm. Show that the Goldilocks zone of Star-A2 is approximately 0.60 AU to 1.11 AU.

You may assume that Star-A2 has a surface temperature identical to the Sun. You should give your value as the radius from the centre of the star. Assume that planets in this zone has a similar albedo to Earth, and that the surface temperature of Earth is at 15° C.

A planet was recently discovered to be located at approximately 0.70 AU from Star-A2. However, upon thorough investigations conducted by the military advisers, they realised that this planet is inhabitable.

(x) [1 point] The military advisors claim you have made an error with your calculations. You disagree; the advisors have not looked at the assumptions of the calculation. But you need to convince them! Explain why this planet is inhabitable even though it lies within the Goldilocks zone.

Part 3: Life as an Engineer in an Astronomy Unit (Sub-total: 7 points)

While the military advisers were figuring out the suitability of Star-A2, Kylo Ben had convinced Supreme Leader Snake to check on the suitability of Star-B3. Several astronomers were commissioned to determine certain details about Star-B3. For the surveying process, Starslayer Base was positioned within the Goldilocks zone of Star-B3, and is confirmed to be habitable (remember, the base is a planet!).

It should be noted that Starslayer Base has a moon orbiting it (by some alien technology). Starslayer Base's moon is similar to Earth's moon in terms of mass and radius. It was also determined that Starslayer Base has a circular orbit about Star-B3 with an orbital radius of 0.5 AU. Starslayer Base's moon has an orbital semi-major axis of 1.4 times the orbital semi-major axis of the Earth's moon about the Earth. The orbital tilt of the moon is 15°.

After your harrowing experience as part of the weapons division team, you successfully requested to be transferred to the astronomy unit. Immediately, you found several tasks waiting for you.

(xi) [1 point] State two conditions for an eclipse to be annular.

(xii) [3 points] Determine the eccentricity value of the orbit of Starslayer Base's moon such that the angular size of the moon is the same as the angular size of Star-B3 during aphelion.

(Hint: To prevent you from failing this task, you secretly bought a 'hint' from Kylo Ben. He gave you the following diagram.

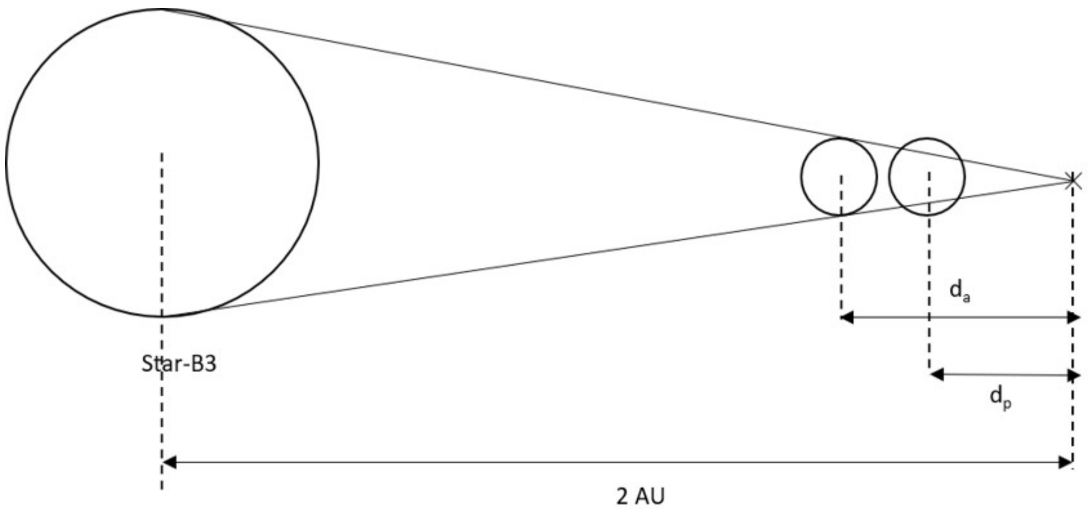

Figure 6: Kylo Ben has good drawing skills.

d_a is the distance of Starslayer Base from its moon at aphelion, and d_p is the distance of Starslayer Base from its moon during perihelion. Use geometric arguments to derive your answer.)

(xiii) [1 point] What is the astronomical interpretation of the value calculated in Part (xii)?

(xiv) [2 points] Compare the frequency at which solar and lunar eclipses are observed on Starslayer Base to that as observed on Earth. You may assume that the value of eccentricity of Starslayer Base's moon is the same value as calculated in Part (xii).

Question 4
Qualitative Astronomy and Cosmology
(Total: 20 points)

The following questions are meant to test your understanding of astronomy and cosmology. It is recommended that you answer the following questions in prose to demonstrate your understanding on the subject matter tested, especially for Part 1.

To be awarded the full score, you should discuss how relevant concepts are used and their contextual implications. You may draw labelled diagrams to assist you in your explanations. Marks may be allocated to relevant diagrams. Purely quantitative answers may not be awarded the full score, unless the question is specifically quantitative in nature.

Part 1: Don't Underestimate the Greeks (Sub-total: 12 points)

The Ancient Greeks were around long before Neil Armstrong first landed on the moon, but they were not in the dark as to the vastness of space. By completing the questions below, you should be able to tell that in spite of the minimal information preserved from that era, we know today that the Greeks were aware of certain astronomical facts.

(i) [1 point] The Greeks were aware of the phenomenon that objects in the distance appear to be smaller. Why do objects in the distance appear smaller?

(Note: You are encouraged to draw a diagram to help you explain.)

(ii) [1 point] The notion that the Earth is round is commonly attributed to Pythagoras of Samos. Give a plausible argument as to how Pythagoras might have known that the Earth was round.

(Hint: Your answer in Part (i) may help.)

Eratosthenes of Cyrene was a Greek astronomer. He is famous for being the first person to calculate the circumference of the Earth.

(iii) [1 point] Eratosthenes knew that the moon was very large. Using your understanding and/or answer to Part (i), explain how Eratosthenes also knew the moon was far away from the Earth.

(iv) [1 point] Eratosthenes also knew that the Sun larger than and further than the moon. Using your understanding of Parts (i) and (ii), explain how may have Eratosthenes might have known these two facts.

(v) [3 points] Using the knowledge that the sun was very far away from Earth, and the distance between two chosen points on Earth, Eratosthenes was able to calculate the circumference of the Earth. How did he do it?

(Note: You are encouraged to draw a diagram to help you explain.)

Hipparchus of Nicaea was able to measure the distance of the moon from the Earth using Eratosthenes's value for the circumference of the Earth.

(vi) [1 point] Draw a diagram to show your understanding of the umbra and penumbra.

(vii) [3 points] How was Eratosthenes able to obtain the distance of the moon from the Earth?

(Hint: Hipparchus was aware that the shadow of the Earth at the moon's orbit was 2.5 times larger than the moon. How did he know this?

He was also aware the shadow cast by the Earth stretches back 108 times the diameter of Earth. Again, how did he know this?

You are encouraged to draw a diagram to help in your explanation.)

It turns out that a century before Hipparchus's successful determination of the Earth-Moon distance, Aristarchus of Samos had devised a method to measure the Earth-Sun distance. Below is a simplified explanation of his method.

- Aristarchus made use of the moon and its phases. If the Sun was at an infinite distance, then the 1^{st} and 3^{rd} quarter phases would occur when the Moon-Earth line formed a right angle to the Earth-Sun direction.
- If the sun was not at infinity, then the quarter phases would occur at a smaller angle than $90°$.

The diagram below provides an exaggerated explanation of the above.

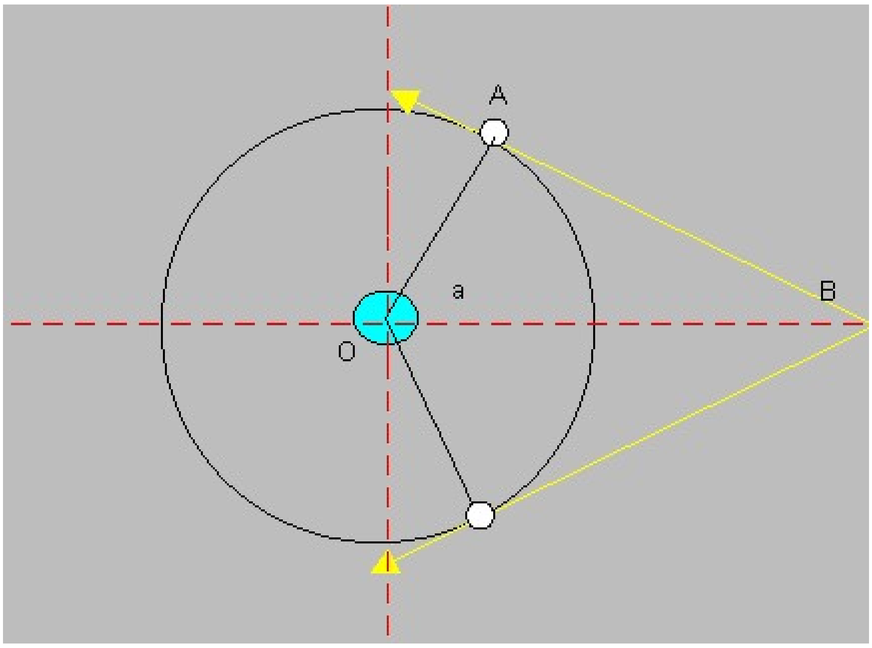

Figure 7: Diagram of Aristarchus's method.

Using this method, Aristarchus obtained a value of the Earth-Sun distance. Today, we know that the value he obtained was approximately 20 times too small.

(viii) [1 point] Where could Aristarchus have gone wrong?

Part 2: Cosmological Primer **(Sub-total: 8 points)**

This section contains questions that are meant to help you understand why certain cosmology theories have gained widespread acceptance. You may answer either qualitatively or quantitatively, wherever relevant.

Fritz Zwicky was a Swiss Astronomer who discovered the Coma cluster. Viewing the Coma cluster today (as Zwicky did back then), we realise that its member galaxies are all moving faster than the expected escape velocity. However, the member galaxies within the Coma cluster are not escaping from the cluster.

(ix) [2 points] Given that the gravitational effects within the Coma cluster can be resolved using Newton's law of gravitation, how might we resolve this apparent paradox? Assume that distance and velocity measurements are accurate.

Today, the Big Bang model has become the most accepted model to explain the evolution of the universe. The Big Bang model was one of the solutions to Einstein's field equations in general relativity. The following is a solution commonly used to describe the Big Bang model, and is called Friedmann's equation. It defines how the universe derives its expansion.

$$H^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2},$$

where

- H is the Hubble parameter today (i.e. the rate of expansion of the universe today),
- K is a value that accounts for the geometry (curvature) of the universe; if $K = 0$ the universe is flat, if $K > 0$ the universe is closed (similar to a sphere), and if $K < 0$ the universe is open (similar to a saddle),
- a is the scale factor (i.e. it describes the relative expansion of the universe), and
- ρ is the total energy density of the universe.

Suppose the universe is flat, i.e. $K = 0$.

(x) [2 points] Modify Friedmann's equation such that ρ is the subject of the equation, and explain the significance of this value of ρ.

(xi) [1 point] ρ is a theoretical value and we want to compare it to observational results. Let Ω be the ratio of ρ' to ρ, where ρ' is the observed total energy density of the universe. Express Ω in terms of ρ'.

(xii) [3 points] Ideally, we expect that $\Omega = 1$. However, we have found that $\Omega < 1$. Derive a similar equation for H^2, taking into account the missing energy density.

(Hint: You may want to work backwards from Part (xi), to Part (x), to the equation in the preamble.)

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Question 5
Practical Astronomy
(Total: 20 points)

Part 1: The Night Sky **(Sub-total: 13 points)**

Refer to the image of the night sky at an unknown location on the next page.

(i) On the image, answer the following questions.

(a) [2 points] Identify the cardinal points.

(b) [1 point] Trace out the 'Little Dipper' and label it accordingly.

(c) [1 point] Trace the 'Great Square of Pegasus' and label it accordingly.

(d) [2 points] Trace out two major complete IAU constellations that are visible (other than Cygnus, Ursa Major, and Ursa Minor).

(e) [2 points] Mark the positions of two prominent nebulae that are visible and label them accordingly.

(f) [2 points] Mark the positions of two prominent open clusters that are visible (except Hyades) and label them accordingly.

(g) [2 points] Mark the approximate positions of two prominent galaxies that are visible and label them accordingly.

(*Note: You should detach the image provided and attach it to your answer script.*)

(ii) [1 point] Approximately, what is the latitude of this location?

Part 2: Looking at the Moon **(Sub-total: 7 points)**

Tired of looking at deep-sky objects, you decide to view a 3 km wide crater on the Moon with your 50 mm telescope of focal length 600 mm.

(iii) Determine the following quantities if possible. If it is not possible, explain why.

(a) [2 points] The focal ratio of the telescope.

(b) [2 points] The magnification of the crater when viewed though the telescope.

(iv) [3 points] Considering the angular diameter of the crater, can your telescope resolve the crater? Explain your answer. You may assume that light in the visible spectrum has a wavelength of 500 nm and that the Moon is 384400 km away from Earth.

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Detach this page and attach it to your answer script.

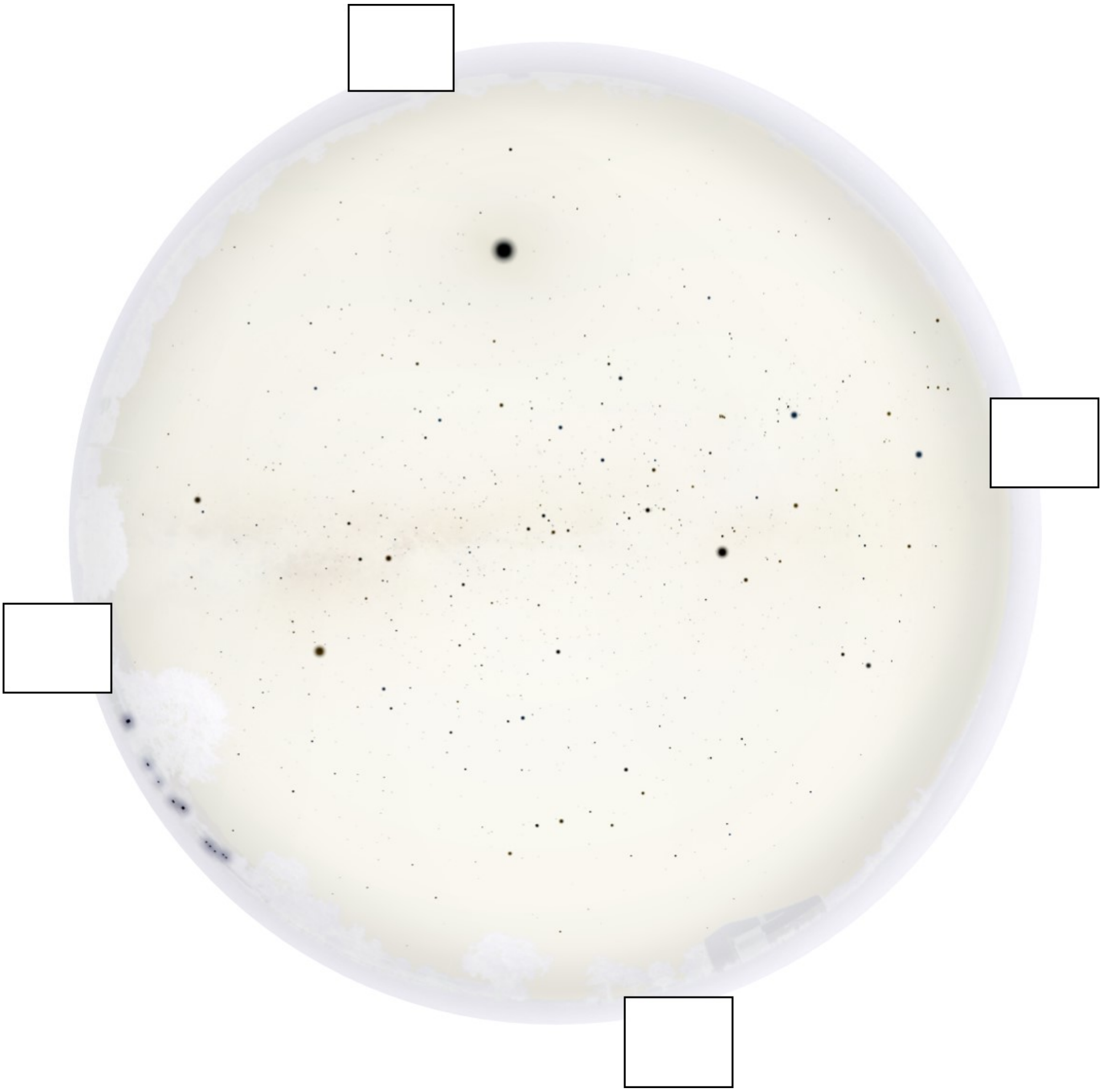

Figure 8: The night sky at some location.

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Appendix A

The following is the radioactive decay chain of uranium-238.

$$
{}^{238}_{92}\mathbf{U} \xrightarrow[4.468\times10^{9}\ \mathrm{y}]{\alpha} {}^{234}_{90}\mathbf{Th} \xrightarrow[24.1\ \mathrm{d}]{\beta^-} {}^{234\mathrm{m}}_{91}\mathbf{Pa} \left\{ \begin{array}{c} \xrightarrow[1.17\ \mathrm{min}]{0.16\%} {}^{234}_{91}\mathbf{Pa} \xrightarrow[6.7\ \mathrm{h}]{\beta^-} \\ \xrightarrow[1.17\ \mathrm{min}]{99.84\%\ \beta^-} \end{array} \right\} {}^{234}_{92}\mathbf{U} \xrightarrow[2.445\times10^{5}\ \mathrm{y}]{\alpha} {}^{230}_{90}\mathbf{Th} \xrightarrow[7.7\times10^{4}\ \mathrm{y}]{\alpha} {}^{226}_{88}\mathbf{Ra} \xrightarrow[1600\ y]{\alpha} {}^{222}_{86}\mathbf{Rn}
$$

$$
{}^{222}_{86}\mathbf{Rn} \xrightarrow[3.8235\ \mathrm{d}]{\alpha} {}^{218}_{84}\mathbf{Po} \xrightarrow[3.05\ \mathrm{min}]{\alpha} {}^{214}_{82}\mathbf{Pb} \xrightarrow[26.8\ \mathrm{min}]{\beta^-} {}^{214}_{83}\mathbf{Bi} \xrightarrow[19.9\ \mathrm{min}]{\beta^-} {}^{214}_{84}\mathbf{Po} \xrightarrow[164.3\ \mu\mathrm{s}]{\alpha} {}^{210}_{82}\mathbf{Pb} \xrightarrow[22.26\ \mathrm{y}]{\beta^-} {}^{210}_{83}\mathbf{Bi} \xrightarrow[5{,}013\ \mathrm{d}]{\beta^-} {}^{210}_{84}\mathbf{Po} \xrightarrow[138.38\ \mathrm{d}]{\alpha} {}^{206}_{82}\mathbf{Pb}
$$

Each of the arrows here represents one decay reaction. The nuclear equations for the first three steps are given below.

$$
\begin{aligned}
{}^{238}_{92}\mathrm{U} &\to {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}, \\
{}^{234}_{90}\mathrm{Th} &\to {}^{234\mathrm{m}}_{91}\mathrm{Pa} + {}^{0}_{-1}\beta, \\
{}^{234\mathrm{m}}_{91}\mathrm{Pa} &\to {}^{234}_{92}\mathrm{U} + {}^{0}_{-1}\beta.
\end{aligned}
$$

Note here that we approximate that ${}^{234\mathrm{m}}_{91}\mathrm{Pa}$ decays straight into ${}^{234}_{92}\mathrm{U}$.

Example

To determine the energy released from one nuclear reaction, for instance, in the equation

$$^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He},$$

We first calculate the mass defect Δm. This is calculated by taking the total mass of the reacting nuclei, subtracted by the total mass of the product nuclei. For the equation above,

$$\Delta m = 238.051u - 234.044u - 4.002602u = 0.004398u.$$

Note that by the formula booklet, $1u = 1.660539 \times 10^{-27}$ kg. The total energy released is calculated using Einstein's mass-energy equivalence equation

$$E = (\Delta m)c^2.$$

Hence, for the reaction above,

$$E = (0.004398 \times 1.6605 \times 10^{-27})(2.9979 \times 10^8)^2 = 6.564 \times 10^{-13}\,\mathrm{J}\,.$$

Multi-stage Reactions

For a multi-stage nuclear reaction like

$$\begin{aligned} {}^{238}_{92}\mathrm{U} &\rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}, \\ {}^{234}_{90}\mathrm{Th} &\rightarrow {}^{234\mathrm{m}}_{91}\mathrm{Pa} + {}^{0}_{-1}\beta, \end{aligned}$$

instead of calculating the total energy released in each equation individually, is there a shorter way of calculating the total energy released in this equation without using the mass of $^{234}_{90}\mathrm{Th}$?

Figuring out this "shortcut" will help you in your task for Question 2 Part (iii).