AstroChallenge 2021
Junior Team Round

SOLUTIONS

Monday 7th June 2021

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. This paper consists of **42** printed pages, including this cover page.
2. You are required to keep your microphone and camera on at all times throughout the round.
3. You are not allowed to use your keyboard at all times, but you may use your mouse to scroll through the question paper as well as switch to the formula booklet.
4. Any materials other than the Question Paper, Formula Booklet, and **ONE** A4-sized cheat sheet held by **ONE** team member only, are strictly prohibited.
5. You have **2 hours** to attempt all questions in this paper.
6. Write your answers on blank pieces of A4 paper or graph paper. Do **NOT** mix solutions for different questions on the same sheet of paper.
7. You will be given time after the paper to collate your answers. You should collate your answers into **separate PDF files** for each question.
8. It is *your* responsibility to ensure that your answer scripts have been submitted.
9. The marks for each question are given in brackets in the right margin, like such: [**2**].
10. The **alphabetical** parts (i) and (l) have been intentionally skipped, to avoid confusion with the Roman numeral (i).

Question 1 Short Answer Questions

Part I HR Diagrams

For this part, you are given the following H-R diagram.

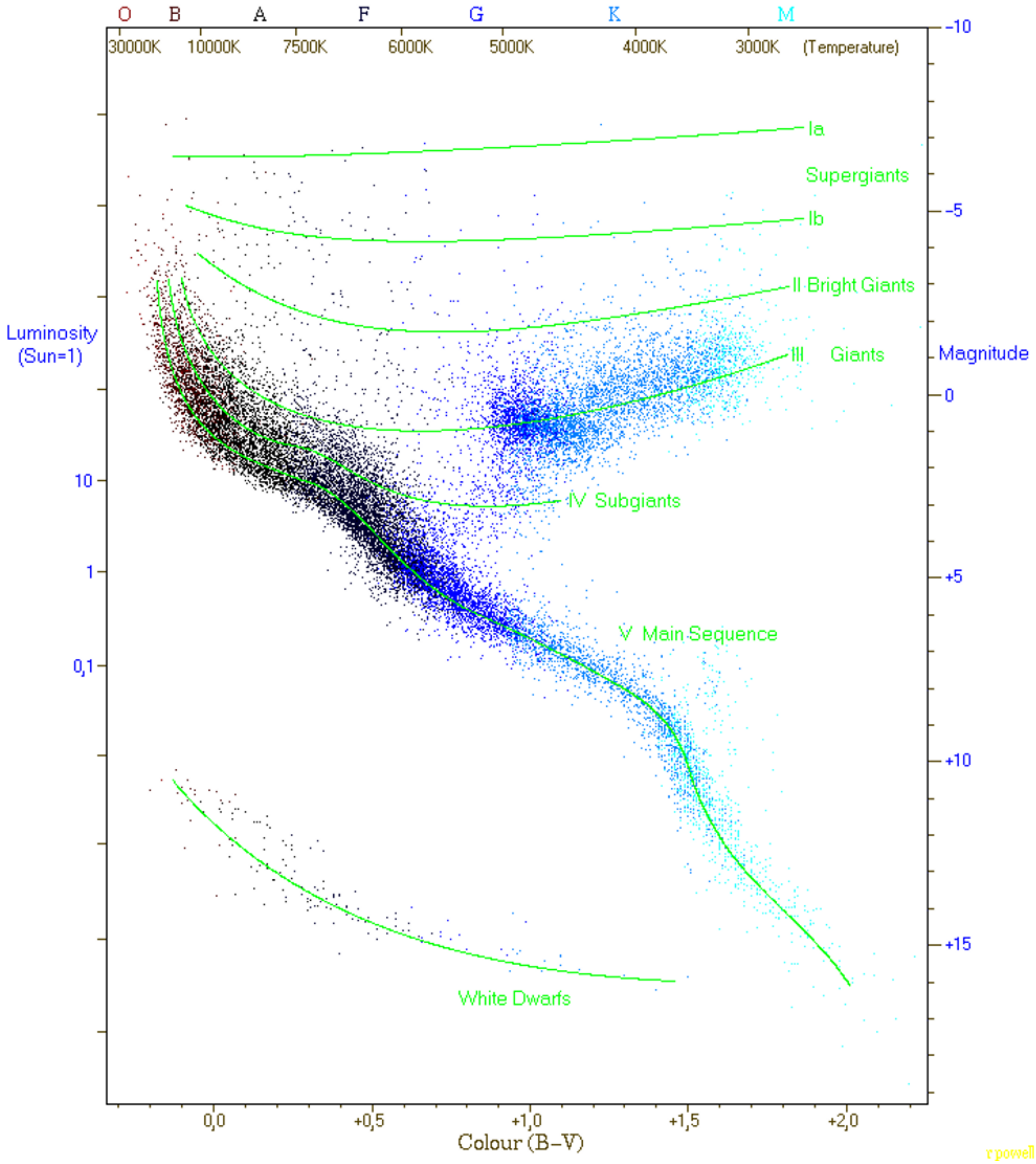


Figure 1: H-R diagram.

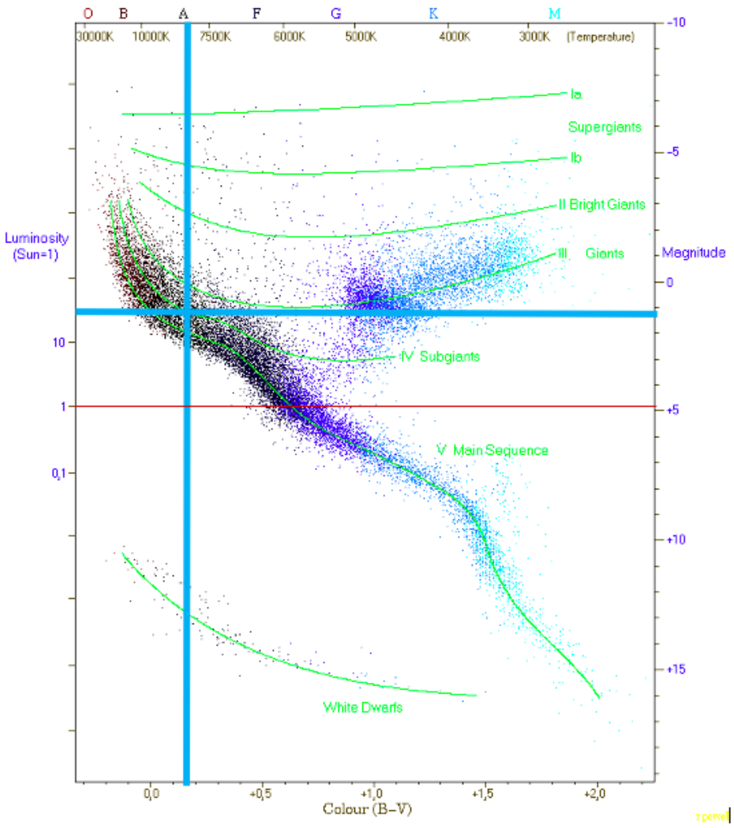
You are also given the following star.

Name	X Astrochallengae
Right Ascension	2h 30min 49s
Declination	67°28′43″
Distance	142ly
Apparent Magnitude	4.5
Absolute Magnitude	1.31
Magnitude Difference (B-V)	0.15

Table 1: Information regarding X Astrochallengae.

(a) With the aid of the H-R diagram (Figure 1), deduce the spectral class as well as the luminosity of X Astrochallengae[1]. **[2]**

Solution:



See the diagram above. We can read off the luminosity and spectral class.

(b) Name the spectral class of the Sun. **[1]**

[1] X Astrochallengae is actually Iota Cassiopeiae.

Solution:

G2V.

(c) Using the information in Table 1 as well as your own knowledge, compare the Sun with X Astrochallengae in terms of radius as well as lifespan. **[3]**

Solution:

From the H-R diagram, we observe that X Astrochallengae is bluer in colour. Furthermore, it is a subgiant and thus is expected to have a larger radius than that of the Sun. Since it is more massive in space, its lifespan is expected to be shorter.

Note: The information for radius and mass can be obtained from the H-R diagram as well.

(d) How can one use a H-R diagram such as Figure 1 to determine the distance to a star cluster? **[2]**

Solution:

Spectroscopic Parallax. We can observe the star cluster spectrum to obtain the spectral type. Its absolute magnitude can then be determined because stars with similar spectral classes have similar luminosities. We can also obtain the star cluster's apparent magnitude, and then use the distance modulus to calculate its distance.

An alternative answer is main sequence fitting.

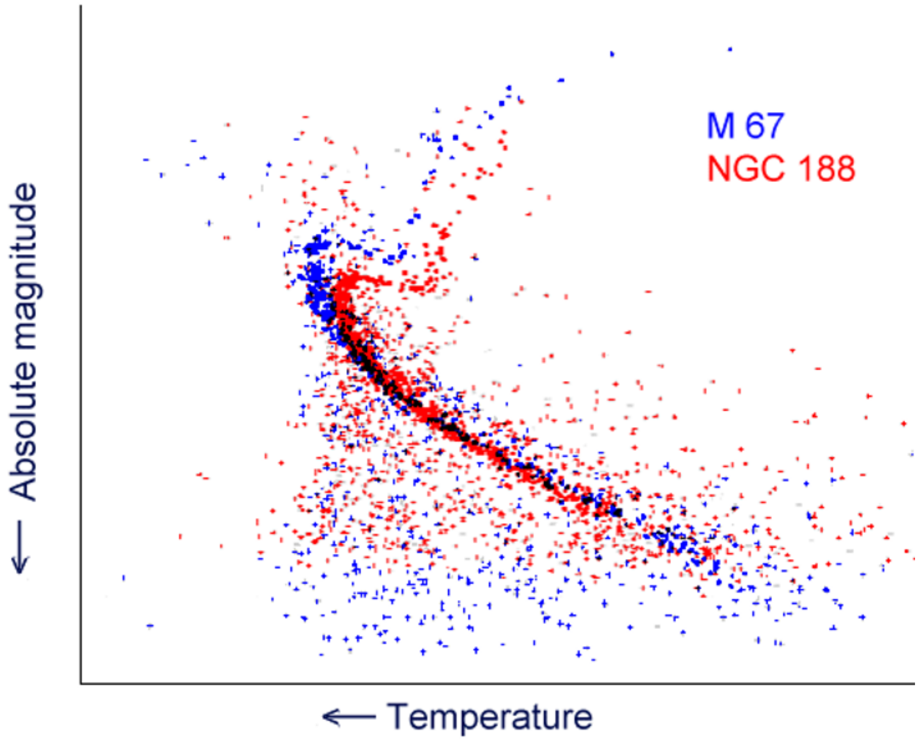


Figure 2: Main sequence turnoff points for two different star clusters

(e) Figure 2 shows examples of a main sequence turnoff point for two different star clusters. How can one deduce the age of a star cluster using the turnoff point? **[2]**

Solution:

As the age of a cluster decreases, the mass of starts located on the main sequence turnoff point decreases. As such, the older the cluster, the further down the Main Sequence (more down-right) the turnoff point will be observed.

Part II Dubious Statements

This part comprises 5 statements. For each statement, indicate clearly whether it is **TRUE** or **FALSE**.

Support your answer with no more than 3 sentences, including any assumptions where required. You may draw up to one additional diagram if they aid your explanation.

Mathematical working is not required, and there are no errors in any of the statements below.

Each statement is worth 2 marks, **attributed only to the quality of the justification**.

(f) The most important factor to determine the visibility of an object is its absolute magnitude. **[2]**

Solution:

The surface brightness of an object is the most significant factor affecting its visibility, especially in light polluted regions. Notably, apparent magnitude is sometimes wrongly cited as the answer.

(g) Pluto is not a planet because it has a large moon orbiting around it. **[2]**

Solution:

This statement is false. Pluto is not considered a planet because it does not fulfil the conditions stipulated by the International Astronomical Union (IAU). To be considered a planet, three conditions must be satisfied:

1. The body must orbit the Sun
2. The body must have sufficient mass to attain hydrostatic equilibrium
3. The body must have "cleared" its neighbourhood

(h) Hydrogen is the most abundant element everywhere in the Universe, including on Earth. **[2]**

Solution:

Hydrogen is not the most abundant element on Earth. However, when considering our solar system as a whole, hydrogen is the most abundant element. Similarly, this consideration applies for other star systems.

(j) Generally, because nuclear fusion produces more energy than nuclear fission, stars less massive than the Sun are powered by fission while stars more massive than the Sun are powered by fusion. **[2]**

Solution:

All stars are powered by nuclear fusion.

(k) Exoplanets orbiting a Sun-like star at the orbit of Mercury cannot have an atmosphere. **[2]**

Solution:

Exoplanets with a significant atmosphere have been detected orbiting their parent start at a close proximity. One such example is Hot Jupiters, which are a class of gas giant exoplanets that have very short orbital periods.

Question 2 History of Astronomy

Humans have been looking up at the sky since antiquity and have always attempted to make sense of what rules the heavens. In this question, you will be taken through a brief journey of the history of astronomy and explore the discoveries and inventions made by historical figures, all of which eventually led to the domain of astronomy that we are familiar with today. While we will be focusing mostly on astronomical progress in Western civilisations, we must also appreciate that several other civilisations too have made outstanding contributions to modern astronomy.

Part I We're So Very Small, in the End

Astronomy has had a long history. In the West (European civilisations), astronomy emerged as the first branch of natural sciences to form a robust (but not necessarily correct) system of explanations for how the night sky behaved. The first to do so comprehensively were the ancient Greek philosophers, who had a keen eye in observing the heavens. They noticed some stars in the heavens always seemed to be moving against the other stars, hence naming them *πλανήτης* (planetes), meaning 'wanderer'. This eventually evolved to the word 'planets' which we are familiar with today. The ancient Greeks eventually came up with a system of explanation for the motion of the planets, now known as geocentrism. The below diagram is an exemplification of the Ptolemaic system of geocentrism, known to many as the height of geocentrist models.

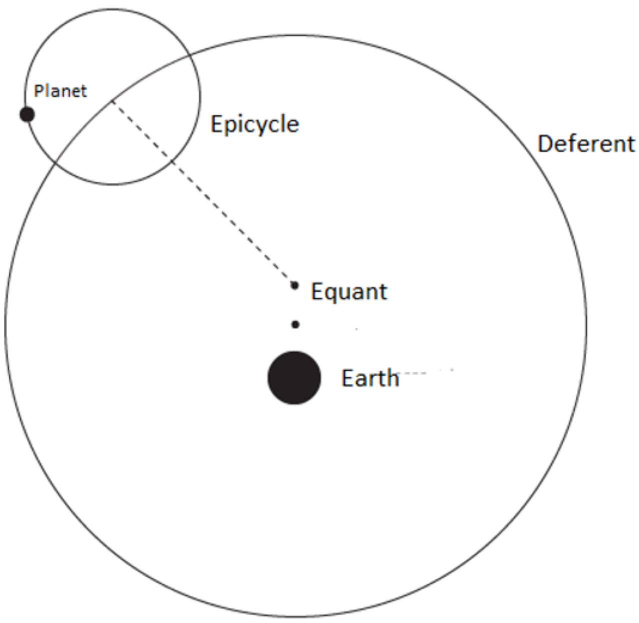


Figure 3: The Ptolemaic system.

As mentioned above, the ancient Greeks observed that planets curiously seem to move backwards at times, counter to their general direction of motion. This effect is now known as *retrogression.* In order to explain the observed phenomenon of retrogression, the ancient Greeks came up with the idea of *epicycles.* Epicycles are circles located along the larger orbits (formally called the *deferent*).

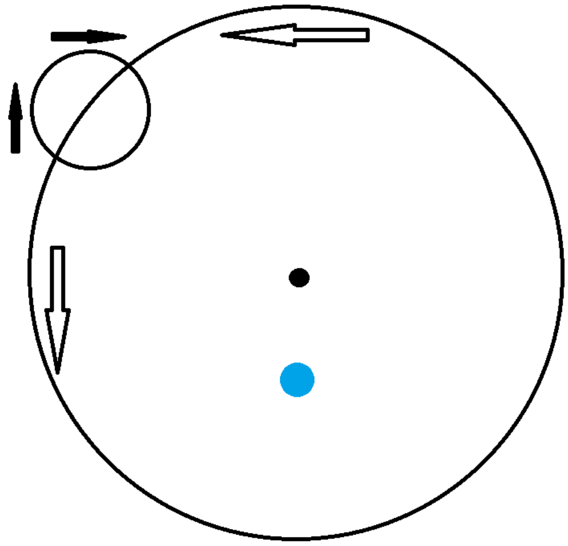
(a) In Figure 3, we assume that we are viewing Earth from the North pole of the ecliptic. Copy Figure 3 and do the following:

(i) Indicate with arrows the direction of the epicycle along the ecliptic and the motion of the planet around the ecliptic which results in retrogression.

(ii) Mark out the part of the epicycle which will see the planet in retrogression.

[2]

Solution:



(b) Using your current knowledge of astronomy, describe how the phenomenon of retrogression relative to Earth-bound observers actually occurs. You may use a diagram to aid your explanation. **[2]**

Solution:

As Earth and an exterior planet orbit the Sun, both their orbital positions will be similar relative to the Sun. However, Earth will be travelling at a higher tangential velocity relative to the planet superior of Earth. Since the Earth travels at a higher velocity in the same resolved vector component's direction, Earth will "overtake" the planet, and the planet will appear to be moving backwards against the background stars, which is the phenomenon of retrogression.

Due to the apparent simplicity and elegance of the geocentrist model, it was recognised as the mainstream model of the universe for over a millennium. It was not until the 16th century that the *De revolutionibus orbium coelestium*, written by Nicolaus Copernicus (1473–1543), was published, reintroducing the neglected idea of heliocentrism to the world.

Copernicus' model of the universe, known as the *Copernician system*, was such that the Sun was at the centre of the Universe, with the planets, including Earth, revolving around the Sun. The diagram below is a representation of the Copernican system.

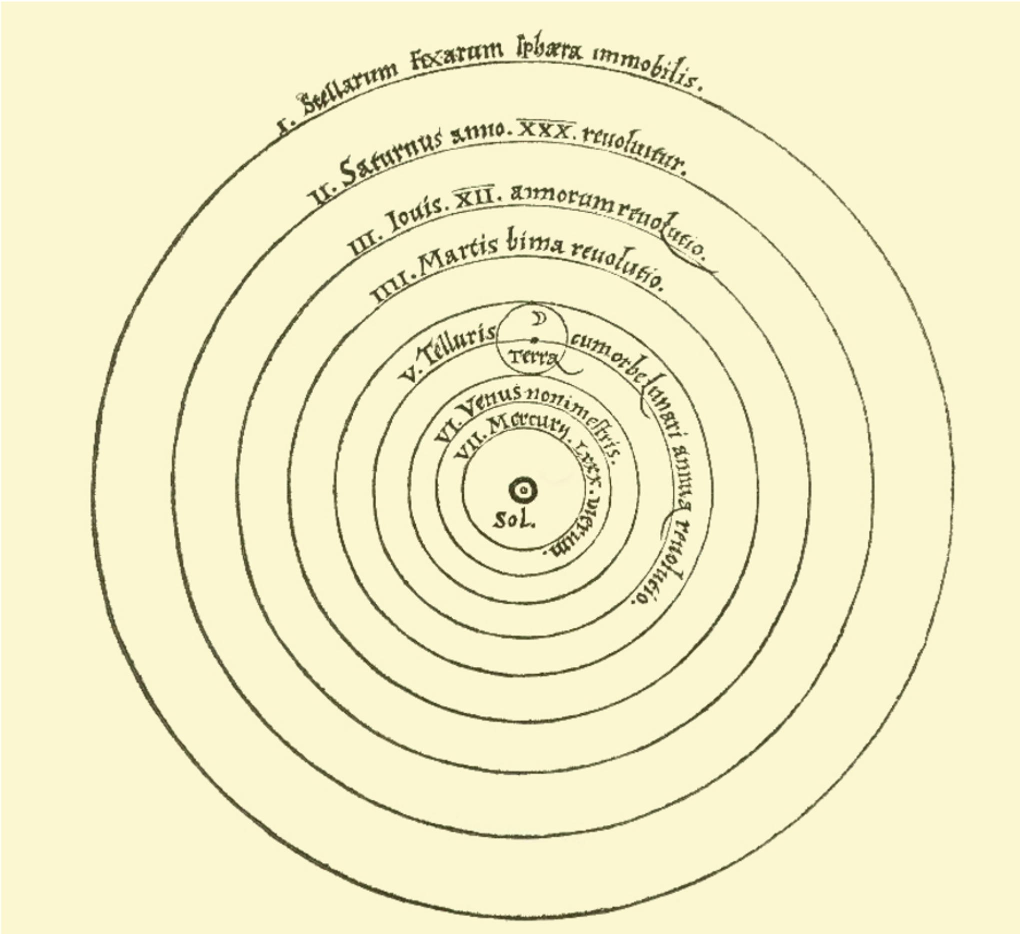


Figure 4: The Newtonian system.

During that early era, heliocentrism was met with heavy criticism. One notable opponent was Tycho Brahe (1546–1601). Tycho Brahe was one of the greatest observational astronomers of the pre-telescopic era, possessing sophisticated instruments to aid him in his observation of stellar positions. He argued that since he was unable to observe parallax in any of the stars, hence Copernicus' theory must necessarily be wrong. It was not until 1838 that German astronomer Friedrich Bessel (1784–1846) performed the first observation of stellar parallax and estimated the distance to the star system 61 Cygni.

(c) In 1838, over the course of a year, Friedrich Bessel observed 61 Cygni constantly, eventually detecting a maximum displacement of 0.314 arcseconds in its position in the night sky. You are given that the Earth orbits the Sun at approximately 150 million kilometres. Using this information, draw a diagram to exemplify the idea of parallax and use it to calculate the approximate distance to 61 Cygni as obtained by Friedrich Bessel. Leave your answer in light years. Note that one light year is approximately 9.461×10^{12}km. **[3]**

Solution:

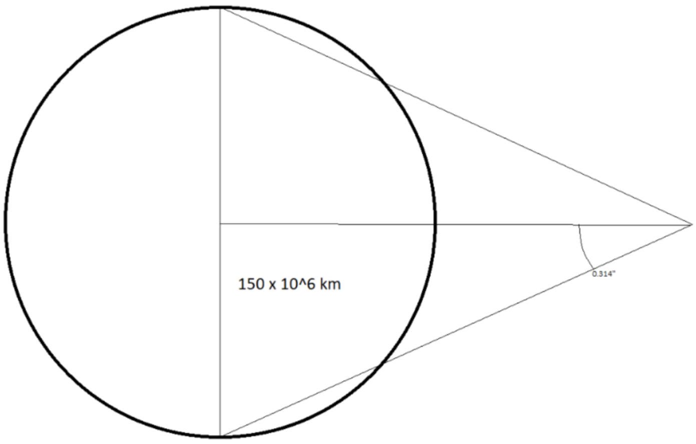
We first convert to degrees. This then becomes a trigonometry problem.

$$0.314'' = 8.7222 \times 10^{-5\circ}$$

$$\tan\left(8.7222 \times 10^{-5\circ}\right) = \frac{160 \times 10^6}{D}$$

$$D = 9.8534 \times 10^{13}\text{km}$$

$$= 10.4\text{ly}$$



(d) The true distance to 61 Cygni is now known to be closer to 11.4 light years. Give a reason why this discrepancy might have arisen. **[1]**

Solution:

Atmospheric diffraction of light from the star.

OR

Imprecision of instrument.

Part II The Good Ol' Days

Let us now return to the earlier days of astronomy. We turn our attention to another notable genius of that era, Johannes Kepler, best known for developing the three laws of planetary motion. Let us focus on his third law which eventually leads to the theorem that, for a circular orbit, $T^2 \propto r^3$, where T is the orbital period of a planet and r is the radius of said planet's orbit.

(e) Assume for simplicity that the orbits of Earth, Jupiter, and Saturn are all perfectly circular. Based on Kepler's third law, given that Jupiter is 5.2044 AU from the Sun with an orbital period of 11.862 Earth years, and that Saturn's orbital period is 29.457 Earth years, what is the distance of Saturn from the Earth at opposition? Give your answer in AU. **[1]**

Solution:

We can simply use Kepler's third law for this. We can avoid the proportionality constants if we take the ratio so

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

$$\left(\frac{29.457}{11.862}\right)^2 = \left(\frac{r_1}{5.2044}\right)^3$$

$$r_1 = 9.54\text{AU}$$

Since the question is asking for Saturn's distance from Earth, we substract the Earth-Sun distance

$$9.54\text{AU} - 1\text{AU} = 8.54\text{AU}$$

As time progressed, telescopes were invented and Galileo Galilei (1564–1642) became the first person to turn the invention towards astronomical observations in 1609. He was the first person to note craters on the Moon and note the satellites of Jupiter, uncovering concrete evidence that would ultimately overturn geocentrism. Telescope technology spread widely across Europe in the following decades, with many more adopting them to peer into the heavens.

(f) Jupiter's satellite Io is known in modern astronomy to be the most volcanic world in the Solar System. Explain how its volcanism arises. **[1]**

Solution:

Tidal heating from gravitational interactions with Jupiter as well as Europa and Ganymede in orbital resonance.

(g) Other major satellites of Jupiter are not known to be as volcanic in nature. Explain the difference. **[1]**

Solution:

There is little to no tidal heating due to orbital resonance occurring within the orbits and less gravitational interactions without the orbit.

(h) At that point in time, telescopes were mostly simple refractors which suffered from many issues. After telescopes became widespread, its technology and diversity also advanced rapidly, with Sir Isaac Newton inventing the Newtonian telescope in 1668. This design marked a significant improvement over its predecessors for a variety of reasons. Name two such reasons. **[2]**

Solution:

No chromatic aberration.

OR

Cost-effectiveness.

OR

Weight-effectiveness.

OR

Longer focal lengths more easily created due to internal reflection.

Note: Any 2 of the above is sufficient.

The first renowned usage of the Newtonian telescope was by Sir William Herschel, a musician-turned-astronomer. He developed skills for mirror polishing, and eventually built telescopes for himself and his sister, Caroline Herschel, to pursue their astronomical work. From 1785 to 1789, Herschel built the world's largest telescope of the time, with an aperture of 48 inches (1.22m) and a focal length of 40 feet (12.2m), giving a focal ratio of f/10. The primary mirror was craft out of speculum, an alloy of copper and tin, polished in such a way to make it reflective. A speculum mirror can reflect up to 66% of light that hits it.

(j) Modern mirrors, commonly made of fused quartz, regularly reflect up to 94% the amount of light that hits the primary in a reflecting telescope. Compared to the speculum mirror of the 40-foot telescope, what aperture of a fused quartz mirror would reflect the sameamount of light? You may leave your answer in either inches or metres for this question only. **[2]**

Solution:

The units of light reflected off the speculum mirror is 0.66. The units of light reflected off the quartz mirror is reflected off the quartz mirror is 0.94. This is proportional to the half of the aperture squared so

$$\frac{0.66}{0.94} = \frac{\left(\frac{A}{2}\right)^2}{\left(\frac{48}{2}\right)^2}$$

$$A = 40.2\text{inches}$$

$$= 1.02\text{m}$$

(k) However, the speculum mirror, with its aperture, will still have one advantage over the quartz mirror due to its larger aperture. What advantage is it? **[1]**

Solution:

The larger aperture will allow for an improved resolution of small objects.

A notable legacy of William and Caroline Herschel, as well as William's son, John Herschel, is their records of celestial objects which would eventually be compiled to form the New General Catalogue (NGC). The NGC remains one of the most notable and comprehensive catalogue of deep-sky objects till this day.

Another renowned catalogue that is widely referenced in the astronomy community is the Messier Catalogue (M). It was compiled by French astronomer Charles Messier (1730–1817) and his assistant, Pierre Méchain (1744–1804). This catalogue started out as an endeavour by Messier to sieve out celestial objects that resembled comets, but were eventually found not to be comets.

(l) State one reason why the observation of a comet could potentially be confused with that of a deep-sky celestial object. Then state how the comet can be distinguished from the deep-sky object. **[1]**

Solution:

A comet often appears as a faint, fuzzy object when observed, just like deep-sky objects (DSOs). Howveer, a comet will appear to move against background stars, while DSOs are stationary.

Part III The Rabbit Hole is Truly Deep

In 1924, Edwin Hubble (after whom the Hubble Space Telescope is named) discovered that the distance to the Andromeda Galaxy (M31) was far greater than ever had been thought, extending far beyond the Milky Way. This conclusion was made through repeated measurements of a special type of star in the Andromeda Galaxy, called a Cepheid variable. In particular, the star he observed was a classical Cepheid known as V1. Cepheid variables are a class of stars that are known to oscillate regularly in observable brightness, corresponding to its luminosity.

The diagram below shows the brightness data for the star V1, as measured by the American Association of Variable Star Observers (AAVSO) and the Hubble Space Telescope.

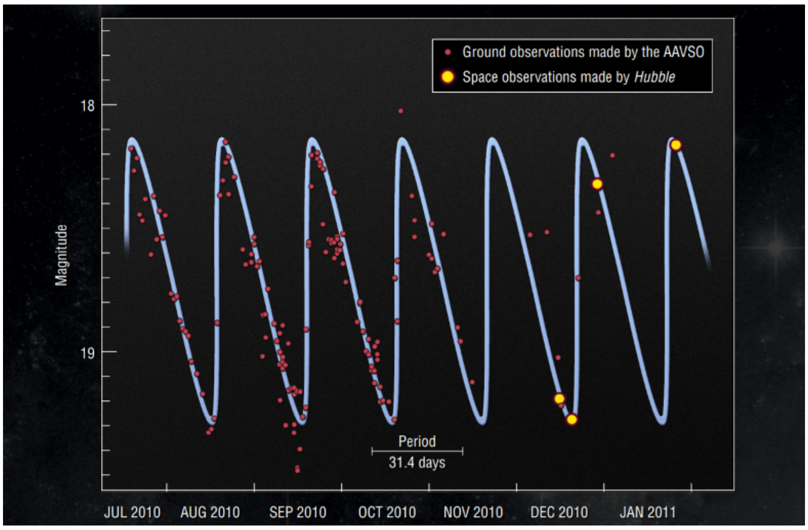


Figure 5: Brightness data for star V1.

The diagram below shows the average relationship between the period of a Cepheid's magnitude oscillation and its magnitude.

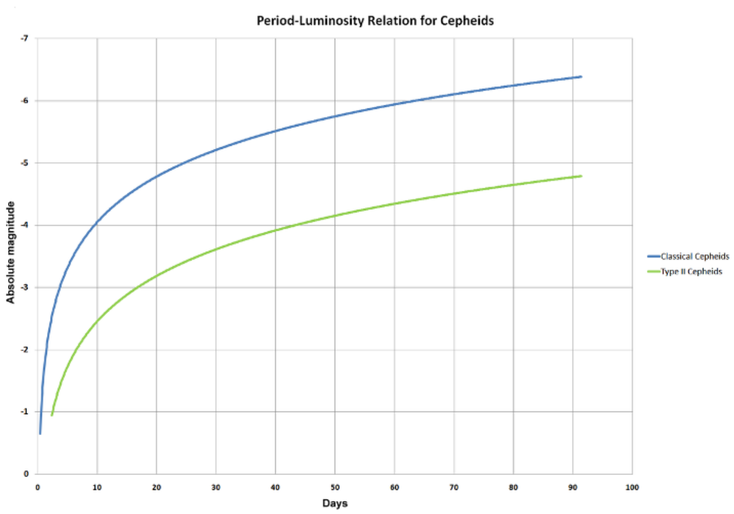


Figure 6: The average relationship between the period of a Cepheid and its magnitude.

(m) Approximate the average apparent magnitude of V1, as well as its absolute magnitude. Hence by comparing these two values, estimate the distance to V1 and therefore to the Andromeda Galaxy. **[3]**

Solution:

The average apparent magnitude is 18.7 and the estimated absolute magnitude is -5.3. The difference in magnitude is $18. - (-5.3) = 24$, which corresponds to a difference in flux of 2.5118^{24} times. By the inverse square law, the distance to the Andromeda Galaxy is given by

$$\frac{1}{r^2} = \frac{1}{2.5118^{24}}$$

This gives $r = 63070$ which is the ratio of the distance to Andromeda against 10 parsecs, where absolute magnitude is measured. The approximate distance is hence $630700\text{pc} = 2.05 \times 10^6\text{ly}$.

OR

The distance modulus gives

$$m - M = 5\lg\left(\frac{d}{10}\right)$$
$$18.7 - (-5.3) = 5\lg\left(\frac{d}{10}\right)$$
$$d = 631000\text{pc} = 2.05 \times 10^6\text{ly}$$

Note: Any similar answers may be accepted so long as the student has displayed understanding of the premise of the question and the underlying concepts.

Question 3 Stephan's Quintet

Part I Cosmological Redshift

Stephan's Quintet is a visual grouping of five galaxies, four of which are physically interacting with one another. We will name the galaxies arbitrarily as A, B, C, D, and E for now. Their redshifts are given in Table 2 below.

Galaxy	Redshift z
A	2.20×10^{-2}
B	2.21×10^{-2}
C	1.92×10^{-2}
D	2.25×10^{-2}
E	2.63×10^{-3}

Table 2: Redshifts of the 5 galaxies

(a) Which of the five galaxies is the odd one out? **[1]**

Solution:

Galaxy E.

Note: It has significantly less redshift than the other four galaxies.

(b) Suggest a reason for its difference from the others. **[1]**

Solution:

It is a foreground galaxy.

OR

It is significantly nearer to us than the other four galaxies.

OR

It is leaving the main group (at high velocities) and approaching us / moving in our direction.

Note: Hubble's law states that the further the galaxies they are from us, the greater the velocity at which they recede from us (and thus the greater their cosmological redshift). Therefore we can deduce that Galaxy E, with a lower redshift value, must be much closer to us than the other four galaxies. Another possible reason for the lower redshift (though not the actual reason inreality) is that the galaxy is actually travelling in our direction. Accept any other answers in the same vein. (I.e. The other four galaxies are in the background of the image.) "It has a lower redshift" is not accepted as that is simply stating the difference without suggesting the reason behind it.

(c) Calculate the recessional velocity in $\mathrm{kms^{-1}}$ for the galaxy you have chosen in part a. **[1]**

Solution:

To calculate the recessional velocity for galaxy E, we can just use

$$zc = v$$

where z is the redshift, c is the speed of light and v is the recessional velocity. This gives $v = 789\mathrm{kms^{-1}}$ when we substitute in the numbers. Note that the value of redshift, z is dimensionless because it is the ratio of change from the emitted to observed wavelength to the emitted wavelength.

Note: There will be e.c.f, so marks are warded if the correct formula was still used.

We shall now assume that emitted light from galaxy you have chosen in part a is being redshifted solely due to the cosmological expansion of the Universe.

(d) Based on this assumption, and the recessional velocity calculated in part c, calculate the proper distance between Earth and this galaxy. Give your answer in light years up to 3 significant figures. **[2]**

Solution:

We can use Hubble's law here. This gives

$$\begin{aligned} v &= H_0 D \\ D &= \frac{v}{H_0} \\ &= \frac{789\text{kms}^{-1}}{67.80\text{kms}^{-1}\text{Mpc}^{-1}} \\ &= 11\frac{72}{113}\text{Mpc} \\ &= 3.80 \times 10^7\text{ly} \end{aligned}$$

Note: There will be e.c.f here as well, unless the value obtained in the previous question is off by too many magnitudes to the existent that it is physically impossible.

(e) Explain why this assumption must be made to achieve a more accurate value of the proper distance of this galaxy from Earth. **[1]**

Solution:

The galaxy could also be affected by local gravitational interactions that will change its radial velocity and redshift.

OR

The galaxy's redshift consists of gravitational and relativistic redshifts as well.

(f) Assuming that this galaxy was formed immediately at the start of the Universe, and that it had maintained a constant recessional velocity throughout its lifetime, calculate the estimated age of the Universe using the proper distance calculated in part d. Give your answer in years up to 3 significant figures. **[2]**

Solution:

$$\text{AgeofUniverse} = \frac{\text{ProperDistance}}{\text{RecessionalVelocity}}$$

Substituting in values from the previous two parts gives 1.44×10^{10} years.

From part f, we can glean some insights into the relation between the age of the Universe and the Hubble constant.

(g) Show that the age of the Universe can be expressed by the following equation, where H_0 is the Hubble's constant:

$$\text{Age of Universe } = \frac{1}{H_0}$$

[1]

Solution:

Using the formula

$$\begin{aligned} \text{AgeofUniverse} &= \frac{D}{v} \\ &= \frac{1}{H_0} \end{aligned}$$

We can see that the age of the universe can indeed be expressed as a reciprocal of Hubble's constant.

(h) Assuming our Universe's mean density is slightly lower than the critical density, state how Hubble's "constant" will change as the age of our Universe tends towards infinity. **[2]**

Solution:

The value of Hubble's "constant" will decrease at a decreasing rate but will never hit 0.

Note: The agre of the universe can be plotted over the Hubble's constant as a $y = \frac{1}{x}$ graph where the asymptote is at $x = 0$.

Part II Galaxy Wars

Assuming that your answer in part a is correct, the galaxy you have chosen is actually NGC 7320, thought to be a galaxy discordant from the main group of NGC 7317, NGC 7318a, NGC 7318b and NGC 7319. In particular, NGC 7318b is infalling into the other three galaxies at high velocities and currently colliding with them.

The quintet is also associated with the galaxy NGC 7320c, which lies 3 arcminutes east-northeast from NGC 7319. Twin tidal tails run from both NGC 7319 and NGC 7320 to NGC 7320c, suggesting past or ongoing gravitational influences. The former (the inner tail shown in the image below[2]) is narrower, less diffuse and has a higher surface brightness than the latter (the outer tail shown in the image below).

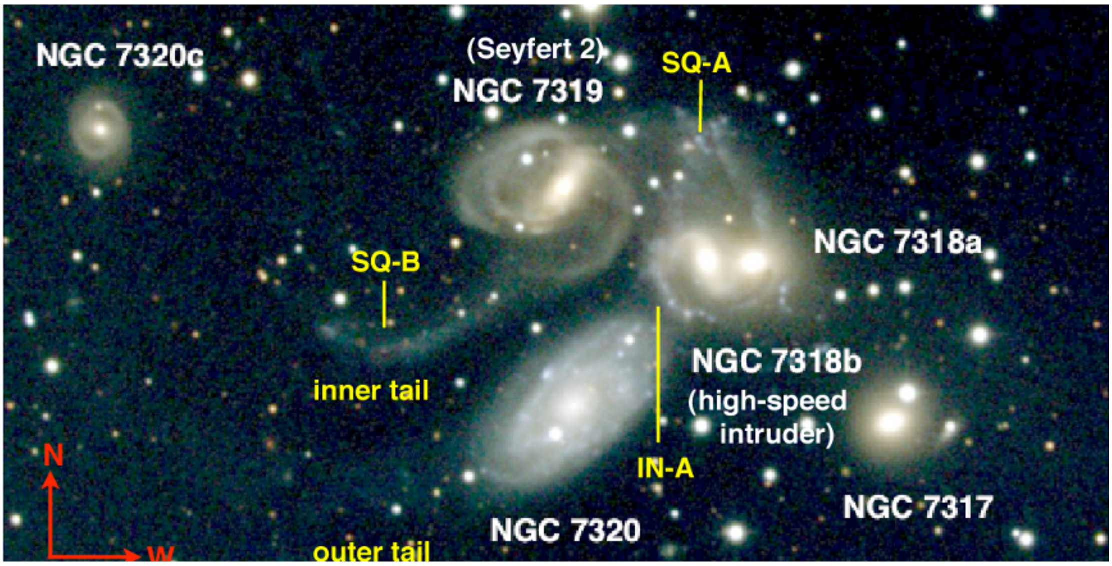


Figure 7: Image of Stephan's Quintet constructed from telescopic data, showing the five members NGC 7317, NGC 7318a, NGC 7318b, NGC 7319, and NGC 7320, as well as a neighbouring galaxy NGC 7302c.

(j) Based on the aforementioned information, rank the following interactions between the galaxies in chronological order.

I NGC 7320c becomes loosely bound to the group.

II NGC 7320c interacts gravitationally with NGC 7320.

III NGC 7317, NGC 7318a, and NGC 7319 become bound together, forming the group's core.

IV NGC 7318b collides with the trio of NGC 7317, NGC 7318a, and NGC 7319.

V NGC 7320c interacts gravitationally with NGC 7319.

[3]

Solution:

The events are III, II, I, V, IV in order from the past to the most recent.

We know that NGC 7318b is "colliding with [the other three galaxies] currently", and hence can put Event IV as the most recent event.

We can also assume that NGC 7317, NGC 7318a and NGC 7319 are the original members of the group, as their places in the group have remained constant and there was no mention of outside interactions with this triplet. Thus, we can place Event III as the earliest event.

Since the inner tail between NGC 7320c and NGC 7319 is "narrower, less diffuse and has a higher surface brightness" than the outer tail between NGC 7320c and NGC 7320, it is likely that NGC

[2]©NAOA/AURA/NSF (Original Image). ©Jeong-Sun Hwang, "Models of galaxy collisions in Stephan's quintet and other interacting systems", 2010 (image edited with annotations).

7320c has collided with NGC 7319 after it has collided with NGC 7320. Thus Event II has to occur before Event V.

The absence of interactions between NGC 7320 and NGC 7319 suggests that a decent amount of time must have elapsed between Event II and Event V. Hence we place Event I in between Event II and Event V, as it also makes more sense for NGC 7320c to first approach the group as a whole before brushing past NGC 7319 in the group.

Note: 1 mark is awarded if Event III and Event IV are placed as the first event and the last event respectively. Another mark is awarded if Event II is placed before Event V chronologically.

The group as a whole is massive enough to attract visitors like NGC 7318b episodically. Due to frequent gravitational interactions with their neighbours, the galaxies in the group have their gas stripped, shocked, and spewed into intergalactic space.

(k) Suggest how this will affect the rate of star formation in the group. Explain your answer briefly. **[2]**

Solution:

The rate of star formation will decrease in the galaxies due to star-forming gas-dense areas being blown apart by gravitational interactions between galaxies.

OR

The rate of star formation will increase in the intergalactic space between galaxies as they share an intergroup space of hot gas that came from each other.

OR

The overall rate of star formation will decrease as the gas spewed into space becomes too diffuse and cold for mass of new stars.

Note: Award 1 mark for stating how the rate of star formation will change AND in which location.(The location matters in this case as the rate of star formation can vary based on the location in the group.) No marks are awarded if this portion of the question is not answered.Award the other mark for supporting his/her answer with a valid reason.

(m) Computer simulations predict that in the near future, despite gravitational attractions among the galaxies, the group is unlikely to merge into one galaxy. Briefly explain what might be keeping them apart. [1]

Solution:

Intruding neighbours tug at components of the group frequently and disrupt their merger.

OR

High-speed neighbours collide/brush past members of the group, imparting enough momentum to escape merger after colliding with/brushing past each other.

OR

The galaxies are in stable orbits around each other.

(n) State the **least** likely shape or type of the final galaxy if the group of galaxies were to eventually merge into one galaxy. [1]

Solution:

Spiral galaxy.

NGC 7319 is a large spiral galaxy. While merging with the others, its disk will be stripped of the spirals by collisions that transform dust and gas into new stars as well as tidal interactions that will fling large amounts of loose matter into space. Thus the end result is unlikely to be a spiral galaxy.

Note: Accept all forms of spiral galaxies as answers. E.g. barred spiral, bulge-less spiral etc.

NGC7319 is a Type II Seyfert galaxy. Seyfert galaxies are galaxies that have active galactic nuclei, which often contain a supermassive black hole in their centre.

(o) Suggest an **observable** characteristic of NGC 7319 that might have given its status as a Seyfert galaxy away to astronomers. **[1]**

Solution:

Relativistic / superluminal jets are detected from the galactic nucleus of NGC 7319.

OR

The galactic nucleus of NGC 7319 is extra luminous compared to other galaxies of similar size when viewed at most wavelengths of the electromagnetic spectrum.

OR

Emission lines on the spectra of NGC 7319's galactic nuclei show strong Doppler broadening.

Note: Accept any other valid answers that discuss detectable signs of a supermassive black hole.

NGC 7320 is found to be significantly less red than the other galaxies in the quintet. Some reasons could be that NGC 7320 is moving towards us, or it is much closer to us than the other galaxies. Therefore, the light it emits is redshifted less.

(p) Suggest **another** possible reason for this. **[1]**

Solution:

The other four galaxies are older and hence contain a greater proportion of low-mass red stars in their stellar populations. / Their age also shows in their relative lack of star-forming regionsand therefore fewer young blue stars.

OR

The other four galaxies are interacting gravitationally with one another and kick up a veil of dust and gases of hydrogen which are ionised by nearby stars to emit red light.

Question 4 Aim for the Stars; If You Miss, You'll Drift in Interstellar Space

Episode IV A New Orbit

The first satellite placed in a geostationary orbit was Syncom 3, launched by a Delta D rocket in 1964. The 39kg satellite was able to transmit live coverage of the 1964 Summer Olympics from Japan to America, something which was impossible to do before Syncom 3 was launched.

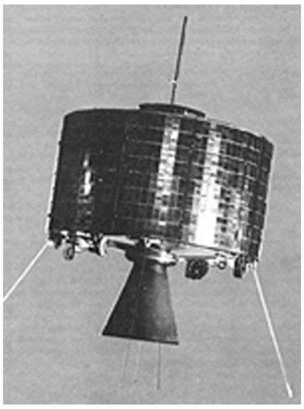


Figure 8: Syncom 3 satellite. (NASA)

Both geostationary and geosynchronous satellites have the same orbital period of 24 hours. Due to their orbital characteristics, geostationary satellites usually perform certain roles for certain countries.

(a) State the difference between a geostationary orbit and a geosynchronous orbit. **[1]**

Solution:

Geostationary orbits must have an inclination of zero to ensure the orbit remains over the equator at all times, whereas geosynchronous orbits can have any inclination.

(b) Briefly explain why high latitude countries such as Russia and Japan are unable to have geostationary satellites over their land. **[1]**

Solution:

For a geostationary satellite with zero inclination, its ground path does not intersect high latitude countries since the countries do not lie on the equator.

(c) The Russians got around this problem by launching satellites in highly eccentric elliptical orbits called Molniya orbits (Figure 9). Suggest why satellites in Molniya orbits can perform roles for high latitude countries similar to those roles performed by satellites in geostationary orbits. **[2]**

Solution:

According to Kepler's 2nd law, an orbiting body moves in its ellipse such that the line between it and the parent body sweeps out equal areas in equal times. Hence a Molniya satellite is able to dwell over the high latitude region for a long time every orbit.

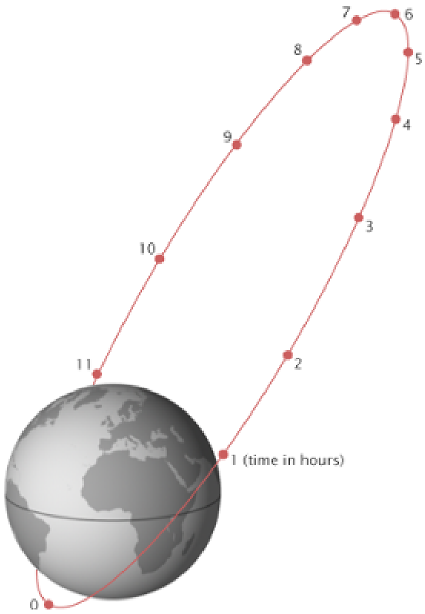


Figure 9: Molniya orbit around the Earth with time marks. (NASA)

Episode V The Jebediah Strikes Out

Jebediah Aerospace wants to put a 123kg satellite named Jeblink into geostationary orbit, but the company doesn't know what the required altitude is. Wernher von Kerman advises them that Kepler's 3rd law states that the square of the period of the orbit of a satellite about its parent body is proportional to the cube of the semi-major axis of the satellite's orbit. That is,

$$T^2 \propto a^3,$$

where T is the satellite's period and a is the semi-major axis (long side) of the orbit.

For the special case of semi-major axis being equal to the semi-minor axis, the orbit is perfectly circular and hence the semi-major axis can be taken as the orbit's radius r.

(d) The Formula Booklet shows the general Kepler's 3rd law for any elliptical orbit and non-negligible relative masses. Derive Kepler's 3rd law in the case of a satellite in a circular orbit around a much more massive body. **[2]**

Solution:

The centripetal force is equal to the gravitational force which is given by Newton's universal law of gravitation

$$m\frac{v^2}{r} = G\frac{Mm}{r^2}$$
$$v^2 = \frac{GM}{r}$$

We know the velocity is related to the period T by $v = \frac{2\pi r}{T}$. Substituting this into our earlier expression gives

$$T^2 = \frac{4\pi^2}{GM}r^3$$

as desired.

(e) Using the relevant values in the Formula Booklet, show that if Jeblink is in geostationary orbit, its altitude (height above sea level) is 35870km. Assume a perfectly circular orbit (0 eccentricity) and a spherical Earth. **[2]**

Solution:

Kepler's third law gives

$$T^2 = \frac{4\pi^2}{GM}r^3$$
$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

From here, we just substitute in the relevant values. This gives $r = 42240$km. Since we are looking for the altitude, we have to subtract the radius of the Earth. This gives the altitude as 35870km, as desired.

(f) Hence, using the relevant values in the Formula Booklet, show that if Jeblink is in geostationary orbit, its orbital speed will be 3071.8ms^{-1}. **[3]**

Solution:

As derived earlier,

$$m\frac{v^2}{r} = \frac{GMm}{r^2}$$

$$v^2 = \frac{GM}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

We use our value of r from earlier, and substitute in the relevant numbers to give 3071.8ms^{-1}.

(g) Using the altitude given above and the fact that low Earth orbit (LEO) ranges from 80km to 2000km in altitude,

(i) state two different uses of a satellite in a geostationary orbit, and **[1]**

Solution:

Communications, Meteorology, Navigation, Observation, Spying

(ii) give a reason why satellite operators prefer geostationary orbits instead of LEOs, even though it is easier and cheaper to place satellites in LEO. **[2]**

Solution:

Ground stations do not need to move their antennas to track the satellites throughout the day.

OR

Only one satellite is required because it is visible for the whole 24h period.

Now that Jebediah Aerospace has figured out the required altitude to put their satellite Jeblink into geostationary orbit, the company decides to build a rocket to execute their plans.

(h) The company puts their satellite Jeblink aboard the largest rocket booster they can find and launches it straight upwards from the ground to geostationary orbit altitude. Upon arrival, the speed of the satellite is precisely the orbital speed given in part f.

However, instead of orbiting Earth in geostationary orbit as expected, the rocket instead reaches a certain apoapsis point before crashing back to Earth. Based on this scenario, suggest and explain a reason why Jeblink failed to enter geostationary orbit. **[1]**

Solution:

Placing an object in orbit means it needs to travel sideways at the orbital velocity once the rocket is in high altitudes, not just pointing straight up from the planet's surface.

Episode VI Return of the Jeblink

Following this failure, Jebediah Aerospace quickly rebuilds a Jeblink 2 satellite and decides to outsource the launch to SpaceZ, a leading rocket launch provider. SpaceZ is able to provide an appropriately-sized rocket booster and the correct flight profile to put Jeblink 2 into equatorial LEO at 100km altitude, avoiding the catastrophe that was the original Jeblink.

However, SpaceZ refuses to design the upper stage to cover the remaining journey to geostationary orbit. It needs to be designed by Jebediah Aerospace. To perform this transfer, Jebediah Aerospace decides to use the Hohmann transfer orbit.

The Hohmann transfer orbit is an elliptical orbit used to transfer a spacecraft between two circular orbits of different radii around a central body in the same plane. The orbital manoeuvre to perform the Hohmann transfer uses two engine burns, one to move the spacecraft from the initial orbit onto the elliptical transfer orbit, and a second burn to circularise at the desired altitude.

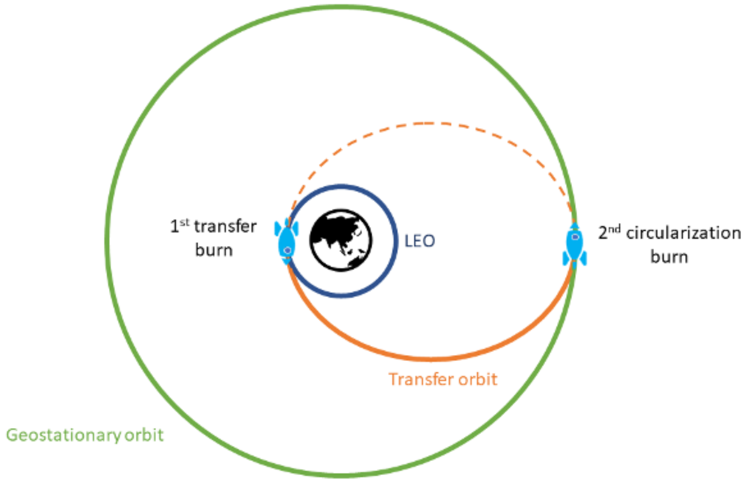


Figure 10: Hohmann transfer manoeuvre from geostationary LEO (dark blue) to geostationary orbit (green) via the Hohmann transfer orbit (orange)

For any satellite in an orbit about a planet, the vis viva equation

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

allows one to compute the instantaneous orbital speed v of the satellite given the semi-major axis a of the orbit and the instantaneous distance r between the spacecraft and the planet's centre of mass.

(j) It is given that the upper stage's orbital velocity in LEO at 100km altitude is 7848.7ms^{-1}. Calculate Δv, the total change in speed of the satellite needed to bring the satellite from 100km equatorial LEO to geostationary orbit. **[3]**

Solution:

We can find the semi-major axis of the transfer orbit

$$a_{\text{transfer}} = \frac{1}{2}(R_{\text{Earth}} + R_{\text{geo}})$$

Using the vis viva equation, we can find the exit velocity of the first orbit

$$v_1 = \sqrt{GM\left(\frac{2}{R_{\text{Earth}}} - \frac{1}{a_{\text{transfer}}}\right)}$$

The change in velocity required is then $v_1 - v_{\text{LEO}}$. To find the exit velocity of the second orbit, we can use the vis viva equation again

$$v_2 = \sqrt{GM\left(\frac{2}{R_{\text{geo}}} - \frac{1}{a_{\text{transfer}}}\right)}$$

The change in velocity required is $v_2 - v_1$. The total change is thus $v_2 - v_{\text{LEO}}$, which is 3976.1ms^{-1} when all the known values have been substituted in.

After a certain amount of research, Jebediah Aerospace engineers have found three different possible propulsion systems to use for the upper stage.

	Option 1	**Option 2**	**Option 3**
Name	BACC SRB	48-7S LFE	IX-6315 EPS
Type	Solid fuel booster	Liquid fuel engine	Electric ion engine
Mass of Engine System	1.5 tonne	0.13 tonne	0.3 tonne
Mass of Fuel	6.25 tonne	1 tonne	0.04 tonne
Thrust (vacuum)	300 kN	20 kN	2.0N
Specific Impulse (vacuum)	210s	320s	4200s
Burn Time	42s	157s	825000s

Table 3: Details about the three possible propulsion systems.

It is given that effective exhaust velocity v_e (units ms^{-1}) is given by

$$v_e = g_0 I_{sp},$$

where $g_0 = 9.80665\text{ms}^{-2}$ is standard gravity and I_{sp} is the specific impulse of the rocket engine.

(k) Jeblink 2's mass is also identical to Jeblink, at 123kg. Choose and explain, with relevant calculations and explanations, which of the three propulsion systems is the only sensible choice to bring Jeblink 2 from 100km equatorial LEO to geostationary orbit from 100km LEO.

(**<u>Hint:</u>** Can the given propulsion systems reach the desired orbit reasonably? Consider Δv, the thrust-to-weight ratio, and any peculiarities of the engine systems.) **[2]**

Solution:

Option 2, liquid fuel engine. Only option 2 has sufficient Δv required and a reasonable thrust-to-weight ratio of more than 1. For option 1, solid rocket boosters cannot be turned off once activated, making a proper 2-burn Hohmann transfer impossible. For option 3, the electric ion engine has absymal thrust-to-weight ratio and it is impossible to do a proper 2-burn Hohmann transfer as well.

Note that there are 2 burns at play here. Let Δv_1 be the required change in velocity for the LEO to transit orbit burn, and Δv_2 be the required change in velocity for the transfer orbit to geostationary orbit burn. Let v_{e1} and v_{e2} be the exhaust velocites for the same. Let m_0 be the initial mass, and m_1 and m_2 be the fuel burnt for the same. Using the rocket equation twice gives us

$$\Delta v_1 = v_{e1} \ln \frac{m_0}{m_0 - m_1}$$

$$\Delta v_2 = v_{e2} \ln \frac{m_0 - m_1}{m_0 - m_1 - m_2}$$

If $m_1 + m_2$ is less than the fuel mass, the Hohmann transfer can be done. We can use this to verify that only option 2 has the required Δv for both burns.

Epilogue

"From our calculations, we will go with Option [REDACTED]", says Valentina Kerman, a rocket scientist.

The rocket was built and prepped for an early morning launch. SpaceZ's Eagle 9 rocket launched the upper stage into 100km LEO without any hiccup. The upper stage continued the journey to geostationary orbit and successfully circularised. The payload was released and Jeblink 2 was able to link up with Ground Control. Hooray!

Question 5 An Ancient Sky Atlas

Apollonius of Perga was said to have invented the astrolabe, an ancient astronomical device that had major significance in observational astronomy. It has many functions: from measuring the altitude of celestial bodies, to identifying visible stars in the celestial sphere.

Carefully read and understand how the astrolabe works before attempting the questions.

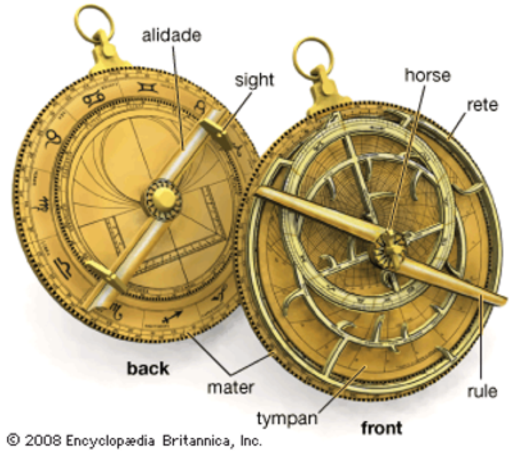


Figure 11: Components of an astrolabe.

The astrolabe has a front and back. Its main components consist of the following:

Mater The main panel to which all other components are attached to. It has markings on its outer circle to indicate the time (24-hour clock written in Roman numerals, with midnight starting clockwise from the bottom).

Tympan A plate inserted into the mater. It has line engravings that show the visible section of the night sky at a particular time (Anything within the grid lines will be visible to the observer). The concentric circles represent the altitude (the centre represents the zenith), while the intersecting lines represent the azimuth. Each line interval is 5°. The boundary grid line represents the horizon.

Rete A frame placed over the tympan that is free to rotate. The main circle (upper-most circle represented as an astronomical clock) indicates the ecliptic line that the Sun lies on. Its pointers (protrusions) indicate the position of bright stars. If rotated clockwise, it is similar to how the celestial sphere rotates with reference to the observer, with one complete rotation representing a single day.

Rule A rotatable pointer used to align the rete, tympan, and mater.

Horse A pin holding everything in place.

Alidade with Sight A rotatable pointer used to measure the altitude of celestial objects.

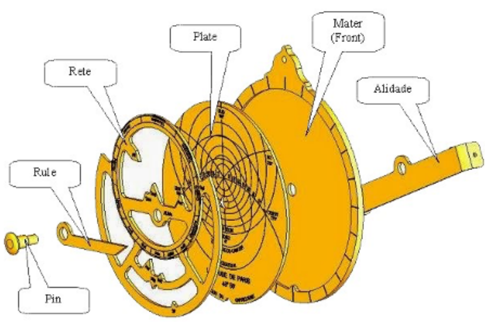


Figure 12: Anatomy of an astrolabe.

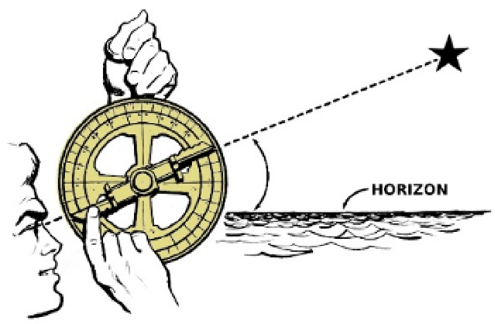


Figure 13: Measuring the altitude of a star using the alidade with sight.

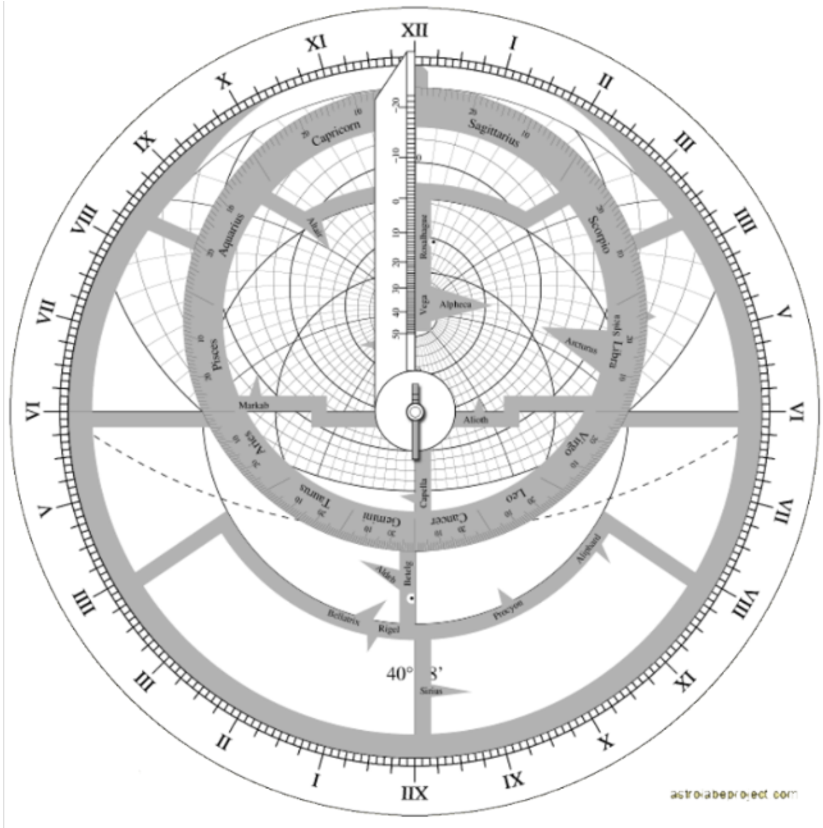


Figure 14: Default position of the astrolabe.

How to find the celestial sphere currently visible to the observer using an astrolabe:

1. The rule is first shifted to the Sun's current position indicated on the rete. The figure below is an example.

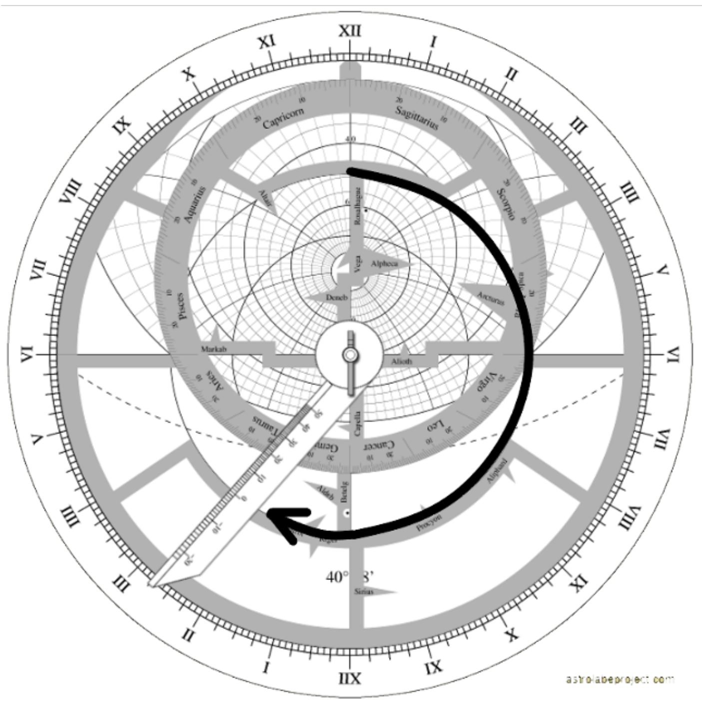


Figure 15: Astrolabe with rule shifted to 20 Taurus.

2. The rete is then rotated together with the rule (with the alignment obtained in Step 1) to the current time indicated on the outer circle of the mater.

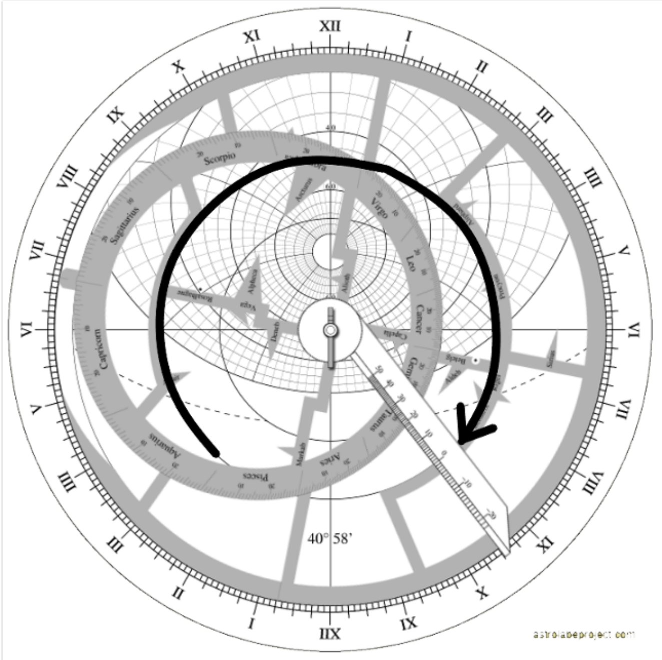


Figure 16: Astrolabe with rule and rete aligned to time 2130.

3. The current celestial sphere visible to the observer is read off from the tympan.

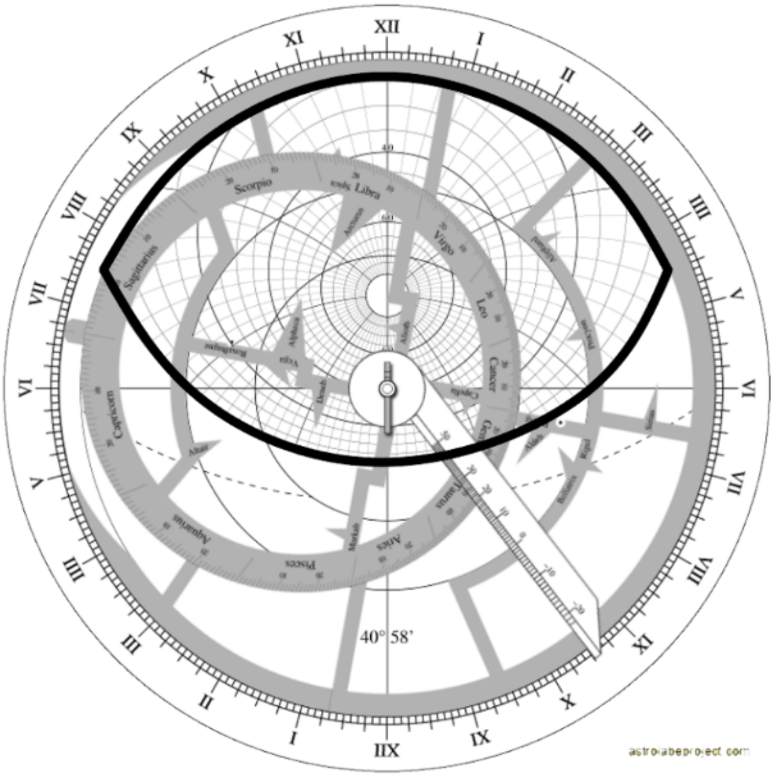


Figure 17: Celestial sphere visible to observer (marked out with black boundary).

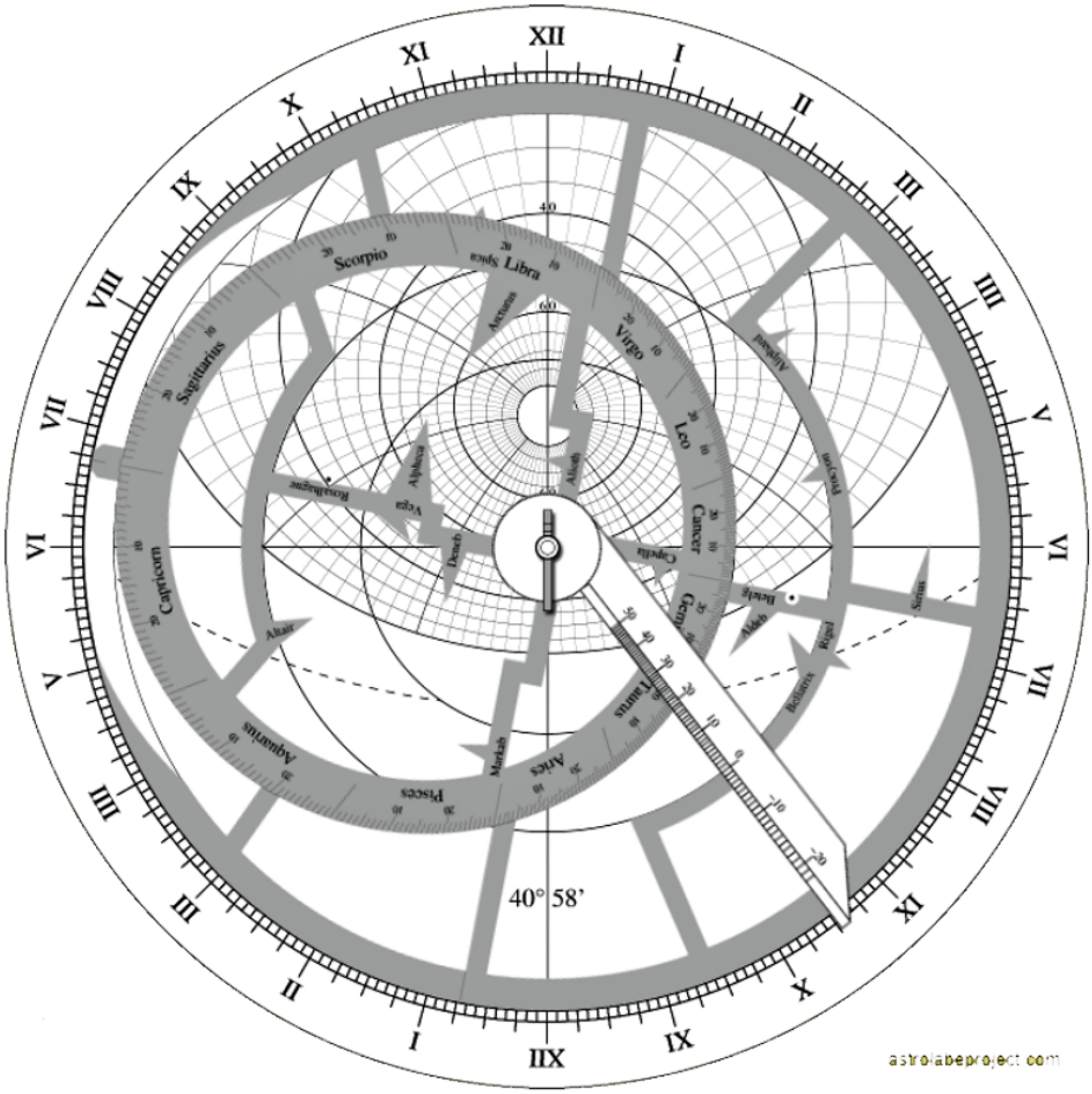


Figure 18: Front view of an astrolabe.

Figure 18 shows an astrolabe indicating that the Sun is currently within 20° of Taurus. The time is 2130. It is given that the tympan was made for a location at 40°58′ latitude.

(a) Gacrux has a declination of approximately −57°. State and explain if it is visible from this location. **[1]**

Solution:

Gacrux is not visible from this location. The Declination - Latitude $= -57° - 40° = -97° < -90°$. This means Gacrux is constantly below the observer's visible horizon.

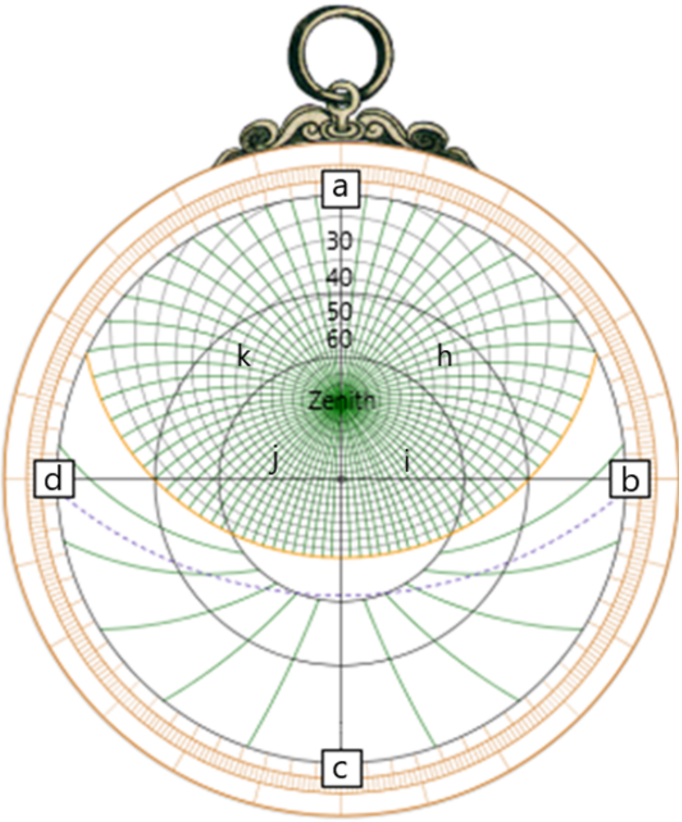


Figure 19: The same astrolabe, labelled for parts b to e.

(b) Based on Figure 18 and the relevant information given, state the cardinal directions labelled as 'a','b','c', and 'd' respectively on the tympan shown in Figure 19. **[1]**

Solution:

'a' is south , 'b' is west, 'c' is north and 'd' is east.

Participants have to consider their latitude of 40°58′ and their knowledge of the declination of any of the stars shown in the astrolabe on figure 14 to deducethe North-South cardinal directions. (E.g. Spica has approximate declination $-11°$, so when viewed from the northern hemisphere, it'll be in the southern section of sky.)The East-West direction is given in the description above where it is stated the Rete rotates clockwise; stars rise from the East (Left) and sets in the West (Right).

(c) The summer triangle is a popular western asterism. State the name of its constituent stars and explain whether it is currently fully visible to the observer. **[2]**

Solution:

Deneb, Altair, and Vega.The asterism is not fully visible. As seen from the Rete, Altair is currently below the observer's visible horizon.

(d) Deduce the approximate time Procyon will set below the horizon. **[1]**

Solution:

Procyon will set below the horizon at approximately 2230.

Participants have to observe from the astrolabe that when Procyon is rotated clockwise until it reaches the horizon, the Rete (together with the rule) will have rotated an angle equivalent to 1 hour (A ruler or finger can be used to extend a line from the centre of the astrolabe, through Procyon, up to the clock on the outer rim). Knowing that the current time is 2130, the time it sets can be deduced to be 2230.

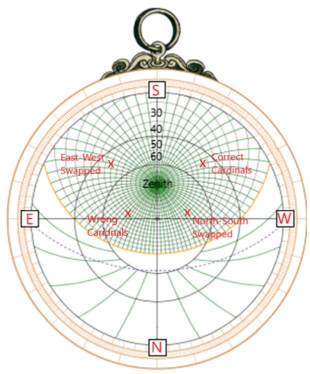
(e) Currently, it is given that Regulus is approximately 235° in azimuth and 40° in altitude with reference to the observer. State which of the four positions 'h', 'i', 'j', and 'k' in Figure 19 is the correct position of Regulus. **[1]**

Solution:

The correct position of Regulus is labelled as 'h'.

It is stated in the description that each grid line is equivalent to 5°. Knowing the cardinal direction from the previous question,participants have to move 235° clockwise starting from North and 40° starting from the horizon (On the tympan) to find regulus.

Note: If participants got the cardinal directions wrong, their answer should be taken with reference to their cardinal directions.



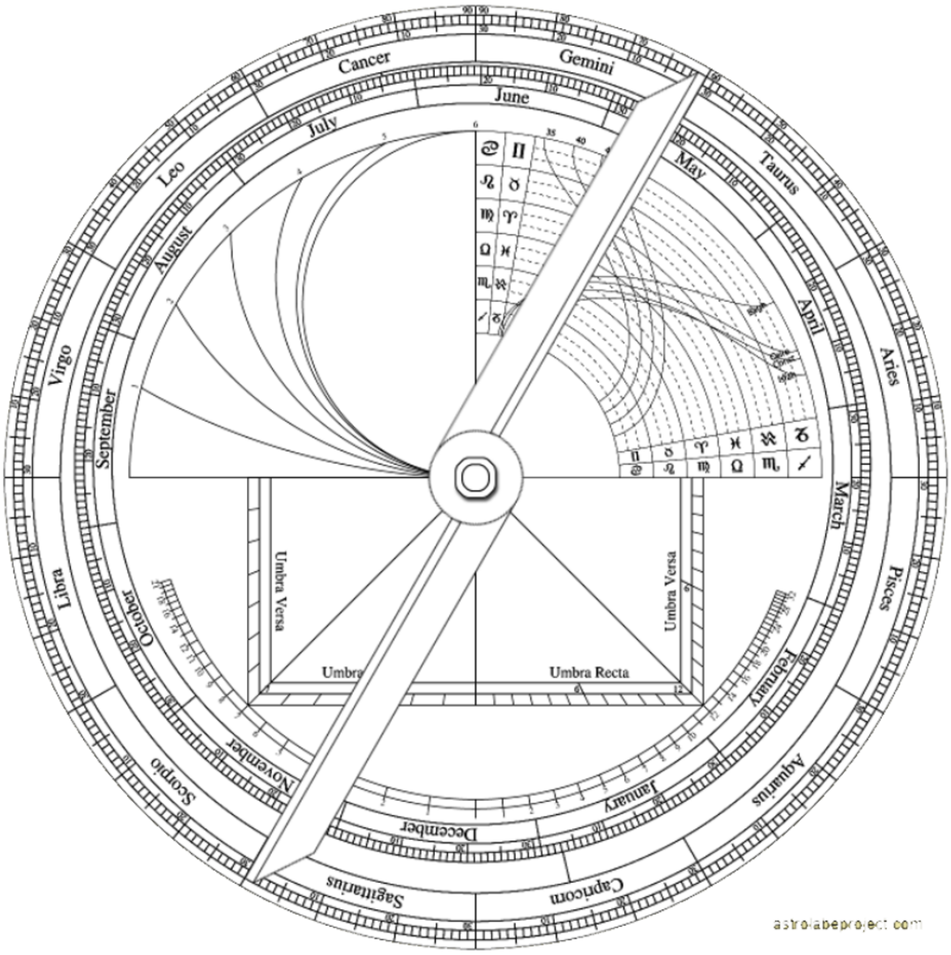


Figure 20: Back view of the same astrolabe.

Figure 20 shows the back view of the same astrolabe after measuring the altitude of a bright star. For simplicity, ignore the complex diagrams in the inner circle and focus only on the outer circles, starting from the circle with the months.

(f) With reference to the front and back views of the astrolabe, state the measured altitude and deduce the name of the bright star. **[1]**

Solution:

The measured altitude is 60°. The star is Arcturus.

Participants have to read the angle off the rule in the upper-right quadrant. Based on the Rete, only 1 "significant" star is at an altitude of 60°, Arcturus.

The astrolabe also allows users to easily find the current date given the Sun's position amongst the constellations and vice versa. This done by aligning the alidade to the given value and reading off the corresponding date. You are given that the current date is 11 December.

(g) Deduce the Sun's current position amongst the constellations. **[1]**

Solution:

The Sun is currently within 20° of Sagittarius.

Participants have to find the corresponding date written on the Mater by extending a line from the outer ring (Astronomical date) to the inner ring (Current date).

(h) Another useful feature of the astrolabe is the ability to deduce the time of sunrise or sunset. State the time of sunrise and explain how it can be obtained by using both the front (Figure 18) and back of the astrolabe (Figure 20).

(**Hint:** In order to find the timings using the front of the astrolabe, the "point at which the rule aligns to the Sun's position on the rete" must lie at the position of the sunrise indicated on the mater. **[2]**

Solution:

The time of sunrise is 0730. The astronomical date corresponding to 11 December is 20 Sagittarius, as read from the Mater on the back of the astrolabe. On the front of the astrolabe, the Rule has to be aligned to 20 Sagittarius. The point at which the Rule aligns with 20 Sagittarius is positioned along the horizon line and the time is read off the Mater.

Participants have to understand that the current date corresponds to a specific position of the Sun (i.e. 20 degrees of Sagittarius), then deduce that the rule must be shifted to that position on the front of the astrolabe (Similar to how they find the current celestial sphere). Finally they should use the given hint to deduce the time.

The names of most celestial bodies we know today were derived from Greek mythology. One famous folklore references Orion, the Hunter, and Scorpio, the Scorpion. Orion was so great that Gaia set Scorpio against him to put an end to his arrogance. The Scorpion succeeded and chases Orion through the night sky to this day.

Figure 21: View of a section of the night sky.

(j) State the proper name of α Scorpii and the month in which it culminates at exactly 0000 hrs. **[2]**

Solution:

The proper name is Antares. The month is June.

(k) State the name of the star that represents the stinger of the scorpion's tail. **[1]**

Solution:

The star is Shaula.

(m) Near Scorpius lies a open star cluster named after a famous astrnomer. State the name of the star cluster and explain how it can be found with reference to Scorpius. **[2]**

Solution:

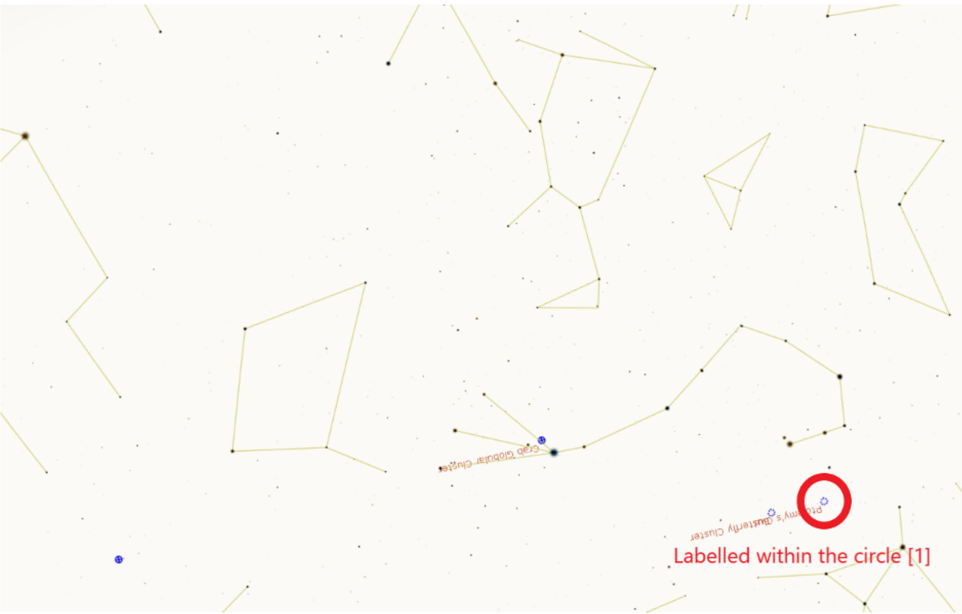
The star cluster is Ptolemy's Cluster. Extend an imaginary line from Lesath through Shaula.

Note: Any other reasonable mathods participants know are accepted.

(n) State one other constellation that is fully visible in Figure 21. **[1]**

Solution:

Lupus, Norma, Libra or Ara.



Although the hunter Orion is often depicted as facing the charge of Taurus, the bull, few myths directly relate the two. One myth, however, references Orion and how he fell in love with the Seven Sisters and pursues them across the night sky.

Figure 22: View of a section of the night sky

(o) State the name or catalogue number of two deep-sky objects (DSOs) in Figure 22 that are not in the constellation Orion. **[2]**

Solution:

1 mark is awarded each for any 2 DSOs correctly stated.

(p) The Mayans had a myth that described the creation of mankind. Within this myth, the Orion Nebula plays a central role as the flames of creation that burned in the centre of the heavenly hearth, marked by three stars in Orion: Alnitak, Saiph, and one other star.

Deduce the name of that star. **[1]**

Solution:

The star is Rigel.

Note: Participants must deduce that Rigel is the only other star in Orion that can form a triangle with the Orion Nebula in the centre.

(q) Given that Aldebaran crossed the meridian at 2351 on 18 Dec, deduce the approximate time at which it will cross the meridian on 21 Dec. **[1]**

Solution:

Aldebaran will cross the meridian at 2339.

Note: Aldebaran will rise approximately 4 minutes earlier each day, so it will cross the meridian 12 minutes earlier than 2351.

∼ **FIN** ∼