AstroChallenge 2017
Data Response Questions

(SENIOR)

PLEASE READ THESE INSTRUCTIONS CAREFULLY

1. This paper consists of **26** printed pages, excluding this cover page.
2. Do **NOT** turn over this page until instructed to do so.
3. You have 2 hours to attempt all questions in this paper.
4. At the end of the paper, submit this booklet together with your answer script.
5. Your answer script should clearly indicate your school (and team number) on **EVERY** page, as well as the individuals in the said team on the first page.
6. It is your team's responsibility to ensure that all pages of your answer script have been submitted, including pages to be detached from this booklet.

DRQ 1: Active Galactic Nuclei [20 marks]

Active Galactic Nuclei (AGN) refer to energetic phenomena in central regions of some galaxies not directly attributable to stars. The two largest subclasses of AGNs are Seyfert galaxies and quasars, differing primarily in the amount of radiation emitted by the compact central source. In a typical Seyfert galaxy, the central source emits roughly the same amount of radiation as all its stars (i.e., ~ 10^{11} $L_{\odot}$), while in a quasar it is nearly a hundredfold more.

NGC5548 is one of the most extensively studied Seyfert in literature and today, we will make use of NGC5548 to determine some of the key features of an AGN.

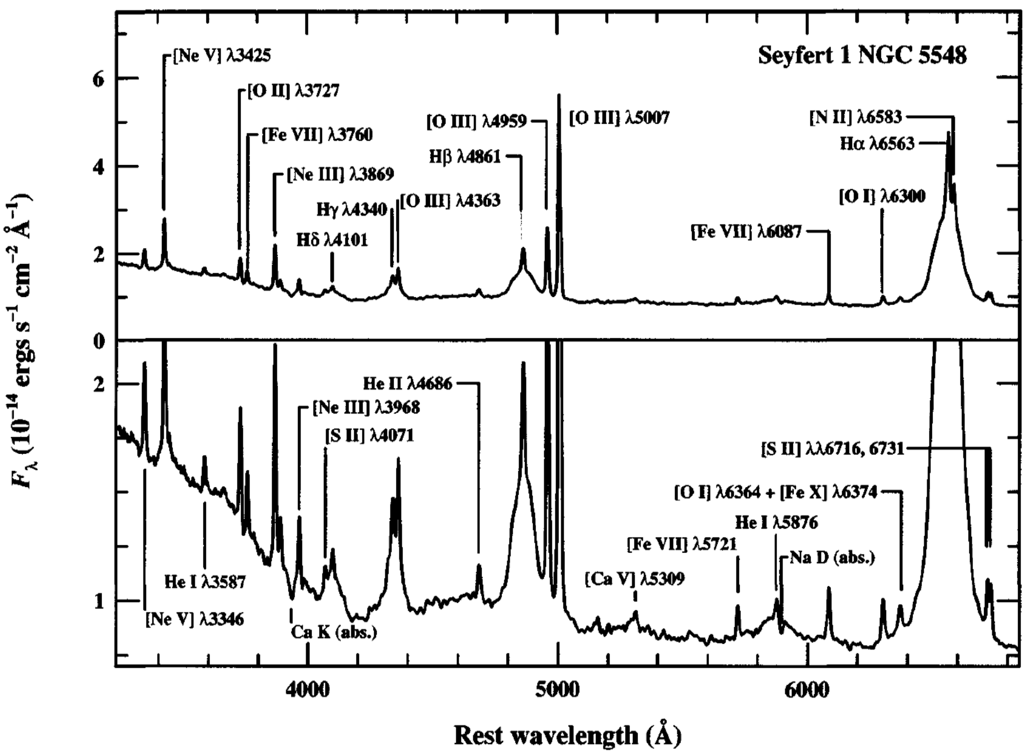


Figure 1 *The optical spectrum of the Seyfert 1 galaxy NGC 5548. Prominent broad and narrow emission lines are labelled, as are strong absorption features of the host galaxy spectrum. The vertical scale is expanded in the lower panel to show the weaker features. Note that the transition lines wavelength values provided are measured at rest.*

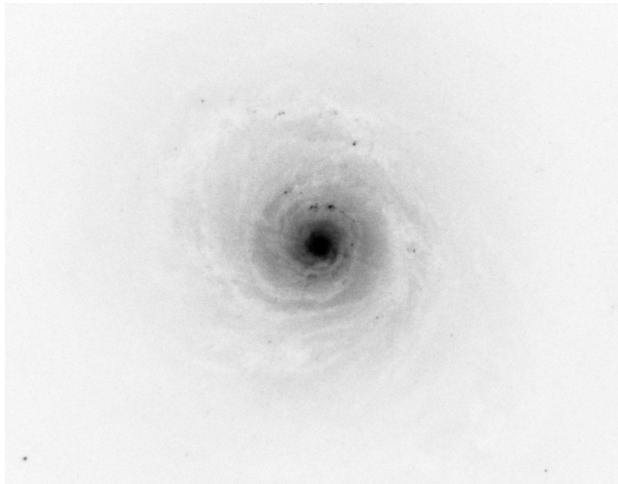


Figure 2 *NGC 5548 photographed by the Hubble Space Telescope. (Image inverted in black and white.) Credit: ESA/Hubble and NASA. (Acknowledgement: Davide de Martin)*

Part I [15 marks]

NGC 5548 is a spiral galaxy that is found in the constellation Bootes. It is also notably first identified in 1943 by Carl Seyfert to possess an active galactic nucleus, a Seyfert 1 galaxy. It is found that the radial velocity required for full width at half maximum (FWHM) of the broad components is about 5900 km s^{-1}, and the width of the narrow components is about 400 km s^{-1}.

Let's attempt to calculate all that is to be known from a spectral graph of such an object.

i) Which region(s) of the Seyfert galaxy do the Hβ transition line the O III **primary transition line** lines originate from? Hence or otherwise, compute the upper and lower bound of the FWHM and the width lengths using the appropriate radial velocities. [3 marks]

ii) Given that the observed spectral wavelength of the O III primary transition line is 5089 angstroms, calculate the redshift for NGC 5548 and hence its distance away from earth. State any assumptions used in your calculations. [3 marks]

iii) It is known that NGC 5548's core is so luminous such that looking through a telescope it appears to be a star, hence 'quasi-stellar', albeit an unresolved star. By calculating the maximum resolution of the NGC 5548 by the 2.4m Hubble Space Telescope, observed in the visible range, suggest an appropriate upper bound to the diameter of the galactic nucleus in pc. Recall that the visible spectrum lies between 350nm to 700nm. Explain your answer. [2 marks]

iv) Calculate the absolute magnitude of NGC 5548 given that it has an apparent magnitude of 13.3. [1 mark]

v) Given that NGC 5548 looks as shown above in Figure 2, suggest if the Tully Fisher relation would be useful in determining the luminosity of the object. [1 mark]

vi) It is understood that the broad emission lines observed in the spectrum of AGNs are formed in a region within/near the galactic nucleus, while that of narrow emission lines lie beyond galactic nucleus. Using Kepler's Laws, explain how can broad and narrow emission lines coexist for the same object (NGC 5548). [2 marks]

vii) The culprit for the broadening of the spectral lines in NGC 5548 is due to the accretion of matter onto supermassive black holes. Hence or otherwise, calculate the mass of the supermassive blackhole responsible for this object. [1 mark]

viii) Suggest why using the smaller orbital velocity value (found for the narrow emission lines) will lead to a poorer estimate, even if all other necessary quantities are known. [1 mark]

ix) From literature, the mass of the supermassive blackhole is approximately $M = 6.5{\times}10^{7}\mathrm{M}_{\odot}$ Compare the literature value against the value you found in (vii) and explain your answer. [1 mark]

Part II [5 marks]

It is known that the torus of the AGN is formed at a distance r_d pc away from the core due to evaporation and it is proportional to the absolute luminosity, L (Barvainis, R. 1987, ApJ, 320, 537) of the nucleus. Thus, it is expected that there would be a time delay Δt days as the incident wavelength is re-emitted by the torus, which is simply the time taken for the photons emitted by the nucleus to reach the torus at a distance r_d pc away.

x) Assume that the region between the nucleus and the torus is fully cleared of materials (i.e. it is a vacuum) and that the time delay $\Delta t = 49.5$ days, what does this tell us about the nucleus radii and the mass of the supermassive blackhole? [2 marks]

xi) Following which, we may check for mass loss by calculation of what is known as the Eddington limit, for beyond this limit, mass loss occurs for a spherically symmetric object in equilibrium. Hence or otherwise, use supporting calculations to determine if there is any possibility that the black hole region is losing mass. You are given that the Eddington Limit is $L_{ED} \cong 1.5{\times}10^{31}\frac{M}{M_{\odot}}L_{\odot}$ for the supermassive blackhole. [3 marks]

DRQ 2: An Astronomical Mystery: A Case of Identity [20 marks]

An astrophotographer has taken the following image over the course of a night.

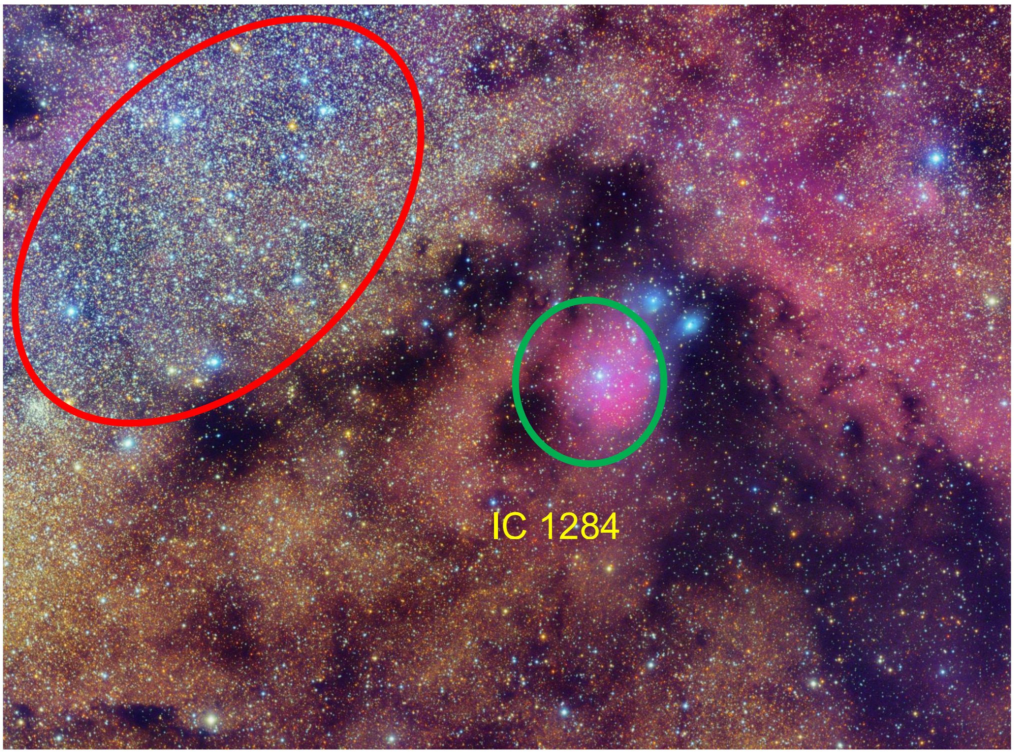


Figure 1 *IC 1284 and surrounding regions (Photograph by Ivan Bok)*

Much is going on in this image; there are at least two key objects of interest here, as circled. Let us first focus on IC 1284 (the circle). We seem to have a bright star, surrounded by a spherical halo of glowing reddish pink gas.

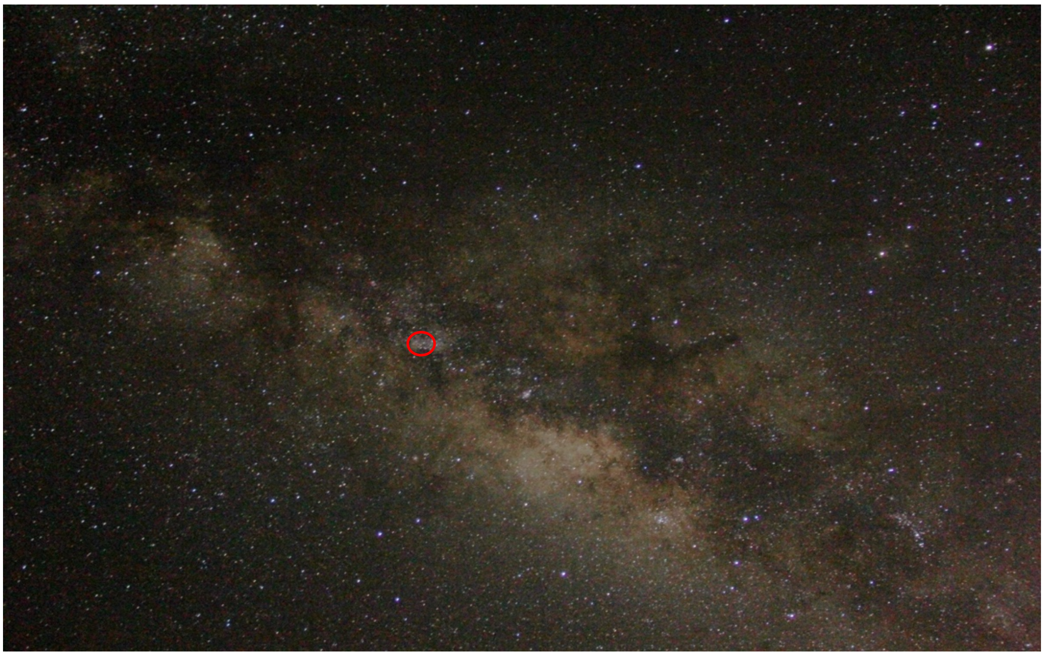


Figure 2 *A wide field shot of the Milky Way. Oval drawn to approximately the same scale. Taken by Kia Yee*

Part I: The Star [5 marks]

The central star of IC 1284 (HD 167815) is poorly studied in the existing literature. Here are some key data from several general surveys, as obtained from the SIMBAD database.

Parallax	2.80 milliarcseconds
Spectral Classification	B1.5III
Apparent magnitude (B)	7.70
Apparent magnitude (V)	7.61
Apparent magnitude (K, near-infrared)	6.957

i) Just by looking at its spectrum, we know that HD 167815 is a B-type giant star with a surface temperature of around 20,000K-25,000K. How do astronomers quickly differentiate a B-type star (from say, a K-type star), solely by looking at a spectrum? [1 mark]

ii) Similarly, how do we infer a star's luminosity class, solely by looking at a spectrum (in other words, differentiate a giant star from a dwarf star)? [1 mark]
NB: We are only looking for a general list of feature(s), not a lengthy explanation of why these feature(s) work to distinguish these stars.

A star with the same stellar classification as HD 167815 is α Lupi, the brightest star in Lupus. The table below summarizes some key information about this star

Parallax	7.02 milliarcseconds
Luminosity	25,000 $L_\odot$
Absolute magnitude (V)	-4.3
Apparent magnitude (B)	2.126
Apparent magnitude (V)	2.286
Apparent magnitude (K, near-infrared)	2.668

iii) Due to intervening dust, α Lupi is visually dimmed by an extinction factor, A(V). A(V) is expressed in terms of magnitudes, and is related to the apparent and absolute magnitude by the following equation:

$$m(V) = M(V) + 5\log_{10} d - 5 + A(V)$$

Where d is in parsecs as usual.

Given the similar spectral classes of α Lupi and HD 167815, we may assume that both stars have similar luminosities and absolute magnitudes. Further, we know that due to interstellar extinction, only a certain fraction of a star's total light reaches us. Hence, compute the amount of light that reaches us from both stars. Express your answer as a percentage of the original unextincted brightness. [3 marks]

Part II: The Nebula [9 marks]

How about IC 1284 itself? Unlike far more famous nebulae, IC 1284 is almost perfectly spherical. This suggests that IC 1284 is a classic example of a special type of nebulae, whose formation process was first described 80 years ago.

Let us start with a star inside a cloud of hydrogen gas of uniform density. The star emits high-energy photons, which completely ionises the hydrogen in a shell around the star. Some of the free protons and electrons then recombine and emit photons (see equation below), generating the light that illuminates the nebula.

$$p^+ + e^- \rightarrow H + \gamma$$

Under ideal conditions, the result is what we see right now: a glowing sphere of ionized hydrogen surrounding a star.

Let us consider a $1\,m^3$ unit volume of the resultant nebula, containing a number of free electrons and protons (n_e and n_p) respectively. The rate of net recombination in that unit volume is then given by the rate equation:

$$Rate = \alpha(T) n_e n_p$$

$a(T)$ represents the net recombination coefficient, equalling $2.6\times10^{-19}\,m^{-3}\,s^{-1}$ at a temperature of 10,000K. (If this *looks* like a typical 2nd order rate equation in JC Chemistry Kinetics, that's because it is.)

iv) How does the rate of recombination change with temperature? In other words, does recombination occur faster or slower at higher temperatures? Why? [1.5 marks]
Hint: consider the physical mechanism behind recombination.

v) An astronomer boldly claims that $n_p = n_e$ for all regions of the nebula. Is this a justified assumption? Briefly explain. [0.5 marks]

vi) Suppose that HD 167815 supplies Q ionising photons per second to IC 1284. Hence or otherwise, derive an expression for the equilibrium radius of the nebula, R_S. Assume perfect absorption of ionising photons. [2 marks]

vii) Given that, the imaging setup used has a pixel scale of 2.48 arcseconds/pixel and that IC 1284 spans a diameter of 370 pixels at its narrowest, calculate the approximate radius of IC 1284 in light years. [2 marks]

viii) A B1.5-type blue giant like HD 167815 produces around 4×10^{47} ionising photons per second. Hence or otherwise, find the average number of protons/electrons per unit volume in the nebula surrounding IC 1284. Assume equilibrium. [1 mark]

ix) Given that the Hα line has a wavelength of 656.28 nm and around 40% of all recombining hydrogen gas emits Hα, what is the luminosity of IC 1284 in Hα? Express your answer in terms of solar luminosities. [2 marks]

Part III: The Big Picture [6 marks]

Now we return to the oval. This oval is large (spanning nearly 2 degrees across), and boasts of several curious properties. What is its true nature?

x) An astronomer plots out a color-magnitude diagram for surveyed stars in the oval brighter than an apparent magnitude of 14 (V). See the figure below. The resultant color-magnitude diagram has extremely high scatter and does not display an obvious main sequence. Given that the data is complete and accurate, suggest the most important reason for this [2 marks]

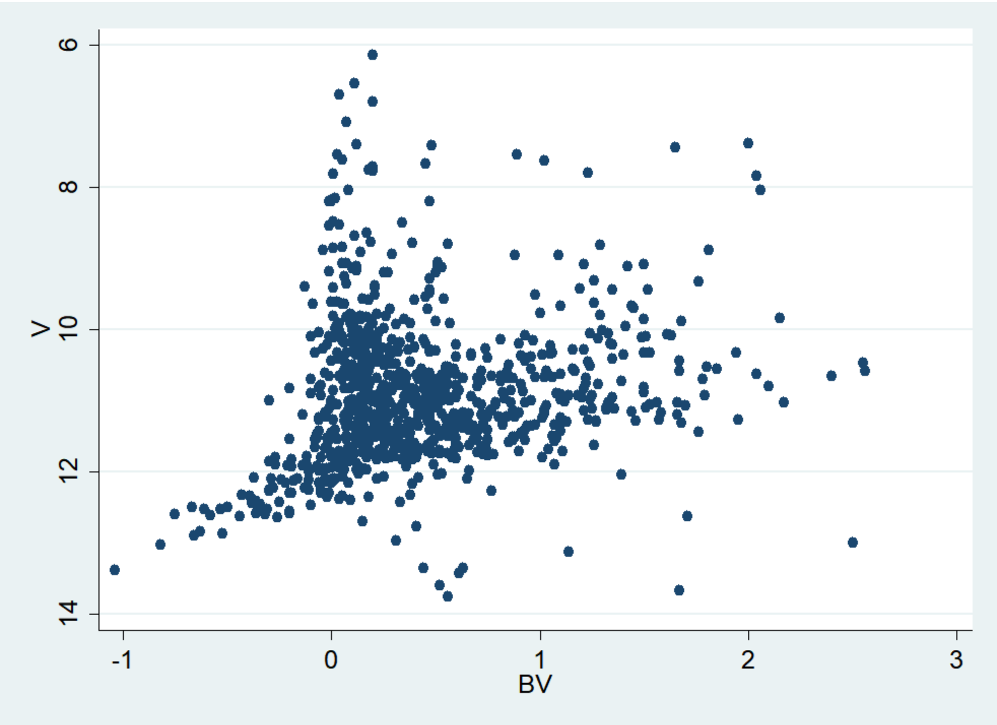


Figure 3: *CMD for a sample of stars within 1 degree from the object centre, brighter than mag +14 (V). Note that the X axis measures B-V.*

xi) Refer to Figure 1. Observe that other than the stars in the oval, most of the stars appear highly yellow. Explain why these stars are so yellow. [1 mark]

xii) In the 1800s, this oval was described as "a large nebulosity containing many stars". With better optics, we know that this is false: this oval is not a nebula. Hence or otherwise, discern the true nature of the object in this oval. Your answer should reference specific evidence in the images and/or answers in the previous questions. [3 marks]

DRQ 3: In a Coma over Coma [20 marks]

The Coma Star Cluster (Melotte 111) is one of the more prominent clusters in the northern spring sky. Located in Coma Berenices, the cluster center has approximate coordinates RA: 12h 25m and Dec: +25° 51′. It should not be confused with the Coma Cluster, a rich galaxy cluster that also lies in Coma Berenices. Using publicly available data, we've provided 2 copies of the Coma Star Cluster's colour-magnitude diagram (CMD), attached below.

Part I: A quick and dirty estimate [8 marks]

i) An astronomer points out that interstellar gas between us and the Coma Star Cluster will absorb light from these stars, thus interfering with our measurements. This will thus cause the observed CMD to differ from the true CMD of the cluster, affecting the conclusions that we will draw. Are these concerns significant? Explain. [2 marks]

Suppose that the CMDs provided have been corrected for the concerns raised in question (i). Appendix I contains data for the simulated Zero-Age Main Sequence (ZAMS) for stars between 0.1 to 7 solar masses. The ZAMS essentially marks where/when stars of each mass first hit the main-sequence.

ii) On the same axes, plot the ZAMS onto the **first** CMD provided [2 marks]. Hence or otherwise, determine the approximate distance to the Coma Star Cluster. Hint: like any good empiricist, you probably should make more than 1 measurement. [1 mark]

iii) State the coordinates of the main-sequence turnoff point [1 mark]

iv) The main sequence lifetime of a star is approximately given by:

$$t_{ms} \propto \frac{M}{L}$$

Using the mass-luminosity relationship, derive an equation linking the main sequence lifetime of any star to the main sequence lifetime of the sun. [1 mark]

Hence or otherwise, estimate the age of the Coma Star Cluster. A ballpark estimate for the main-sequence lifetime of the Sun is 10 billion years. [1 mark]

Part II: Improving the estimate [5 marks]

As you might expect, we can improve on this crude estimate. One of the key concerns we might have over our ballpark estimates is the exact functional form of the mass-luminosity relationship. The mass-luminosity relationship is a general formula: there is strong reason to believe that the mass-luminosity relationship depends on metallicity and the exact stellar masses under consideration.

v) Linearise the mass-luminosity relationship [1 mark].

vi) By plotting suitable values on the blank graph provided, use the ZAMS data to estimate the exact form of the mass-luminosity relationship for values of interest. For your ease, we've plotted the point for a solar mass star. [3 marks]

vii) Hence or otherwise, revise your age estimate in part (iv). [1 mark]

Part III: The actual work [7 marks]

In practice, astronomers determine the age of star clusters by using the concept of stellar isochrones. A stellar isochrone is a curve connecting stars of equal age on a HR diagram. By fitting these curves onto a cluster CMD, we can obtain a much more accurate estimate of a cluster's true age. For this express purpose, we have provided a set of 9 stellar isochrones in Appendix 2, spanning from 250M years to 850M years.

Note: to facilitate your plotting, the isochrones provided in Appendix 2 have been truncated: stars below 1 solar mass and stars that have evolved past the main sequence are **NOT** included in the subsequent isochrones.

viii) Given the information above and the set of theoretical isochrones in Appendix 2, describe a simple way of determining the best fit isochrone. [1 mark]

ix) Implement the procedure stated in part (viii). On the separate diagram provided, clearly plot the best fit isochrone to the Coma Star Cluster CMD. [2 marks].

x) Hence or otherwise, estimate the age of the Coma Star Cluster. Also, re-estimate the distance to the Coma Star Cluster. Clearly state all assumptions used, if any. [2 marks]

xi) Suppose that we used the uncorrected CMDs instead. How would our estimates differ from the true value? Briefly explain (computations are not required). [2 marks]

Appendix I: ZAMS data

Note: Mass, L and R are expressed in terms of the solar units Msun, Lsun and Rsun respectively. Age stands for the age of the star (in years) at the point it hits the main sequence, while ST stands for Spectral Type. Mbol and Mv stand for the absolute bolometric and visual magnitudes respectively.

The ZAMS and isochrones were generated by Siess (1997)

Mass	L	R	Teff (K)	Age	Mbol	B-V	Mv	ST
0.5	4.63E-02	4.29E-01	3829	1.02E+08	8.09	1.43	9.32	M1
0.6	8.29E-02	5.22E-01	4008	8.32E+07	7.45	1.37	8.5	M0
0.7	1.48E-01	6.37E-01	4287	6.58E+07	6.82	1.21	7.57	K6
0.8	2.80E-01	7.56E-01	4698	5.18E+07	6.13	1	6.64	K4
0.9	4.88E-01	8.66E-01	5047	4.16E+07	5.53	0.86	5.91	K2
1	8.10E-01	9.97E-01	5334	3.31E+07	4.98	0.8	5.28	K0
1.1	1.30E+00	1.17E+00	5556	2.63E+07	4.47	0.73	4.71	G8
1.2	2.00E+00	1.33E+00	5788	2.19E+07	4	0.65	4.2	G5
1.3	2.95E+00	1.48E+00	6070	1.89E+07	3.58	0.56	3.75	G0
1.4	4.28E+00	1.66E+00	6284	1.60E+07	3.17	0.49	3.33	F7
1.5	6.07E+00	1.85E+00	6498	1.35E+07	2.79	0.41	2.93	F5
1.6	8.12E+00	1.97E+00	6770	1.17E+07	2.48	0.37	2.59	F3
1.7	1.09E+01	2.06E+00	7120	1.01E+07	2.16	0.32	2.26	F1
1.8	1.41E+01	2.13E+00	7476	8.87E+06	1.88	0.25	1.98	A9
1.9	1.78E+01	2.20E+00	7832	7.81E+06	1.62	0.2	1.74	A8
2	2.23E+01	2.27E+00	8176	6.94E+06	1.38	0.17	1.51	A7
2.2	3.33E+01	2.40E+00	8824	5.56E+06	0.94	0.08	1.13	A3
2.5	5.44E+01	2.51E+00	9705	4.14E+06	0.41	-0.02	0.8	A0
2.7	7.34E+01	2.62E+00	10241	3.44E+06	0.09	-0.04	0.57	A0
3	1.06E+02	2.72E+00	11048	2.68E+06	-0.31	-0.07	0.32	B9
3.5	1.84E+02	2.94E+00	12094	1.82E+06	-0.91	-0.09	-0.09	B8
4	3.17E+02	3.24E+00	13208	1.27E+06	-1.5	-0.11	-0.45	B7
5	6.82E+02	3.53E+00	15334	6.87E+05	-2.33	-0.14	-0.89	B6
6	1.34E+03	3.91E+00	17291	3.99E+05	-3.07	-0.17	-1.32	B4
7	2.37E+03	4.32E+00	19281	2.46E+05	-3.69	-0.2	-1.62	B3

Appendix II: Stellar Isochrones (250M years to 850M years)

Note: Mass, L and R are expressed in terms of the solar units Msun, Lsun and Rsun respectively. ST stands for Spectral Type. Bol and Mv stand for the absolute bolometric and visual magnitudes respectively.

250M years

Mass	L	R	Teff	Bol	B-V	Mv	ST
1	8.58E-01	8.97E-01	5730	4.92	0.67	5.13	G6
1.1	1.36E+00	1.01E+00	6047	4.41	0.57	4.59	G0
1.2	2.08E+00	1.14E+00	6360	3.96	0.46	4.11	F7
1.3	3.05E+00	1.25E+00	6675	3.54	0.38	3.66	F4
1.4	4.30E+00	1.33E+00	7055	3.17	0.33	3.27	F1
1.5	5.90E+00	1.36E+00	7569	2.82	0.23	2.93	A9
1.6	7.84E+00	1.39E+00	8051	2.51	0.18	2.64	A7
1.7	1.02E+01	1.43E+00	8488	2.23	0.11	2.39	A4
1.8	1.29E+01	1.49E+00	8865	1.97	0.07	2.16	A3
1.9	1.62E+01	1.55E+00	9202	1.72	0.03	1.96	A2
2	2.01E+01	1.62E+00	9499	1.49	-0.01	1.83	A1
2.2	3.02E+01	1.78E+00	10012	1.05	-0.03	1.49	A0
2.5	5.31E+01	2.08E+00	10619	0.44	-0.05	1	B9
2.7	7.53E+01	2.36E+00	10890	0.06	-0.06	0.66	B9
3	1.25E+02	3.01E+00	10943	-0.49	-0.06	0.12	B9

325M years

Mass	L	R	Teff	Bol	B-V	Mv	ST
1	8.62E-01	8.99E-01	5732	4.91	0.67	5.13	G6
1.1	1.37E+00	1.02E+00	6049	4.41	0.57	4.59	G0
1.2	2.09E+00	1.14E+00	6360	3.95	0.46	4.1	F6
1.3	3.08E+00	1.26E+00	6678	3.53	0.38	3.65	F4
1.4	4.38E+00	1.34E+00	7062	3.15	0.33	3.25	F1
1.5	5.99E+00	1.37E+00	7570	2.81	0.23	2.91	A9
1.6	7.99E+00	1.41E+00	8045	2.49	0.18	2.62	A7
1.7	1.04E+01	1.46E+00	8453	2.21	0.12	2.37	A5
1.8	1.33E+01	1.52E+00	8798	1.94	0.08	2.13	A3
1.9	1.67E+01	1.60E+00	9109	1.69	0.04	1.92	A2
2	2.08E+01	1.68E+00	9376	1.45	0.01	1.75	A1
2.2	3.17E+01	1.88E+00	9852	1	-0.02	1.41	A0
2.5	5.70E+01	2.32E+00	10296	0.36	-0.04	0.86	A0
2.7	8.24E+01	2.80E+00	10249	-0.04	-0.04	0.45	A0

400M years

Mass	**L**	**R**	**Teff**	**Bol**	**B-V**	**Mv**	**ST**
1	8.66E-01	9.00E-01	5733	4.91	0.67	5.12	G6
1.1	1.38E+00	1.02E+00	6050	4.4	0.57	4.58	G0
1.2	2.11E+00	1.14E+00	6363	3.94	0.46	4.09	F6
1.3	3.12E+00	1.26E+00	6681	3.51	0.38	3.64	F4
1.4	4.46E+00	1.35E+00	7077	3.13	0.33	3.23	F1
1.5	6.09E+00	1.38E+00	7570	2.79	0.23	2.89	A9
1.6	8.14E+00	1.43E+00	8021	2.47	0.18	2.6	A7
1.7	1.06E+01	1.49E+00	8411	2.19	0.13	2.34	A5
1.8	1.36E+01	1.56E+00	8739	1.92	0.09	2.1	A3
1.9	1.72E+01	1.65E+00	9021	1.66	0.05	1.87	A2
2	2.15E+01	1.76E+00	9263	1.42	0.03	1.67	A1
2.2	3.32E+01	2.01E+00	9620	0.95	-0.01	1.31	A0
2.5	6.11E+01	2.69E+00	9718	0.29	-0.02	0.67	A0

475M years

Mass	L	R	Teff	Bol	B-V	Mv	ST
1	8.70E-01	9.02E-01	5734	4.9	0.67	5.12	G6
1.1	1.39E+00	1.02E+00	6051	4.39	0.57	4.57	G0
1.2	2.14E+00	1.15E+00	6366	3.93	0.46	4.08	F6
1.3	3.16E+00	1.27E+00	6685	3.5	0.38	3.62	F4
1.4	4.50E+00	1.35E+00	7082	3.12	0.32	3.22	F1
1.5	6.20E+00	1.40E+00	7572	2.77	0.23	2.88	A9
1.6	8.28E+00	1.45E+00	8001	2.45	0.18	2.58	A7
1.7	1.08E+01	1.52E+00	8375	2.16	0.13	2.31	A5
1.8	1.39E+01	1.61E+00	8662	1.89	0.1	2.07	A4
1.9	1.77E+01	1.72E+00	8914	1.63	0.07	1.83	A3
2	2.23E+01	1.84E+00	9114	1.38	0.04	1.6	A2
2.2	3.48E+01	2.18E+00	9369	0.9	0.01	1.19	A1
2.5	6.88E+01	3.14E+00	9258	0.16	0.03	0.41	A1

550M years

Mass	L	R	Teff	Bol	B-V	Mv	ST
1	8.74E-01	9.04E-01	5735	4.9	0.67	5.11	G6
1.1	1.40E+00	1.02E+00	6054	4.39	0.57	4.56	G0
1.2	2.16E+00	1.15E+00	6370	3.92	0.46	4.06	F6
1.3	3.20E+00	1.28E+00	6689	3.49	0.38	3.61	F4
1.4	4.57E+00	1.36E+00	7084	3.1	0.32	3.2	F1
1.5	6.31E+00	1.41E+00	7564	2.75	0.23	2.86	A9
1.6	8.43E+00	1.47E+00	7974	2.43	0.18	2.56	A7
1.7	1.10E+01	1.56E+00	8301	2.14	0.15	2.29	A6
1.8	1.43E+01	1.66E+00	8594	1.86	0.1	2.03	A4
1.9	1.82E+01	1.79E+00	8796	1.6	0.08	1.78	A3
2	2.31E+01	1.95E+00	8947	1.34	0.06	1.54	A3
2.2	3.63E+01	2.42E+00	9003	0.85	0.06	1.06	A2

625M years

Mass	L	R	Teff	Bol	B-V	Mv	ST
1	8.78E-01	9.05E-01	5736	4.89	0.67	5.11	G6
1.1	1.41E+00	1.03E+00	6056	4.38	0.57	4.55	G0
1.2	2.18E+00	1.16E+00	6372	3.91	0.45	4.05	F6
1.3	3.24E+00	1.28E+00	6693	3.47	0.38	3.6	F4
1.4	4.64E+00	1.37E+00	7085	3.08	0.32	3.18	F1
1.5	6.41E+00	1.42E+00	7551	2.73	0.24	2.84	A9
1.6	8.59E+00	1.50E+00	7952	2.42	0.18	2.54	A7
1.7	1.13E+01	1.59E+00	8252	2.12	0.16	2.26	A6
1.8	1.46E+01	1.72E+00	8507	1.84	0.11	2	A4
1.9	1.88E+01	1.87E+00	8653	1.57	0.1	1.74	A4
2	2.39E+01	2.07E+00	8747	1.3	0.09	1.48	A3
2.2	3.77E+01	2.75E+00	8507	0.81	0.11	0.97	A4

700M years

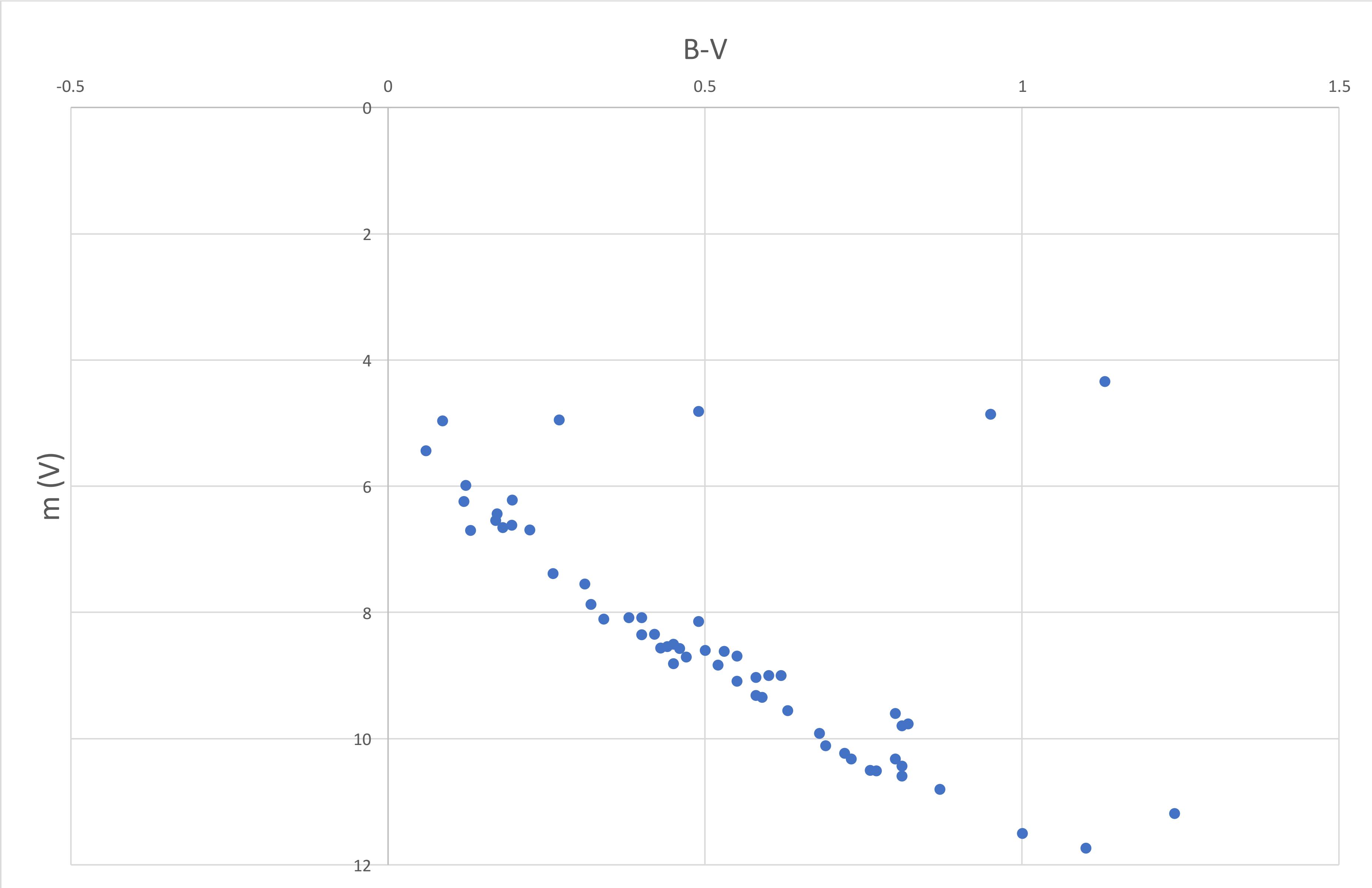
Mass	L	R	Teff	Bol	B-V	Mv	ST
1	8.83E-01	9.07E-01	5738	4.89	0.67	5.1	G6
1.1	1.42E+00	1.03E+00	6058	4.37	0.57	4.55	G0
1.2	2.20E+00	1.16E+00	6375	3.89	0.45	4.04	F6
1.3	3.28E+00	1.29E+00	6696	3.46	0.38	3.58	F4
1.4	4.71E+00	1.38E+00	7092	3.07	0.32	3.17	F1
1.5	6.50E+00	1.44E+00	7543	2.72	0.24	2.82	A9
1.6	8.73E+00	1.52E+00	7907	2.4	0.19	2.52	A7
1.7	1.15E+01	1.64E+00	8185	2.1	0.16	2.23	A7
1.8	1.50E+01	1.78E+00	8396	1.81	0.13	1.96	A5
1.9	1.93E+01	1.97E+00	8484	1.54	0.11	1.7	A4
2	2.47E+01	2.24E+00	8490	1.27	0.11	1.43	A4

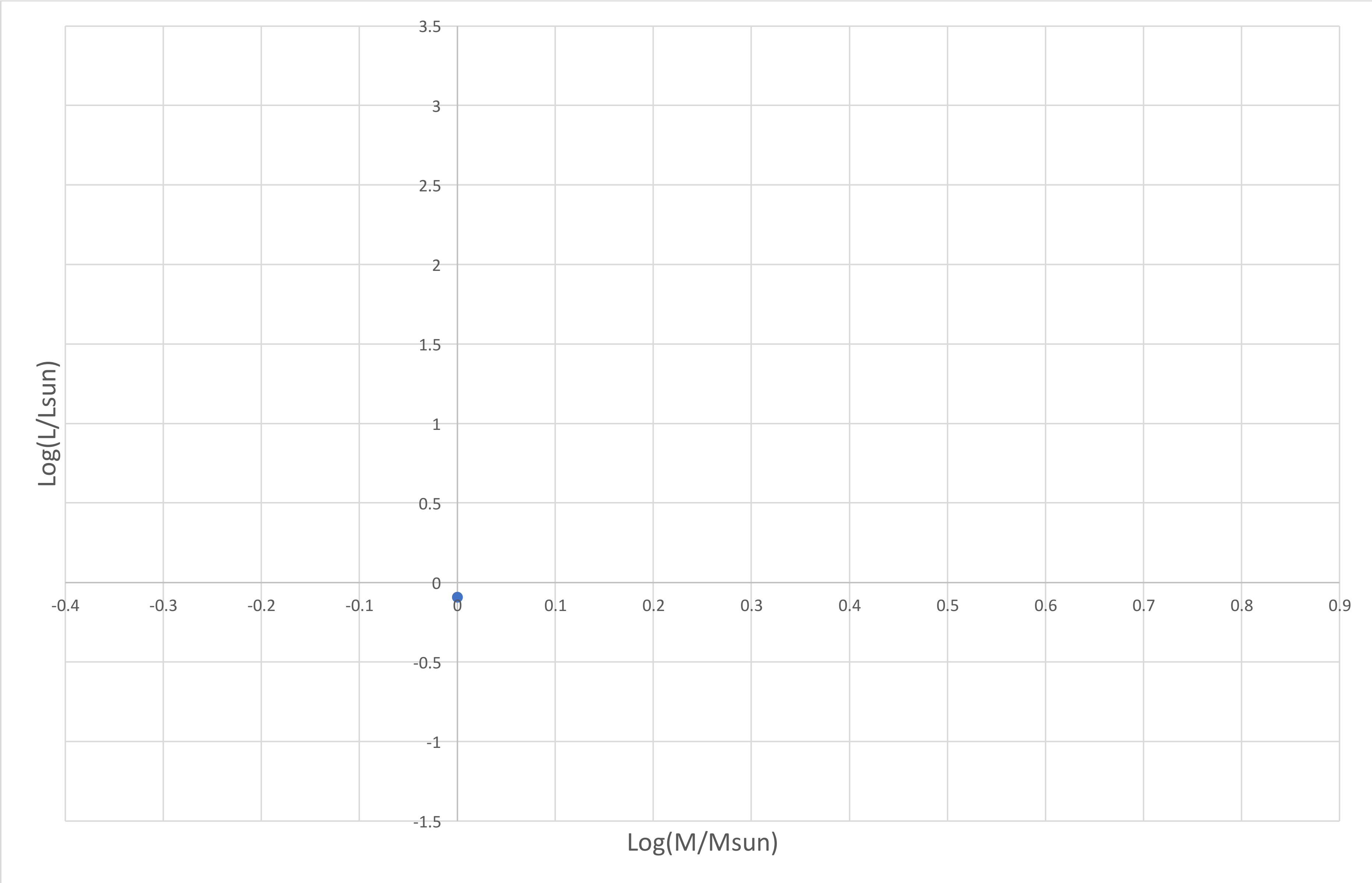
775M years

Mass	L	R	Teff	Bol	B-V	Mv	ST
1	8.87E-01	9.09E-01	5741	4.88	0.67	5.1	G6
1.1	1.43E+00	1.03E+00	6061	4.36	0.57	4.54	G0
1.2	2.22E+00	1.17E+00	6379	3.88	0.45	4.03	F6
1.3	3.32E+00	1.30E+00	6700	3.45	0.38	3.57	F4
1.4	4.78E+00	1.39E+00	7084	3.05	0.32	3.15	F1
1.5	6.61E+00	1.46E+00	7517	2.7	0.24	2.8	A9
1.6	8.91E+00	1.55E+00	7864	2.38	0.19	2.49	A7
1.7	1.18E+01	1.69E+00	8106	2.07	0.17	2.2	A7
1.8	1.54E+01	1.86E+00	8260	1.78	0.15	1.92	A6
1.9	1.99E+01	2.10E+00	8299	1.51	0.15	1.65	A6
2	2.53E+01	2.45E+00	8140	1.24	0.17	1.37	A7

850M years

Mass	L	R	Teff	Bol	B-V	Mv	ST
1	8.92E-01	9.11E-01	5743	4.87	0.67	5.09	G6
1.1	1.44E+00	1.04E+00	6065	4.35	0.57	4.53	G0
1.2	2.25E+00	1.17E+00	6376	3.87	0.45	4.02	F6
1.3	3.37E+00	1.30E+00	6703	3.43	0.38	3.55	F4
1.4	4.85E+00	1.40E+00	7085	3.04	0.32	3.13	F1
1.5	6.71E+00	1.48E+00	7491	2.68	0.25	2.79	A9
1.6	9.06E+00	1.59E+00	7817	2.36	0.2	2.47	A8
1.7	1.20E+01	1.74E+00	8016	2.05	0.18	2.18	A7
1.8	1.58E+01	1.94E+00	8118	1.76	0.17	1.89	A7
1.9	2.04E+01	2.25E+00	8045	1.48	0.18	1.6	A7
2	2.67E+01	2.71E+00	7825	1.18	0.2	1.3	A8


B-V
-0.5
0
0.5
1
1.5
m (V)
0
2
4
6
8
10
12


3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
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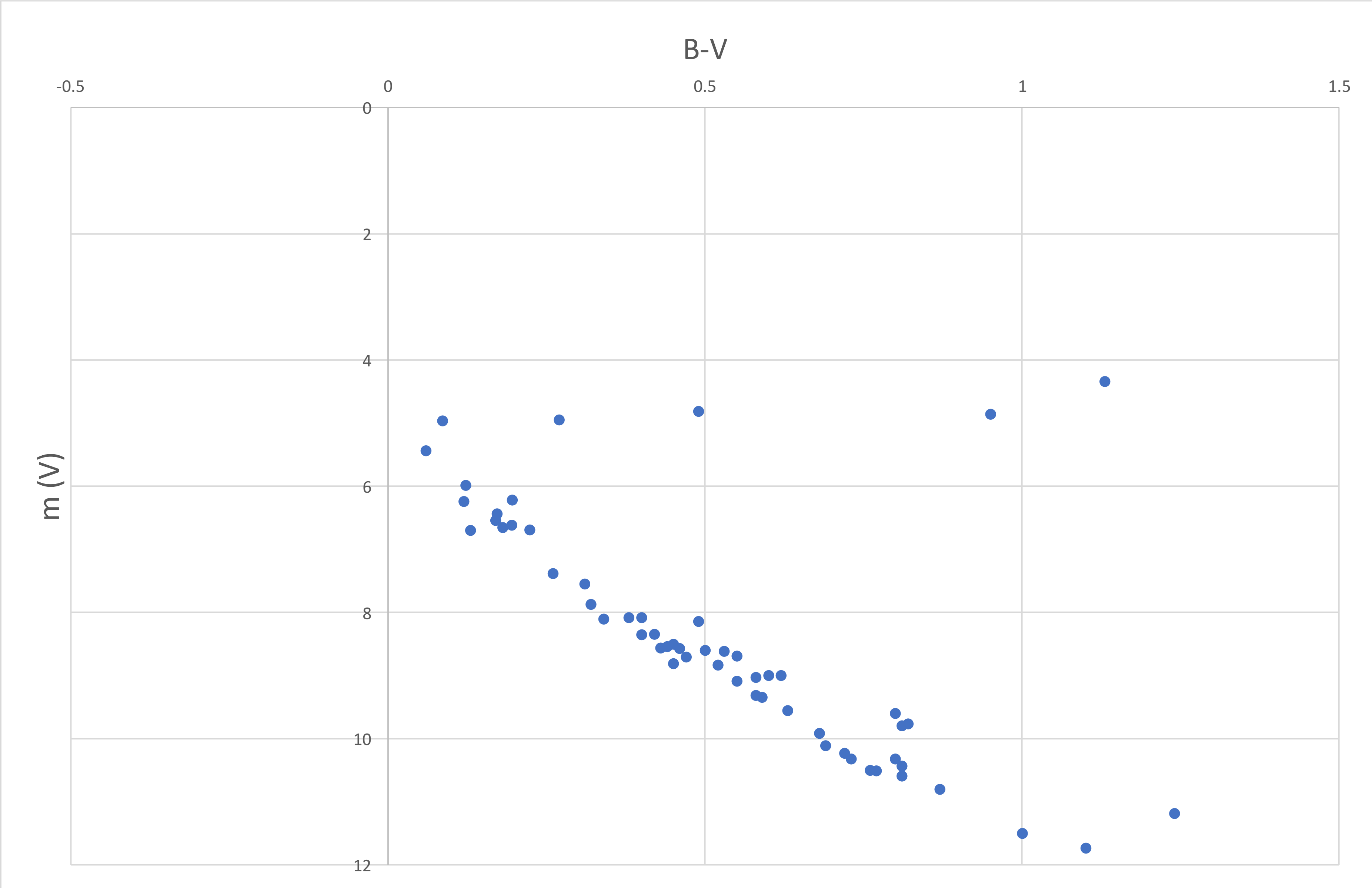
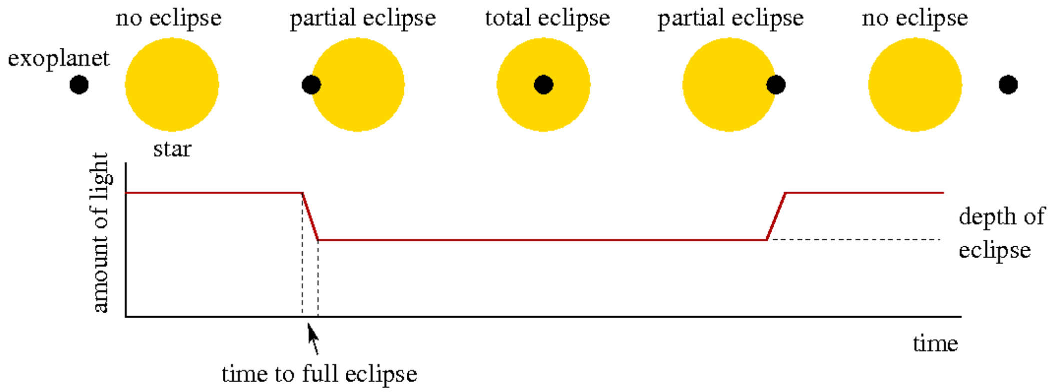
0.8
0.9
Log(L/Lsun)
Log(M/Msun)


B-V
-0.5
0
0.5
1
1.5
m (V)
0
2
4
6
8
10
12

DRQ 4: Finding Exoplanets: IT'S A TRAP(PIST1) [22 marks]

Exoplanets can be detected once they transit a host star, causing a dip in the host star's light curve periodically (Image modified from galileospendulum.org):



Let the stellar mass be M_*, stellar radius be R_*, planet radius be R_p, the orbital semi-major axis be a, and orbital inclination be i.

The transit depth can be expressed as ΔF, or change in flux (due to the planet blocking the host star). Let F_n be normal flux without a transit, and F_t be transit flux observed:

$$\Delta F = \frac{F_n - F_t}{F_n} = \left(\frac{R_p}{R_*}\right)^2$$

(Equation 1)

Since the transit curve can effectively be expressed as a periodic sine function, it can be expressed as a ratio of duration of the flat part of transit (t_F) to the total transit duration (t_T):

$$\frac{\sin\left(\frac{t_F \pi}{P}\right)}{\sin\left(\frac{t_T \pi}{P}\right)} = \frac{\sqrt{\left(1 - \frac{R_p}{R_*}\right)^2 - \left(\frac{a}{R_*}\cos i\right)^2}}{\sqrt{\left(1 + \frac{R_p}{R_*}\right)^2 - \left(\frac{a}{R_*}\cos i\right)^2}}$$

(Equation 2)

Next, the total transit duration, t_T can be expressed as:

$$t_T = \frac{P}{\pi}\arcsin\left(\frac{R_*}{a}\sqrt{\frac{\left(1 + \frac{R_p}{R_*}\right)^2 - \left(\frac{a}{R_*}\cos i\right)^2}{1 - \cos^2 i}}\right)$$

(Equation 3)

To visualise all of the above, this diagram was drawn by Seage & Mallen-Ornelas, 2003[1]:

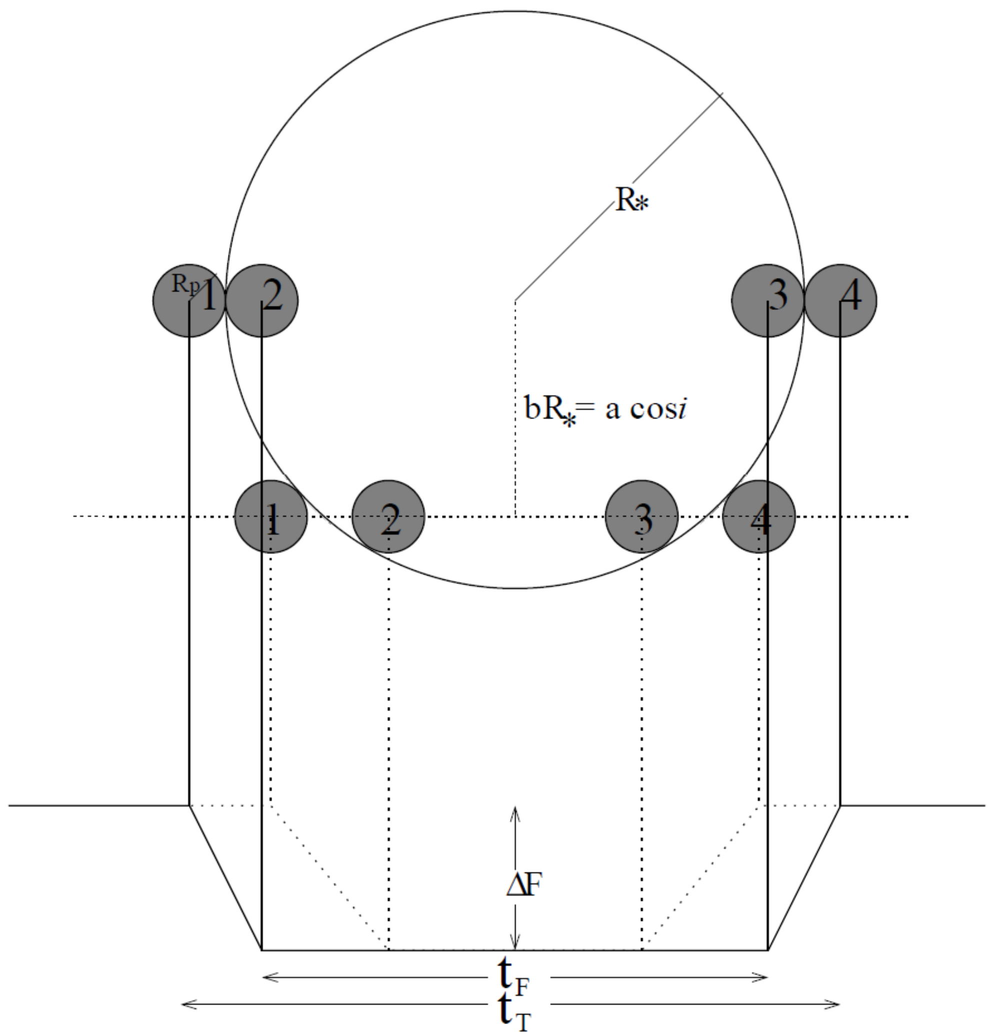


Figure 1. *Definition of transit light-curve observables. Two schematic light curves are shown on the bottom (solid and dotted lines), and the corresponding geometry of the star and planet is shown on the top. Indicated on the solid light curve are the transit depth ΔF, the total transit duration t_T, and the transit duration between ingress and egress t_F (i.e., the " flat part " of the transit light curve when the planet is fully superimposed on the parent star). First, second, third, and fourth contacts are noted for a planet moving from left to right. Also defined are R_*, R_p, and impact parameter b corresponding to orbital inclination i. Different impact parameters b (or different i) will result in different transit shapes, as shown by the transits corresponding to the solid and dotted lines.*

[1] Adapted from: Seager, S., & Mallen-Ornelas, G. (2003). A unique solution of planet and star parameters from an extrasolar planet transit light curve. *The Astrophysical Journal*, *585*(2), 1038.

i) Using the above equations and Fig. 1, show that semi-major axis, *a*, can be expressed as: [3 marks]

$$a = R_* \sqrt{\frac{\left(1+\sqrt{\Delta F}\right)^2 - b^2\left[1-\sin^2\left(\frac{t_T \pi}{P}\right)\right]}{\sin^2\left(\frac{t_T \pi}{P}\right)}}$$

(Equation 4)

ii) Show that, when orbital inclination is assumed to be 90°, i.e. the planet's orbit is exactly on the plane of the host star's ecliptic, the following equations can be obtained: [2 marks]

$$t_T = \frac{P}{\pi} \arcsin\left(\frac{R_*}{a}\left(1+\frac{R_p}{R_*}\right)\right)$$

(Equation 5)

$$a = R_* \frac{\left(1+\sqrt{\Delta F}\right)}{\sin\left(\frac{t_T \pi}{P}\right)}$$

(Equation 6)

Just as you're about to hit the *PANIC button*, you're in luck: Experts have already helped you to derive stellar density, ρ_*, which can be expressed as:

$$\rho_* = \left(\frac{M_*}{R^3}\right) = \left(\frac{4\pi^2}{P^2 G}\right)\left(\frac{\left(1+\sqrt{\Delta F}\right)^2 - b^2\left[1-\sin^2\left(\frac{t_T \pi}{P}\right)\right]}{\sin^2\left(\frac{t_T \pi}{P}\right)}\right)^{\frac{3}{2}}$$

(Equation 7)

iii) What is the key assumption made, in order to obtain equation 7 from equation 4 in the process? (Hint: You can derive equation 7 from equation 4 in case you cannot figure it out, and you will notice a certain term is missing. Actual derivation workings NOT required.) [1 mark]

iv) Briefly state two other key assumptions or conditions that need to be made or currently exists, in order to successfully use the above equations in order to compute parameters (you do not need to explain why). [1 mark]

If you're wondering if this question was indeed a trap… **you're probably right**. Just as you thought things couldn't get any worse, you're presented with a case study of the planetary system, TRAPPIST-1 (Disclaimer: this exists in real life, discovered by the Transiting Planets and Planetesimals Small Telescope in Belgium. Belgium has a famous beer brand named – you guessed it – Trappist Beer).

Spectral type	M8V
Distance	39.5 ly (12.1 pc)
Absolute magnitude (MV)	18.4
Mass	0.08 $M_{\odot}$
Radius	0.114 $R_{\odot}$
Luminosity (visual, LV)	0.00000373 $L_{\odot}$
Surface Temperature	2550±55 K
Metallicity	0.04
Rotation	3.30 days
Age	3–8 Gyr

Table 1 *Data pertaining to host star, TRAPPIST-1a*

Designation of planet	Mass ($M_{\oplus}$)	Semimajor axis (AU)	Orbital period (days)	Eccentricity	Inclination (°)	Radius ($R_{\oplus}$)
b	0.79 ± 0.27	**UNKNOWN**	1.51	0.019	89.65	1.086
c	1.63 ± 0.63	0.01522	2.42	0.014	89.67	1.056
d	0.33 ± 0.15	0.021	4.05	0.003	89.75	0.772
e	0.56 ± 0.24	0.028	6.10	0.007	89.86	**UNKNOWN**
f	0.36 ± 0.12	0.037	9.21	0.011	89.68	1.045
g	0.57 ± 0.04	0.045	12.35	0.003	89.71	1.127
h	0.086 ± 0.08	0.060	18.76	0.086	89.8	0.715

Table 2 *Data about the TRAPPIST-1 planetary system, with some information missing.*

~~(Eeeeek) Using the most complicated formula you've seen, calculate the transit duration of…~~

Just as you're about to hit the *PANIC button* again, your Astrochallenge intellect informs you that Equations 3 and 4, while accurate and important for other planets, aren't necessary for the TRAPPIST-1 star system. The simpler Equations 5 and 6 would suffice.

With this in mind, answer the following questions instead:

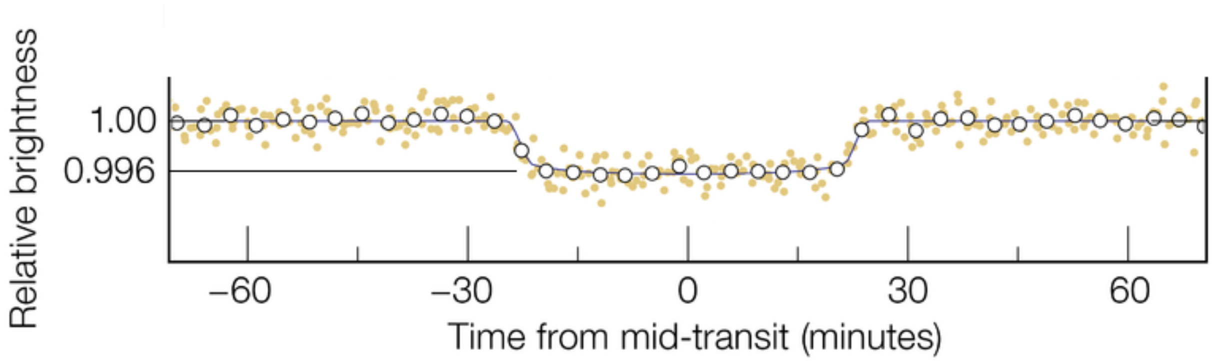
v) Why can you use Equation 5 and 6 for the TRAPPIST-1 system? [0.5 marks]

vi) Given that the transit duration of TRAPPIST-1b is approximately 0.025 days, calculate the semi-major axis of TRAPPIST-1b, leaving your answer in <u>AUs</u>. [1 mark]

vii) Calculate the transit duration of TRAPPIST-1d, leaving you answer in number of <u>days</u>. [1 mark]

viii) Given that the transit duration of TRAPPIST-1e is approximately 0.0395 days, calculate the radius of TRAPPIST-1e, leaving your answer as a ratio compared to Earth's Radius ,$R_\oplus$. [1.5 marks]

The following is a light curve of one of the planets in TRAPPIST-1.



ix) Determine which planet is featured by the above light curve, showing clear evidence why you think so. [2 marks]

Tired of all the calculation questions? Here's some theory and a little controversy:

The inner and outer limits of the Habitable Zone of TRAPPIST-1a is determined to be a distance between 0.024 AU and 0.048 AU by one group of experts (Kopparapu et al., 2013, 2014), according to the following:

Inner limit: "runaway greenhouse limit where a planet's temperature would soar even with no CO2 present and lose all of its water in a geologically brief time in the process".

Outer limit: "maximum greenhouse limit of a CO2-rich atmosphere where the addition of any more of this greenhouse gas would not increase a planet's surface temperature any further".

x) State the importance of the habitable zone in astrobiology and the implications of the above result pertaining to <u>all</u> the planets in the TRAPPIST-1 star system. [1.5 marks]

Other experts disagreed, arguing that due to the combined effect of tidal heating and tidal locking, up to all of the planets might be suitable for life or none of them might be suitable at all. Optimistic experts saw a loophole in the habitable zone's definitions which would allow life on these planets, while Pessimistic experts cited the Rare Earth Hypothesis and why these planets are all hostile for life. <u>You have to pick a side</u>**.**

xi) Briefly suggest how tidal heating and tidal locking EITHER improves OR hampers the feasibility of life for the TRAPPIST-1 system. [1.5 marks]

xii) Suggest two other methods other than the transit method for discovering exoplanets, and briefly state their principles. [2 marks]

One final obstacle awaits you: A dreaded graphing question requiring everything you've seen so far and your wits.

(Hint: It would be helpful to simplify Equation 2 for this following section. Workings are recommended but not strictly required for the final answer, though they can salvage your marks and give far more accurate diagrams. Feel free to use a pen/ highlighter of a different colour if it helps.)

xiii) Sketch the following, with both light curves on the same diagram, labelling each line as accurately and clearly as possible: [4 marks]

A) The transit curve of TRAPPIST-1c, assuming negligible inclination.
Recommended units: Relative brightness/ flux compared to usual (1 $F_{TRAPPIST\text{-}1a}$) and time from mid-transit in minutes

B) The transit curve of TRAPPIST-1c but with twice the current planet radius, with everything else remaining the same. (assume negligible inclination as well)

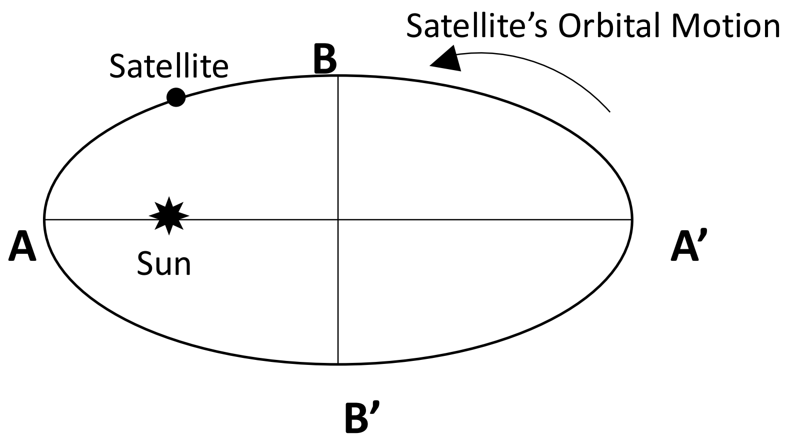
In case you're still trapped by TRAPPIST-1, press this button.
(Disclaimer: the DRQ will not explode)

[DON'T PANIC]*

***Don't throw in the towel. Read the question carefully, move on to an easier section, and/or ask for assistance from your team. You can do it!**

DRQ 5: Elliptical Orbit Analysis [18 marks]

A satellite moves in an elliptical orbit with semi mayor axis ***a***, and eccentricity ***e***. The satellite can be considered as a particle that moves under the influence of Sun's gravity. Its velocities at perihelion and aphelion are denoted by V_{pe} and V_{ape}, respectively.



Based on the information above, please answer the following questions:

i) State 2 quantities that are conserved in the satellite's motion. [1 marks]

ii) Hence or otherwise, show that the velocity of the satellite at any arbitrary distance ***d*** from the sun along its orbits is as follows: [3 marks]

$$V_d^{\ 2} = 2GM_{sun} * \left(\frac{1}{d} - \frac{1}{2a}\right)$$

iii) Derive the expression for perihelion distance d_{pe} in the terms of ***a***, the perihelion velocity V_{pe}, and the aphelion velocity V_{ape}. [3 marks]

iv) State Kepler's 2nd Law [1 mark]

v) In order to simplify our work, we define here the area constant to be the constant of proportionality of the area to the period of orbit. It is known that the area constant is defined as half of the satellite's angular momentum per its unit mass at any arbitrary position, express the satellite's area constant in terms of the other known quantities. [2 marks]

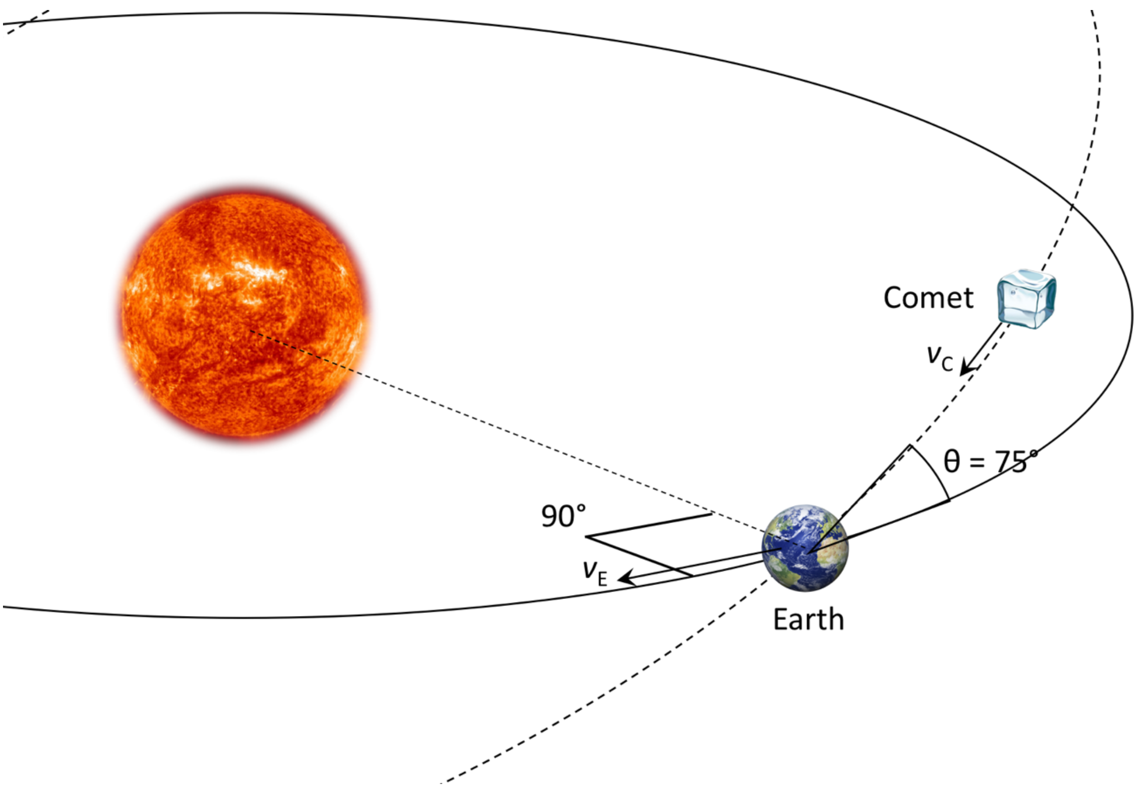
vi) Hence or otherwise, using your results in d) and e), prove that the orbital period P can be expressed as follows. [2 marks]

$$P = \frac{\pi a^2\sqrt{1-e^2} * (V_{pe} + V_{ape})}{V_{pe} * V_{ape}}$$

Recall that the area of an ellipse = $\pi ab = \pi a^2\sqrt{1-e^2}$, where b is the semi-minor axis.

vii) Using the results found above, calculate the velocities at perihelion and aphelion, $\boldsymbol{V_{pe}}$ and $\boldsymbol{V_{ape}}$ for Earth. Comment on the two values obtained. [2 marks]

We will now briefly study the legendary dinosaur killer, Chicxulub impactor. We will employ a model in which it is a massive metallic object with a mass of 5.3×10^{15}kg with a kinetic energy before impact of about 0.7×10^{24}J. In this model, it's believe the impactor was originally a long-period comet in orbit about the sun.



viii) Calculate the angular momentum of the impactor just before impact assuming that it collided tangentially to Earth along its orbit. You may assume circular orbit for Earth here. [2 marks]

ix) Hence or otherwise, calculate the inclination of the final orbit relative to the original orbit should a similar object were to collide Earth in this day. Assume that the impactor approaches at an inclination of about 75.0° before impact, as measured from the plane of orbit of Earth. [2 marks]