ASTROCHALLENGE 2025
JUNIOR TEAM ROUND

SOLUTIONS

Wednesday 4th June 2025

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[6] MedMig YT. *Amelia Watson ground pounds desk-kun [HololiveEN Animation]*. YouTube video. Screenshot taken at 0:03. Jan. 2021. URL: https://www.youtube.com/watch?v=wiMdgOoCBjI.

Question 1 Our Solar System Family

(a) Galaxies are filled with giant gas clouds, and they are mostly stable. Suggest one possible event that can cause the gas clouds to collapse and form stars. **[1]**

Solution:

1. Shockwaves from nearby stars/ supernova
2. A large object passing through the cloud

(b) At first, the gas cloud is barely rotating. As the gas cloud collapses to form a star cluster, it rotates faster and faster. Why? **[1]**

Solution:

Conservation of angular momentum. As the radius of the disk decreases, the velocity increases.

(c) Explain why the Sun and planets of the Solar System are all seen near the ecliptic. **[2]**

Solution:

As the gas cloud collapses, it rotates and forms a disk. Since the sun and planets all form from the same disk, with the same angular velocity, all the planets rotate around the sun on the same plane as the original protoplanetary disk.

(d) Mercury's surface temperature can vary from 430 °C to −180 °C. Explain why this happens. **[2]**

Solution:

Mercury is tidally locked, so one side experiences extreme heating, while the other side receives no sunlight at all. The lack of atmosphere also makes it harder for energy to be evenly distributed around the planet.

(e) All planets of the Solar System orbit the Sun and rotate on its axis in the anticlockwise direction when viewed from the top, except for Venus, which rotates clockwise. Suggest a reason why. **[1]**

Solution:

An impact with another object during the early solar system caused it to change its direction of spin.

(f) The oldest rocks on Earth are estimated to be 4.5 billion years old, and we believe Earth is the same age. Explain why and how we can tell the age of the Earth just by 'looking at rocks'. **[2]**

Solution:

There are radioactive isotopes of elements that decay over time. By knowing the half life of the elements, and the proportion of the isotopes, we can estimate the age of the rock.

(g) Although the moon is around the same age as the Earth, its surface is very heavily cratered compared to Earth. Suggest 2 reasons why. **[2]**

Solution:

Earth has an atmosphere that causes most meteorites to burn up and lose mass before reaching the ground. The Moon, however, does not have an atmosphere. Earth has plate tectonics which causes the surface to be shifted around, covering up old craters. Since the moon does not, old craters will remain on the surface.

Figure 1: The surface of Mars showing long, valley like structures. [2]

(h) Suggest what may have caused the long, valley-like structure on the Martian surface (see Figure 1). **[1]**

Solution:

It might have been flowing water eroding the surface, or plate tectonics.

Accept any plausible answers.

(j) Some people think that Jupiter protects Earth from asteroids, while some think that it sends more asteroids our way. Why does Jupiter have the ability to do such things? **[2]**

Solution:

Jupiter's large mass causes it to have large gravitational attraction on surrounding objects. Asteroids might be attracted towards Jupiter instead of Earth, protecting Earth. Or Jupiter may also cause an asteroid to change its trajectory and move towards Earth instead.

(k) Saturn is very well known for its rings. In March 2025, the rings were not visible for any observers on Earth. Suggest a reason why. **[1]**

Solution:

The plane of the rings was edge on to observers on Earth, and since the rings are very thin, they could not be seen.

(m) Ancient civilizations more than 2000 years ago already knew about the existence of the Sun and the planets Mercury to Saturn, but Uranus was only discovered in 1781. Why did people not know about Uranus before that? **[1]**

Solution:

The planets Mercury to Saturn are very bright, allowing ancient observers to see them. However, Uranus requires a telescope to be seen, and it was not until much later when telescopes were invented, before Uranus was discovered.

(n) Although Neptune has a mass 17 times the mass of the Earth, its surface gravity is actually similar to that of Earth. If you were standing on its surface (if it has a solid surface), the gravity you would experience is very similar to Earth's. Why is this so? **[1]**

Solution:

The radius is 4 times of Earth. GMm/r^2, the square of 4 is 16, which cancels out (approximately) the effect of 17 times mass.

(o) Does your team think Pluto should be considered a planet? Why or why not? **[1]**

Solution:

Any answer on whether Pluto should be considered a planet or not will be given full marks as long as the explanation supports the answer. For example, if the student answer that Pluto should not be a planet, the explanation could be:

- Did not clear orbit
- Different orbital plane
- etc.

(p) Why is it not possible to get a 'family portrait' of all the planets at local midnight, even if you could live long enough to see all the possible arrangements and positions of the planets? Which planets can you not see, and why? Assume that your technology is good enough to capture a decent portion of the sky and can see Uranus and Neptune. **[2]**

Solution:

Mercury and Venus will never be visible at local midnight as they are inferior planets, since the angular distance from the sun is always small.

Question 2 Exploring Colliding Dead Stars

Part I Finding the ripples in space-time

What happens when massive bodies move in the four-dimensional (three-spatial and one-time dimension) spacetime? Are there any perturbations we can observe? 10 years ago, in 2015 we made our first observations of gravitational waves. However, this question will discuss another instance of detection of these waves. On the 17th of August, 2017, Ligo detected gravitational waves (GW170817) from two orbiting objects with the pattern as shown in Figure I-1.

(a) What does the middle graph (labelled LIGO-Livingston) in Figure I-1 tell about the two orbiting objects? **[1]**

Solution:

The two bodies in orbit are getting closer and closer to each other at an accelerating rate and they collide in the end. **[1]**

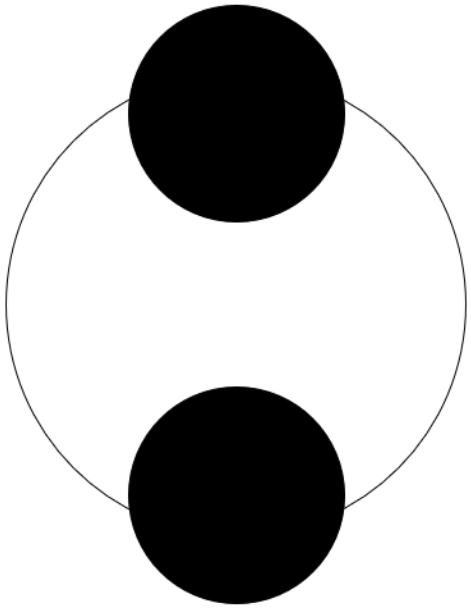

Figure 2: The two bodies of equal mass orbiting around the centre.

(b) Refer to Figure 2. Assuming the two objects are of equal mass, calculate the orbital period of the two objects at the gravitational wave frequency of 100Hz (This is a circular orbit). **[1]**

Solution:

$$T = \frac{1}{0.5f}$$
$$= 0.02\,\mathrm{s}$$

Since both objects are identical, the position of the two objects relative to the observer looks the same every 1/2 of the period, hence 2 gravitational wave periods are identical to one real period of orbit. [1]

(c) Given $m_1 = (1.81 \pm 0.45)M_\odot$ and $m_2 = (1.11 \pm 0.25)M_\odot$, is this system more likely a neutron star-black hole pair or neutron star-neutron star pair? *(Hint: The Chandrasekhar limit is about $1.4M_\odot$.)* **[1]**

Solution:

m_1 is right around the Chandrasekhar limit ($1.4M_\odot$), the maximum mass of a white dwarf, before the object is a neutron star. The limit for maximum mass of a neutron star is uncertain but current estimates put the maximum mass for a neutrons star between 2.2 and 2.9 solar masses. Hence, it is very unlikely to find a black hole of such a low mass. m_2 is right below the Chandrasekhar limit, hence, it is very unlikely for it to be a black hole. The condition in the question requires that at least one object is a neutron star, hence m_2 is very likely a neutron star. It is possible that m_2 has had mass loss after it became a neutron star. Therefore it is likely that it is a neutron star-neutron star pair [1]

Given that m_1=1.81$M_\odot$ and m_2 = 1.11$M_\odot$, and the total distance between the m_1 and the barycenter is 116.32 km, the total distance between m_1 and m_2 is about 300 km. the power loss from gravitational waves can be described using Equation 1:

$$-\left\langle \frac{dE}{dt} \right\rangle = \frac{32G^4 m_1^2 m_2^2 (m_1+m_2)}{5c^5 a^5 (1-e^2)^{7/2}} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right) \tag{1}$$

where e is the orbital eccentricity and a is the length of the semimajor axis.

It is known that 2.5% of the total energy of the system was lost as gravitational waves for a duration of 1.5 s in the collision.

Assuming that bodies involved in the collision are point mass and there was no other loss of energy, Milo tried to calculate the final mass of the remnant. This is his working:

> Since the orbit is circular,
>
> $$e = 0 \tag{2}$$
>
> $$a = 300\,\text{km} \tag{3}$$
>
> Using Equation 1,
>
> $$P_{\text{loss}} = \frac{32G^4 m_1^2 m_2^2 (m_1+m_2)}{5c^5 a^5 (1-e^2)^{7/2}} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right) = \frac{32G^4 m_1^2 m_2^2 (m_1+m_2)}{5c^5 a^5} \tag{4}$$
>
> $$E_{\text{loss}} = P_{\text{loss}}\, t = \frac{32G^4 m_1^2 m_2^2 (m_1+m_2) t}{5c^5 a^5} \tag{5}$$
>
> $$E_{\text{remaining}} = \frac{0.975}{0.025} E_{\text{loss}} = \frac{0.975}{0.025} \times \frac{32G^4 m_1^2 m_2^2 (m_1+m_2) t}{5c^5 a^5} \tag{6}$$
>
> $$M_{\text{remaining}} = \frac{E_{\text{remaining}}}{c^2} = \frac{1}{c^2} \times \frac{0.975}{0.025} \times \frac{32G^4 m_1^2 m_2^2 (m_1+m_2) t}{5c^5 a^5} = 5.12 \times 10^{27}\,\text{kg} \tag{7}$$

However, it turned out that the actual mass of the remnant is 5.45×10^{30} kg.

(d) One of the key steps above contained a mistake, which resulted in the disparity in the calculation outcome. Which step is it? Explain why it is a mistake. **[1]**

Note that there is no mistake in the numerical calculation. You may use the equation number to refer to the step.

Solution:

Equation 3 contains a mistake. [0.5]

The mistake is that the semi-major axis is constant throughout the merger process.

As the stars collide, the semi-major axis of their orbit, a, decreases from 150 km to 0. So it does not make sense that $a = 300$ km through the entire process. [0.5]

(e) What is one technique that you can use to correct for this mistake? Qualitatively explain how this technique will be helpful. **[1]**

Solution:

We should account for the change of semi-major axis throughout the merger process by seeing $a(t)$ as a function of t. [0.5]

We can then use the technique of integration to account for the change in $a(t)$. [0.5]

Specifically, once we write $a(t)$ as a function of t, now we have $-\left\langle \frac{\mathrm{d}E}{\mathrm{d}t} \right\rangle$ as a function of t. That is, we have power loss $P_{\text{loss}}(t)$ as a function of t. Then, in step 5, we can find the total energy loss by

$$\int P_{\text{loss}}(t)\,\mathrm{d}t$$

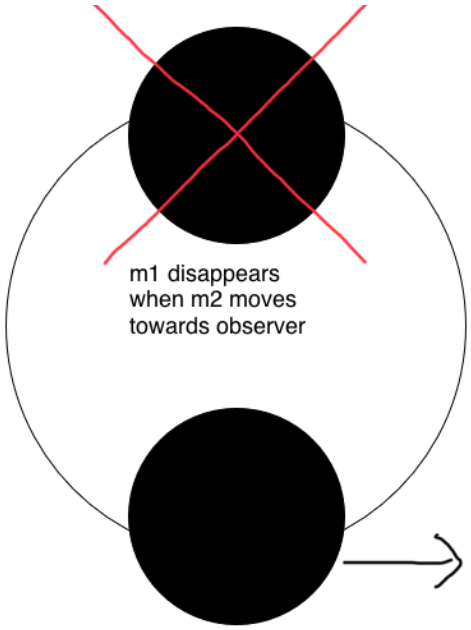

Figure 3: m_1 disappears, m_2 is directly coming towards you.

(f) Suppose before the collision, m_1 disappears when the orbital frequency is 100Hz. Figure 3 depicts the moment when m_1 disappears.

Find the angular displacement of m_2 after m_1 disappeared but before m_2 was flung out.

You may assume the speed of gravitational waves to be the speed of light. **[2]**

Solution:

$$\text{Time for gravity to travel} = \left(\frac{300 \times 10^3}{3.00 \times 10^8}\right) = 0.001\,\text{s} \;\; [0.5]$$

$$\text{Fraction of orbit traversed} = \left(\frac{0.001}{0.02}\right) = 0.05 \;\; [0.5]$$

$$\text{Angular displacement} = 0.05 \times 360^\circ = 18^\circ \;\; [1]$$

Part II The Kilonova

During the merger, atomic nuclei go through rapid neutron capture process (r-process), in which atoms capture neutrons quickly. Due to some of these nuclei being unstable, they decay quickly, releasing light - the kilonova! (The kilonova is accompanied by a short Gamma Ray Burst (GRB) of less than 2s and X-ray/radio synchrotron emission. Additionally there is a GRB afterglow in X rays and radio waves).

The discovery of the GRB together with the gravitational waves, confirmed our theory that gravity propagates at the same speed as light. Figure I-3 shows the emission spectrum of AT2017gfo, the electromagnetic counterpart detection of the merger. Studying the spectra tells the nature of the spherically expanding output material. We will investigate this in more detail in the coming questions.

(g) Explain why there are peaks at certain wavelengths in this light spectrum. **[1]**

Solution:

Atoms or molecules have electrons that drop from a higher energy level to a lower energy level. These discrete energy levels correspond to discrete wavelengths of light emitted and observed as peaks in the light spectrum. [1]

OR

Synchrotron emission at certain wavelengths when electrons interact with magnetic field. [1] (although first answer is preferred)

(h) Explain why the light spectrum looks like a bell-shaped curve instead of sharp emission lines. **[1]**

Solution:

Output material from the kilonova is dispersed in a spherical shell around the center of collision. [0.5]

Hence, some material disperses towards us observers while some material disperses away. This creates a blue shift and redshift respectively of the emission wavelength emitted by the output material. [0.5]

Thus, producing a bell shape of wavelengths longer and shorter than the peak emission wavelength.

(j) Take the 1.58 μm (micrometer) feature in the +8.4 d graph. Estimate the full width at half maximum intensity (FWHM)[1] of the curve. **[1]**

Solution:

The answer in the range 0.06-0.10 μm is acceptable. [1]

(k) Let f_o be the observed frequency, f_s be the source frequency, and v_s be the source radial velocity. The Doppler effect equation relating these quantities is Equation 8.

$$f_o = f_s\left(\frac{c}{c \pm v_s}\right) \tag{8}$$

Assuming spherical expansion of output and taking into account the Doppler broadening, find the expansion velocity of the output material in terms of the speed of light. **[2]**

Solution:

[1] FWHM is the width of a spectrum curve measured between those points on the y-axis which are half the peak amplitude.

The wavelength can be converted to frequency using the equation

$$f = \frac{c}{\lambda} \quad [0.5]$$

Furthermore, manipulating Equation 8 gives us

$$v_s = \pm\left(\frac{cf_s}{f_o} - c\right) \quad [0.5]$$

The acceptable answer is in the range

$$5.70 \times 10^6\,\mathrm{m\,s^{-1}} < v_s < 1.90 \times 10^7\,\mathrm{m\,s^{-1}} \quad [1]$$

The emission spectrum of AT2017gfo shows the presence of tellurium, formed from the decay of heavy elements — these are formed via the rapid neutron capture process (r-process). Tellurium-128 is an isotope formed via the radioactive decay of antimony (Sb). This decay - negative beta decay - releases an electron (e^-) and an electron antineutrino ($\bar{\nu}_e$). The decay scheme of Sb-128 is given as follows:

$$^{128}_{51}\text{Sb} \longrightarrow ^{128}_{52}\text{Te} + e^- + \bar{\nu}_e$$

You are also given the molar mass of Sb-128 and Te-128:

$$\text{molar mass of } ^{128}_{51}\text{Sb} = 127.9091673\,\text{g/mol}$$

$$\text{molar mass of } ^{128}_{52}\text{Te} = 127.9044614\,\text{g/mol}$$

(m) Using $E = mc^2$, calculate the energy released (total energy of electrons and antineutrinos released) in the decay of one atom to 3 significant figures. **[1]**

Solution:

The difference in molar mass is $\Delta M = 0.0047059\,\text{g/mol}$. Therefore, the difference in the mass of the decay in one atom is $\Delta M / N_\text{A}$ where N_A is the Avogadro's number. The energy is then

$$\begin{aligned} E &= mc^2 \\ &= \frac{0.0047059 \times 10^{-3}\,\text{kg/mol}}{6.02 \times 10^{23}/\text{mol}} \times (3.00 \times 10^8\,\text{m s}^{-1})^2 \\ &= 7.03 \times 10^{-13}\,\text{J} \quad [1] \end{aligned}$$

(n) Given that the heliocentric velocity (receding away from the sun) of NGC 4993, the host galaxy of AT2017gfo, is 2930km/s, find the distance to NGC 4993 in Mpc. **[1]**

Solution:

Using Hubble's law $v = H_0 d$,

$$\begin{aligned} d &= \frac{v}{H_0} \\ &= 43.2\,\text{Mpc} \quad [1] \end{aligned}$$

(o) Given that AT2017gfo is observed to have a peak apparent magnitude of 17.22 in the Y0 band, find the peak absolute magnitude of AT2017gfo in the Y0 band. **[1]**

Solution:

Using the distance modulus formula,

$$\begin{aligned} M &= m - 5\log\left(\frac{d}{10\text{ pc}}\right) \\ &= -16.0 \quad [1] \end{aligned}$$

(p) Given that the AT2017gfo peak magnitude of 17.22 is observed over 1 second, estimate the number of Sb-128 atoms that decayed during that second. **[2]**

Hint: Refer to the absolute bolometric magnitude formula in the formula book.

Solution:

The absolute bolometric magnitude has been calculated in part (o). Therefore

$$M_{\text{bol}} = -2.5 \log_{10}\left(\frac{L}{L_\odot}\right) + 4.7554$$

$$\implies L = 7.7207 \times 10^{34}\,\text{W} \quad [1]$$

$$\text{number of decays} = \frac{7.7207 \times 10^{34}}{7.03 \times 10^{-13}}$$

$$= 1.10 \times 10^{47}\,\text{decays} \quad [1]$$

If students have the value of M_{bol} wrong in part (o), ECF is given as long as the steps in part (p) are correct.

Part III Galactic Influence

(q) Galactic quenching is a process in which star formation is cut off in a galaxy. How may supernovae cause quenching? How do Kilonovae compare? **[2]**

Solution:

Supernovae eject high-energy material that heat up cold gas in galaxies, which is optimal for star formation [1]. They also create winds that drive out cold gas [1]. These effects disrupt the collapse of nebulae into protostars and hence disrupt star formation, contributing to the quenching of a galaxy.

(r) A popular astronomer, Carl Sagan, had said "We are made of star-stuff". With our newfound understanding, do you agree with the statement? Explain your answer. **[1]**

Solution:

Any answer that explains we are made of both star-stuff and neutron star-stuff will be given marks [1].

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Question 3 Problem Caused by *Skill Issue*

NOTE: THIS IS A QUESTION DESCRIBING A FICTIONAL SITUATION THAT OCCURRED IN A FICTIONAL UNIVERSE. ANY COINCIDENCE IN NAMES, EVENTS, ORGANISATION AND OTHER ENTITIES ARE PURELY COINCIDENTAL.

Tails, an inhabitant of Earth, was playing a game of *Valorant* and was about to hit the "Radiant" rank, where an Internet outage suddenly happened and made her lose the game! Seeing her rank points drop, Tails got so tilted that she slammed her desk, which to her (and everyone's) surprise, caused the Earth's position to shift and its orbit to become tilted!

Figure 4: A rough illustration of the desk slam that caused this whole incident [6]

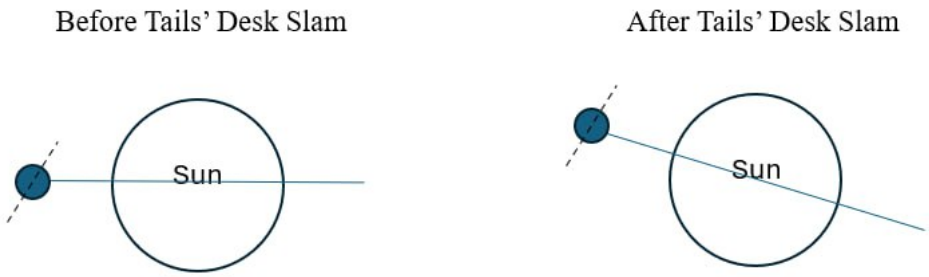


Figure 5: Tails' illustration of the solar system (before and after her tilt)

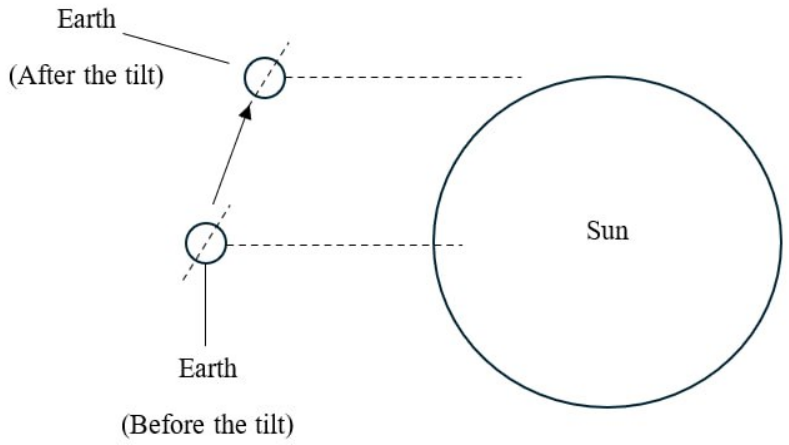
Note: As this question is set in a fictional universe, we shall make the following assumptions to make out lives easier. You are reminded of the need to take these assumptions into account when answering the questions.

1. The Solar System consists of only the Earth and the Sun, as shown in Figure 5.
2. The Earth is in a circular orbit around the Sun.
3. The physical properties of all objects in the Solar System, including the distance between the Earth and Sun, are as found in the Formula Booklet. The Solar System itself is assumed to not be moving.
4. Tail's Valorant tilt has only caused the Earth's position and the inclination of its orbit to change, as shown in Figure 5. The orientation of the rotational axis of the Earth, distance between the Earth and the Sun and other orbital parameters did not experience any changes.

Naturally, being small beings on Earth, it would be relatively difficult to physically feel the shift itself. However, certain objects can still serve as a strong indicator that the shift has happened, such as the Sun!

(a) With the aid of a diagram, describe and explain how one may immediately realise that the Earth's orbit has been perturbed? **[1]**

Solution:



The position of the Sun in the sky would appear to have shifted. As all other parameters were assumed to remain the same, the Earth's sudden change in position would result in a shift in the Sun's position in the sky with respect to its inhabitants.

Other logical answers can be accepted.

Realising that the Earth's orbit has been altered, Tails hurriedly contacted the only astronomers that she knows - us, **who are stationed at a space station far away from the Solar System**. Tails asks us whether the situation is an emergency, since this is the first time such an event has ever happened.

(b) Using your knowledge of astronomy, explain whether the change to Earth's orbit constitute an emergency to the inhabitants of Earth. **[1]**

Solution:

The change in the Earth's orbit will likely constitute an emergency to the inhabitants of Earth if significant enough.
A change in inclination would result in a change in the orbital axis of the Earth and hence the axial tilt of the planet. As seasons are derived from the axial tilt of the Earth, a change in that may result in serious repercussions in the overall climate of the Earth, impacting the survival of its inhabitants.
Any other logical and physically correct answers/explanations/opinions are accepted.

Regardless of whether it is an emergency or not, it was decided that it would be the best to try and rectify the issue. After a bit of deliberation, Tails realised that a simple solution may be able to help her in making things back to how they were again.

> *"If I play enough* Valorant*, get tilted and then slam the table enough times, everything will probably go back to normal."*
> - Tails, probably

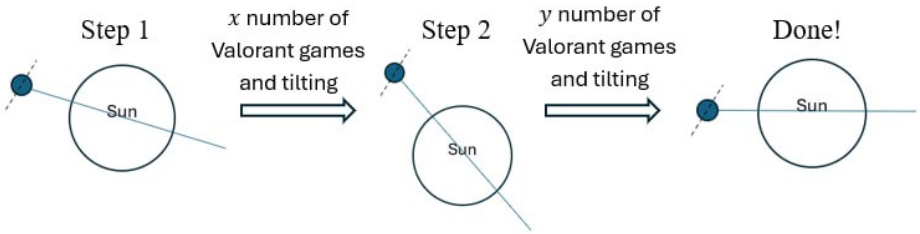


Figure 6: A brief visualisation of Tails' Plan

However, knowing that Tails suffers the chronic condition known by most as *Skill Issue*, she may accidentally cause the destruction of the Earth while conducting her plan. As such, it would be the best for us to track the presence of Earth, for us to see whether we will have to be self-sufficient for the rest of out lives in the space station. So, let's consider using some of the most common ways of detecting the presence of a planet like Earth (other than going to the planet and seeing whether it's there or not) and see what we can expect to see as Tails continues to tile and change the inclination of the Earth!

But before that, we should go through something that not many of us has heard before...

Part I Hold up - Orbits... Can Tilt?!

Just like any objects in 3-dimensional space, orbits can indeed be tilted! Such tilt takes a special name in astronomy (just like in any other sciences) - inclination. Formally, the definition of inclination is as such:

> *"The inclination of an orbit, i, is the angle between the reference plane and the orbital plane."*
> - Encyclopaedia Kushagra[2]

As Tails' legendary *Valorant* tilt caused a sudden tilting of the Earth's orbit, it hence would be of our interest to think about the change in the inclination. Relative to the space station that we are in, orbits with an inclination of 90° are edge on, while orbits with an inclination of 0° are face on. Figure 7 shows an example of what this means.

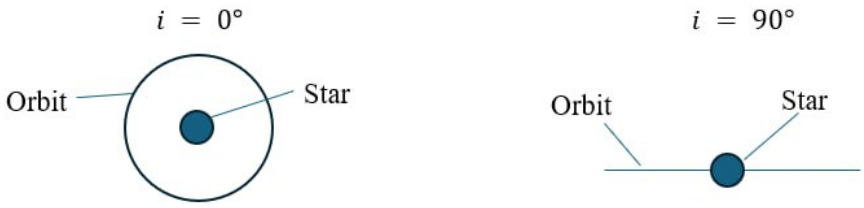


Figure 7: What inclination means relative to us

(c) What is the reference plane that we, the people on the space station, are using to determine the inclination of Earth? **[1]**

[2]This encyclopedia does not exist. Please do not actually search for it under any circumstances.

Solution:

The plane that is perpendicular to our line of sight.

Alright, now that we have explored a little bit about inclination, let's get started on understanding the few methods of determining the presence of the Earth!

Part II Tails Angy, Tails Smash

One of the more common methods is what is known as Transit Photometry, where the presence of planets is detected through the dip in the brightness of stars during a transit. As planets pass in front of the star, they block some of the light emitted by the star they are orbiting about.

(d) Calculate the maximum percentage change in the brightness of the Sun with respect to the spaceship when the Earth transits the Sun, if possible. State any assumptions made in your calculations, if any. **[2]**

Solution:

Since the Sun emits light through its surface, the brightness of the Sun is dependent on the area of the Sun that is not blocked out by the Earth.

Assume that the Sun emits light evenly, we see that the luminosity that would be blocked out by the Earth would be

$$\frac{A_{Earth}}{A_{Sun}}$$

As from the perspective of the space station, the Earth and the Sun are nothing more than discs, we can take $A = \pi r^2$. The above ratio would hence reduce to

$$\frac{R_{Earth}^2}{R_{Sun}^2}$$

The percentage change would therefore be:

$$\text{Percentage Change} = \frac{R_{Earth}^2}{R_{Sun}^2} \times 100\% = 0.00837\% \text{ (to 3 significant places)}$$

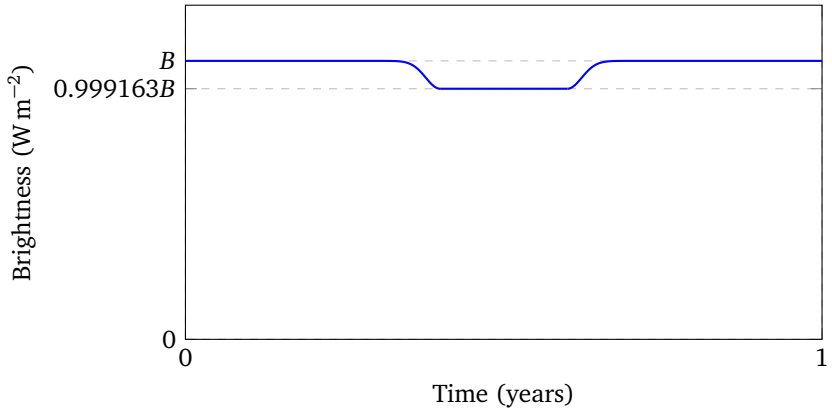
With this data, we can make a rough model of what we can expect to observe in terms of luminosity through a **light curve**, which is simply a graph which shows the brightness of an object throughout time. The light curve should be able to give us a rough expectation of what we should see if Tail's *Skill Issue* does not cause the destruction of Earth.

(e) Sketch the light curve of the Sun over the period of a year before the Earth's inclination was altered by Tail's initial tilt. **[2]**

Solution:

Since the brightness of the Sun would dip by 0.00837% as calculated in (d), and the Earth would only transit the Sun once a year, we can expect the light curve to look something like this:

Taking the initial brightness of the Sun when there is no transit to be B,



Note that the change in brightness is gradual since the Earth is a disc, it takes some time for the entirety of the Earth to block the Sun. During this period, the brightness of the Sun will gradually change as more and more (or less and less) portion of the Earth blocks the Sun.

Naturally, with the change in inclination caused by Tail's tilt, both initially and in her attempt to correct the orbit of the Earth, the light curve of the Sun would change - but exactly how so? To a certain extent, the decrease in inclination would cause the apparent transit path of Earth to shorten relative to an external observer, as shown in Figure 8.

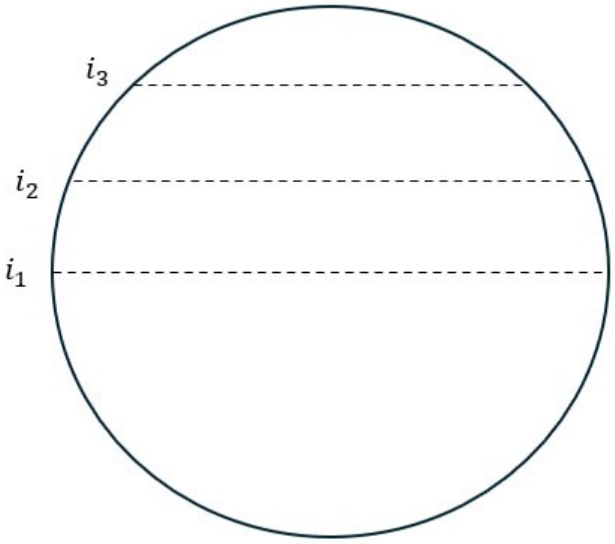


Figure 8: The change in transit path distance with decreasing inclination, from i_1 to i_3

Being the nerds that all of us are, is knowing that the duration of the transit would decrease with decreasing inclination good enough? Of course not! As people of cul- ahem I mean science, we seek to know exactly how the inclination would change the duration of the transit - would it change linearly (like a straight line graph), or will it change in some other weird manner? That is what we probably should (and we are going to) try and figure out.

Let's start by considering how a transit between the Sun and a planet of an orbital inclination i would look like from the space station's perspective. Since a transit is just a planet appearing to move across the Sun, the transit would probably look something like Figure 9 from the perspective of an observer.

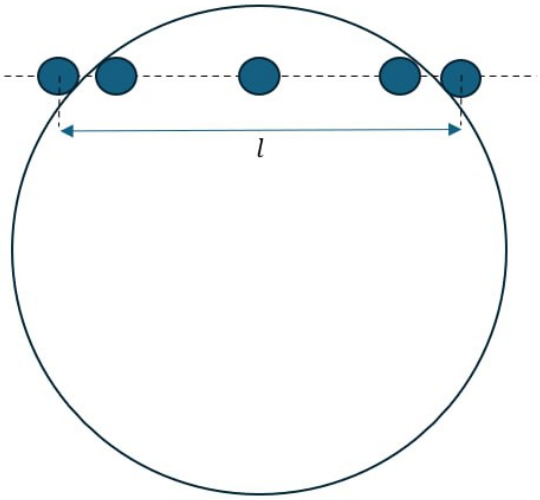


Figure 9: The Earth's transit across the Sun when the orbital inclination is not 90°

The apparent distance, l, shown in Figure 9, refers to the linear distance that the Earth has moved during the transit relative to us.

While it is simple for us to figure out the apparent distance when the inclination of the Earth is 90° (since it is just the diameter of the Sun), it becomes slightly harder to figure out for other inclinations. Thinking of the issue from another perspective could help us make things slightly simpler.

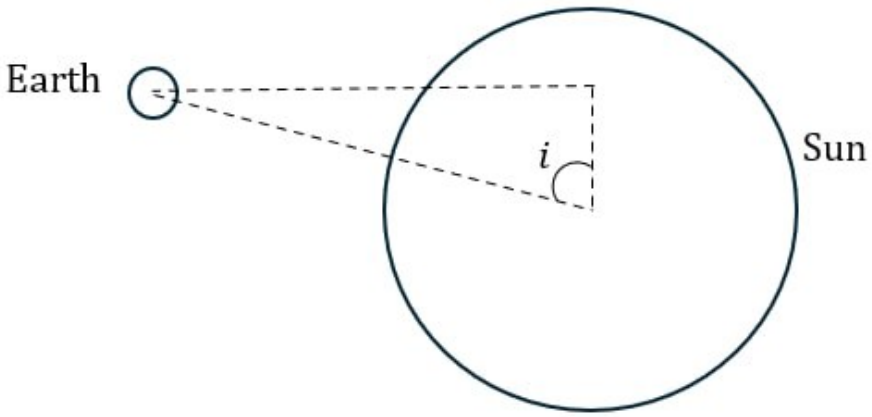


Figure 10: Side view of the Earth's transit when the orbital inclination is i°

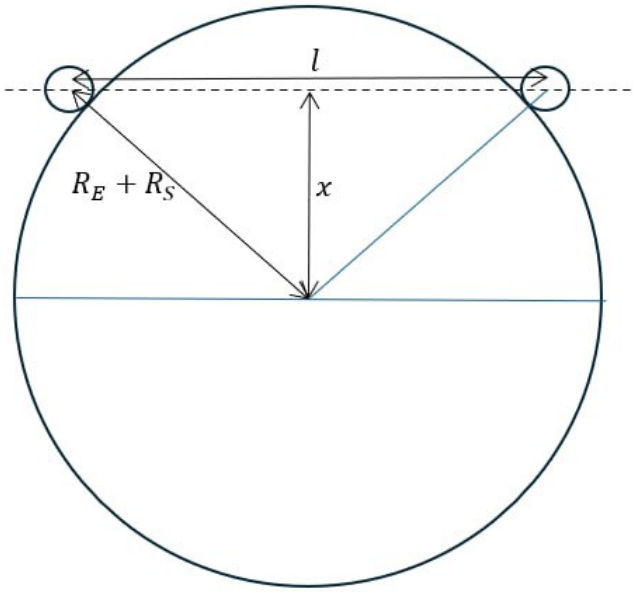
(f) Given that the radius of Earth and Sun are R_E and R_S respectively, the inclination of the Earth's orbit at any point of time is i, and the radius of the Earth's orbit is d, show that the apparent distance travelled by the Earth, l during each transit is given by Equation 9.

$$l = 2\sqrt{(R_S + R_E)^2 - (d\cos i)^2} \tag{9}$$

You are highly encouraged to use diagrams wherever possible to aid in your derivation and explanation. **[2]**

Solution:

We can see Figure 9 and 10 as such:



Note that the distance x is the apparent deviation of the transit path from the centre of the Sun due to the inclination of Earth's orbit caused by Tails' tilt.

From Figure 9, we see that

$$x = d\cos i$$

We hence can calculate the apparent distance that the Earth has travelled during the transit using Pythagoras' Theorem, since the triangle drawn in Figure 8 is two right angled triangles joined together:

$$\frac{1}{2}l = \sqrt{(R_E + R_S)^2 - x^2}$$

Substituting the expression that we derived for x, and solving for l, we get:

$$l = 2\sqrt{(R_S + R_E)^2 - (d\cos i)^2}$$

Now that we have figured out the apparent distance the Earth has travelled, now what? One important thing to note is that we are living in a three-dimensional space, not some ideal two-dimensional world where every family consists of a handsome spy, a pretty assassin and a child who is able to read people's minds. Taking that into account, a planetary transit would look something like this:

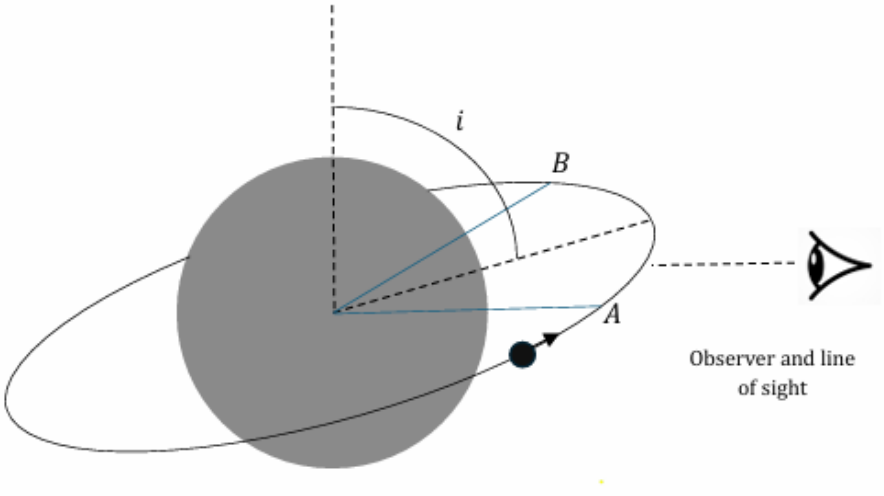


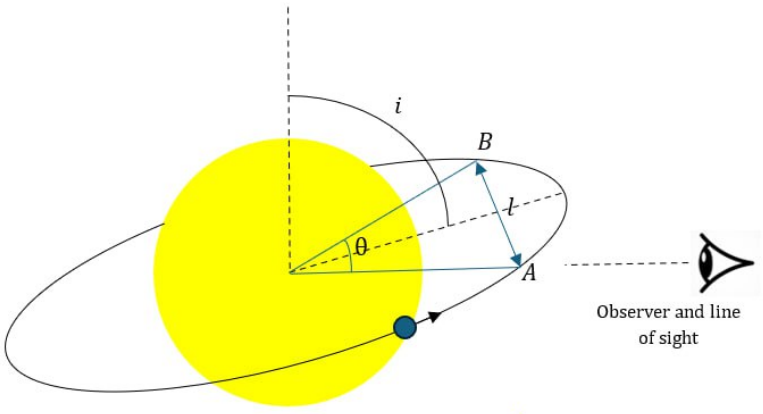
Figure 11: A 3-dimensional consideration of how a transit looks like

Note that in Figure 11, A and B are the position of the Earth at the start and the end of the transit

(g) Using the information provided in the previous parts and your own knowledge, express the duration of Earth's transit in terms of the orbital inclination of the Earth, i. **[2]**

Solution:

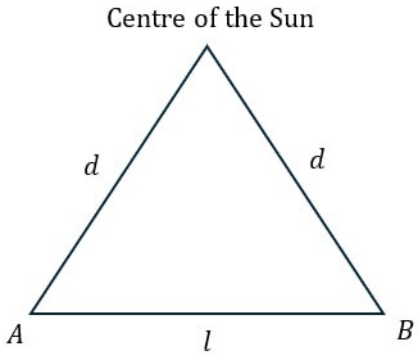
A closer inspection at Figure 11 would reveal that the line AB is the apparent distance travelled by the Earth. We therefore can see that Figure 11 can be seen as so:



As such, the duration of the transit is simply the time taken for the Earth to travel from position A to position B. Since only the position of Earth and the inclination of its orbit has changed, we can still calculate this duration using the angular velocity of the Earth.

$$t = \frac{\theta}{\omega}$$

We also observe that position A and B produces a triangle with the centre of the Sun, as shown in the below Figure:



As the triangle is symmetrical, we can take the triangle above to the two right triangles joint together. The angle θ hence can be calculated as such:

$$\frac{1}{2}\theta = \sin^{-1}\left(\frac{\frac{1}{2}l}{d}\right)$$

$$\theta = 2\sin^{-1}\left(\frac{\frac{1}{2}l}{d}\right)$$

Now that we know the expected angular displacement of the Earth during the transit, we hence can calculate the duration of transit.

$$t = \frac{2}{\omega}\sin^{-1}\left(\frac{\frac{1}{2}l}{d}\right)$$

Substituting the formula for l into the above equation,

$$t = \frac{2}{\omega}\sin^{-1}\left(\frac{\sqrt{(R_E+R_S)^2-(d\cos i)^2}}{d}\right)$$

where ω is the angular velocity of the Earth's orbit.

And that's it! We have managed to figure out a method to keep track of the changing inclination of the Earth as a result of Tails repeatedly slamming the table. However, when we run this through our simulations, a new problem has shown up! No matter what, there is a point of time where the Earth's inclination has changed so much that it no longer appears to move in front of the Sun, making it impossible for us to detect any transits from there on!

Is that it? Are we just done from that point onwards - not knowing whether we have just become an accessory to the murder of 8 billion humans by accidentally allowing Tails to destory the Earth?

Find out in the next episode of *Dragon Ba* - wait! Before we start screaming in incoherent languages, let's consider another way we can potentially see this! Just... relax and hear me out...

Part III Tails tilts harder

An interesting thing to note about orbiting systems is that most objects that may appear to be stationary are, in fact, moving. This may sound weird based on our intuition, which tells us that the Sun should be the one that is causing things to move, not the thing that is moving in an orbit!

(h) Is the idea that the Sun experiences motion within the Earth-Sun system consistent with our understanding of physics? Explain why or why not. **[1]**

Solution:

The idea is consistent with our understanding of physics.

The primary interaction between the Earth and the Sun is gravity, which by nature acts mutually between the two bodies. The Sun hence also moves because of Earth's gravitational influences. This is, however, less pronounced due to the higher mass of the Sun compared to the Earth.

Regardless whether we should continue *cooking*, the reality **is** that the Sun experiences some form of motion within the Earth-Sun system. So let's just live with that and continue our journey of figuring out where exactly we are going with this.

As it turns out, the presence of the Earth itself could shift the centre of mass of the Earth-Sun system slightly from the centre of the Sun. This is similar to how carrying a bag could shift one's centre of gravity slightly, making it sometimes easier for us to fall down.The Earth and the Sun moves around this centre of mass, which in astronomy, is called the barycentre.

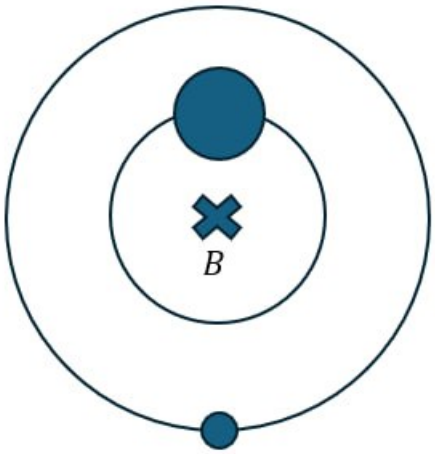


Figure 12: The orbits of a planet and star in a hypothetical stay system similar to the Solar System, with a barycentre marked as B

For simple two body problems such as the one shown in Figure 12 and the Solar System that Tails is in, the distance between one of the objects and the barycentre, r_1, can be easily calculated using the following formula:

$$r_1 = a \times \frac{m_2}{m_1 + m_2} \tag{10}$$

where

- a is the distance between the centres of the two objects in the system
- m_1 and m_2 are the masses of the two bodies

The position of the barycentre would determine the motion of the objects within the system. In the case of a simple 2-body system like the Solar System, objects tend to travel in circular motion. Such motion can be split into two different categories: wobbles (a periodic motion where the path of motion is entirely within the celestial body) and orbits (a periodic motion which follows an external path of motion).

(j) Using Equation 10 to explain your answer, deduce whether the Sun experiences motion in the form of an orbit or wobble. **[1]**

Solution:

Calculating the distance between the Sun and the barycentre of the system (by plugging in the relevant values into the equation provided), we see that

$$r_{sun} = 449168.3265\,\text{m}$$

As the distance between the Sun and the barycentre of the system is well within the radius of the Sun, the Sun would hence appear not to have a clear orbital path. The Sun would hence appear to wobble instead of undergoing an orbit.

How exactly does this motion help us in determining whether the Earth still exists? The key is that as long as the Sun is travelling towards or away from an observer, the frequency of the light from the Sun would increase or decrease respectively. Such a phenomenon is also known as blue-shift/red-shift, which is a result of the Doppler effect.

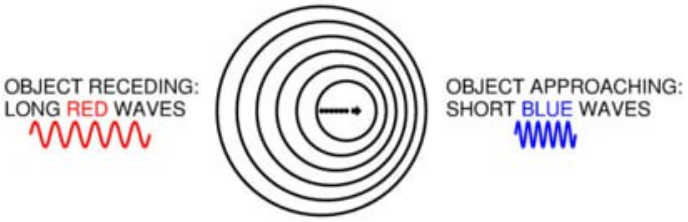


Figure 13: A brief visualisation of the Doppler effect [4]

Since the Sun is undergoing some form of circular motion with the Earth, the Sun would also appear to be travelling towards and away from the space station, resulting in the Doppler effect being observed by the observer.

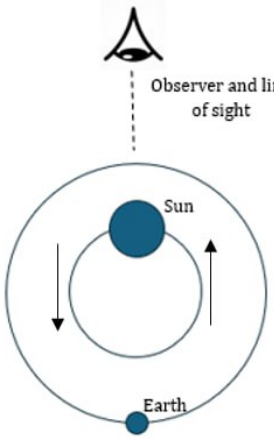


Figure 14: A 2-dimensional top-down view of the circular motion of the Sun in the Earth-Sun system

As Tails continues to change the inclination of the Earth, the inclination of the Sun's motion will change as well. To be more specific, the inclination of the Sun's motion will change by the same amount in the opposite direction of the Earth!

With the change in the inclination of the Sun's movement, the Doppler shift of the Sun's light would change. But again, as people of cul- ahem I mean science, we are more interested in figuring out exactly how this change will occur! We have a rough equation on the space station that claims it can help us, but we need to see whether it is correct!

Something good to note in the process of verifying the equation is that we will have to take barycentre to be the "primary body" of the system since the Sun and the Earth are orbiting around it. This results in slight changes in the various formula that we know and love, such as the Kepler's Third Law, which in this case would take the form of Equation 11.

$$T^2 = \frac{4\pi^2}{G(M_1 + M_2)}(R_1 + R_2)^3 \tag{11}$$

In addition to this, the Conservation of Linear Momentum, which states that the Linear Momentum within any closed system is constant in the absence of any external forces still applies. In other words, the momenta of the Earth and Sun should be equal to each other in value. The linear momentum, p, of an object is the product of the mass and velocity of the object.

$$p = mv$$

(k) Show that the maximum Doppler shift of the Sun's light as observed by us, z_{max}, is related to the inclination of the Earth's orbit, i, in the form of Equation 12.

$$z_{max} = \frac{\frac{M_{\text{Earth}}}{M_{\text{Sun}}} \frac{r_{\text{Earth}} \sqrt{G(M_{\text{Sun}}+M_{\text{Earth}})}}{a^{3/2}}}{c} \sin i \tag{12}$$

where a is the distance between the Earth and the Sun. **[3]**

Solution:

We should first recall the formula for Doppler shift:

$$z = \frac{v}{c}$$

where z is the Doppler shift observed, v is the recessional velocity of the light source and c is the speed of light.

Since the Sun is travelling in a circular orbit caused by gravitational influences, as shown in the diagram below, the motion of the Sun can be treated as an orbit (as hinted in Figure 14). By the Conservation of Linear Momentum, we can see that

$$p_{earth} = p_{sun}$$

$$M_{earth} v_{earth} = M_{sun} v_{sun}$$

$$\frac{M_{earth}}{M_{sun}} v_{earth} = v_{sun}$$

Consider Kepler's Third Law, we see that the square of the period of the Earth's orbit can be expressed as

$$T^2 = \frac{4\pi^2}{G(M_{\text{Earth}} + M_{\text{Sun}})} (R_{\text{Earth}} + R_{\text{Sun}})^3$$

The period of the Earth's orbit is therefore

$$T = \frac{2\pi}{\sqrt{G(M_{\text{Earth}} + M_{\text{Sun}})}} (R_{\text{Earth}} + R_{\text{Sun}})^{\frac{3}{2}}$$

Consider the orbital velocity of the Earth, we have

$$v_{\text{Earth}} = \frac{2\pi r_{\text{Earth}}}{T}$$

Substituting the period of the Earth's orbit

$$v_{\text{Earth}} = \frac{2\pi r_{\text{Earth}}}{\frac{2\pi}{\sqrt{G(M_{\text{Earth}}+M_{\text{Sun}})}} (R_{\text{Earth}} + R_{\text{Sun}})^{\frac{3}{2}}}$$

which would result in

$$v_{\text{Earth}} = \frac{r_{\text{Earth}} \sqrt{G(M_{\text{Earth}} + M_{\text{Sun}})}}{(R_{\text{Sun}} + R_{\text{Earth}})^{\frac{3}{2}}}$$

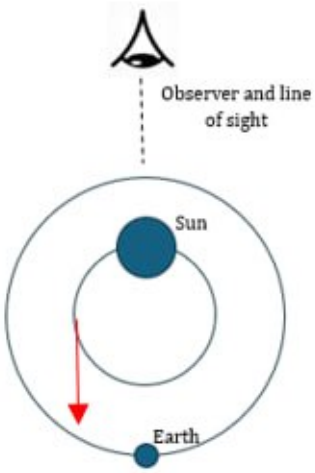
Since $a = R_{\text{Sun}} + R_{\text{Earth}}$, we see that

$$v_{\text{Earth}} = \frac{r_{\text{Earth}} \sqrt{G(M_{\text{Earth}} + M_{\text{Sun}})}}{a^{\frac{3}{2}}}$$

Apply this to v_{Sun} gives

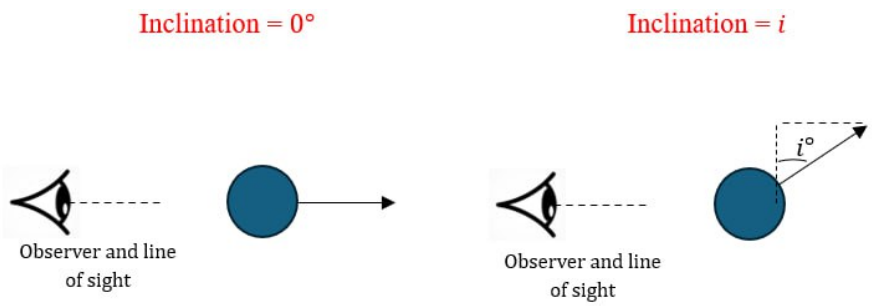
$$v_{\text{Sun}} = \frac{M_{\text{Earth}}}{M_{\text{Sun}}} \frac{r_{\text{Earth}} \sqrt{G(M_{\text{Earth}} + M_{\text{Sun}})}}{a^{\frac{3}{2}}}$$

For the case where the inclination of the Sun's motion is 0°, the maximum recessional velocity of the Sun would be equal to v_{sun}. This is when the Sun is positioned and moving at the origin and direction of red arrow respectively as shown in the diagram below. The position along the Sun's path where it has the highest recessional velocity is the same for the case where the Sun's path is inclined.



This however gets slightly complicated when the Earth's inclination changes, which changes the inclination of the Sun. The tilt of the Sun's orbit relative to the space station results in an additional component of the SUn's velocity in addition to its recessional velocity.

The maximum recessional velocity of the Sun when its movement has an inclination of $i°$ can hence be calculated by resolving the orbital velocity of the Sun into two components: the recessional component as well as its perpendicular.



We hence observe that the maximum recessional velcoity of the Sun for an inclination i would be

$$v_{recession} = v_{sun} \sin i$$

Applying this into the equation for Doppler shift would give us a maximum Doppler shift, z_{max} of:

$$z_{max} = \frac{\frac{M_{\text{Earth}}}{M_{\text{Sun}}} \frac{r_{\text{Earth}} \sqrt{G(M_{\text{Earth}}+M_{\text{Sun}})}}{a^{3/2}}}{c} \sin i$$

Naturally, with most people being allergic to mathematics, it would be much better to describe what changes we expect to see in the data that we would be working with by demonstrating what the data would look like for a certain inclination. Doing so would require some initial data, as shown in Figure 15.

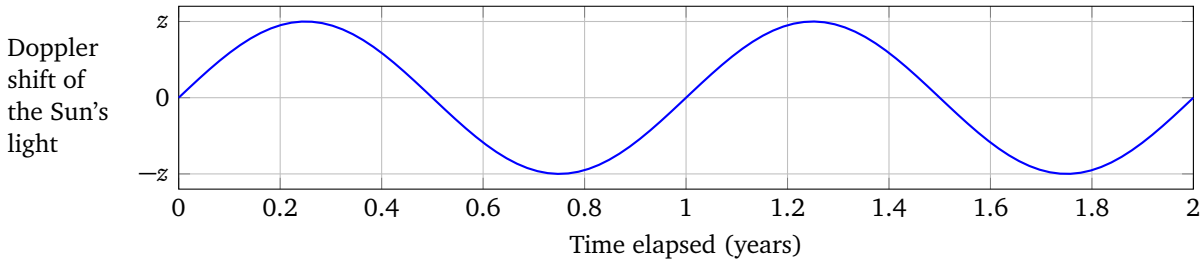


Figure 15: Doppler shift of the Sun's light before the Earth's inclination changed

(m) Taking the maximum Doppler shift of the Sun's light before the inclination of the Earth was changed, that is when $i = 90°$, to be the constant z, sketch the graph of the Doppler shift of the Sun's light against the time elapsed in years when Tails changes the Earth's inclination to 30°.

You are required to label the maximum and minimum value of the Doppler shift in terms of z, and the period of the graph, if any. **[1]**

Solution:

Note that $\sin 30° = \frac{1}{2}$. Based on the equation shown in part k, we see that the graph will be of the same shape as that of Figure 15, with the amplitude of the curve halved and the period remaining the same.

The maximum and minimum doppler shift will hence be $\frac{1}{2}z$ and $-\frac{1}{2}z$ respectively, with the period of the curve remaining the same, and hence the x-intercept of the graph should remain the same.

And that's about it for the Doppler shift method! But before we get all smug and let Tails work her magic, we should consider an important question - is there an inclination where both the transit and Doppler shift method will not help us in figuring out whether the Earth exist? This is especially important to consider, so that we will not get caught off guard.

(n) State an inclination where both transit photometry and Doppler shift will not help us to detect the presence of the Earth. Explain why at such inclination, both methods are ineffective. **[1]**

Solution:

Both methods will not work when the inclination of the Earth is 0°. At such inclination, the Earth will not be moving in front of the Sun relative to us and will be moving only perpendicularly to our line of sight, resulting in no Doppler shift in the Sun's light.

(o) Suggest and explain in detail, another method that we can use to detect the Earth's presence when the Earth's orbit has an inclination of what was stated in n. You are encouraged to use diagrams to aid in your explanation. **[2]**

Solution:

Under such scenario, a potential method that can be used is to detech the Earth's presence by observing the Sun's wobble through astrometric measurements. When the inclination of the Earth is 0°, the inclination of the Sun's motion will also be 0° relative to the space station, as shown in the diagram below.

Such motion would result in the shift in position which can be detected by the space station. The expected maximum angular displacement could be calculated using the following equation

$$\theta = \tan^{-1} \frac{r}{d}$$

where θ is the expected maximum angular displacement of the Sun, r is the distance between the Sun and the barycentre of the Earth-Sun system and d is the distance between the space station and the barycentre.

With that, we are about done with what we can do! Tails has just been notified that she can begin with the plan. As she begins to play her first *Valorant* game to get her tilted enough, there is suddenly one additional question that came to her mind: if Tails slammed her table so hard that it changed the Earth's orbit... and the table did not break, what exactly is her table made of?

BONUS: Suggest what Tails' table is made of. (Interesting answers may be featured)

DETACH THIS PAGE AND SUBMIT IT WITH THE REST OF YOUR ANSWERS.

Question 4 *El Primo*

Pablo El Sancho is a self-proclaimed astronomer working at the European Space Agency (ESA). He is best known for his natural aptitude in astrophysics and astrobiology, particularly his studies on Astronomical Mass Observable Gravitational Units (AMOGUs), which are hypothetical objects found outside space stations. Another part of his work is ESA's new deep sky survey project, Intelligence Mission Programme for Organisation of Star and Telescopic Arithmetic Research (IMPOSTAR). He is probably Spanish... but he won't let me look at his personal file, so who knows.

Today he is giving all of us a little lecture on the night sky, which he is quite new to. Using his latest obsession Stellarium, he starts yapping about the Milky Way and its vast selection of associated deep sky objects (DSOs). Unfortunately, Pablo fails to read the room and spends two hours talking about why Mensa is a bad constellation.

After finally sensing that the crowd tonight is a bit tough and frankly uninterested in hearing an incredibly detailed explanation of why he hates a specific constellation in a "night sky talk", he relents and brings the crowd outside to go stargazing. He asks for any suggestions to start off a cracking night of observation...

Figure 16 shows a part of the night sky as it appears in front of you.

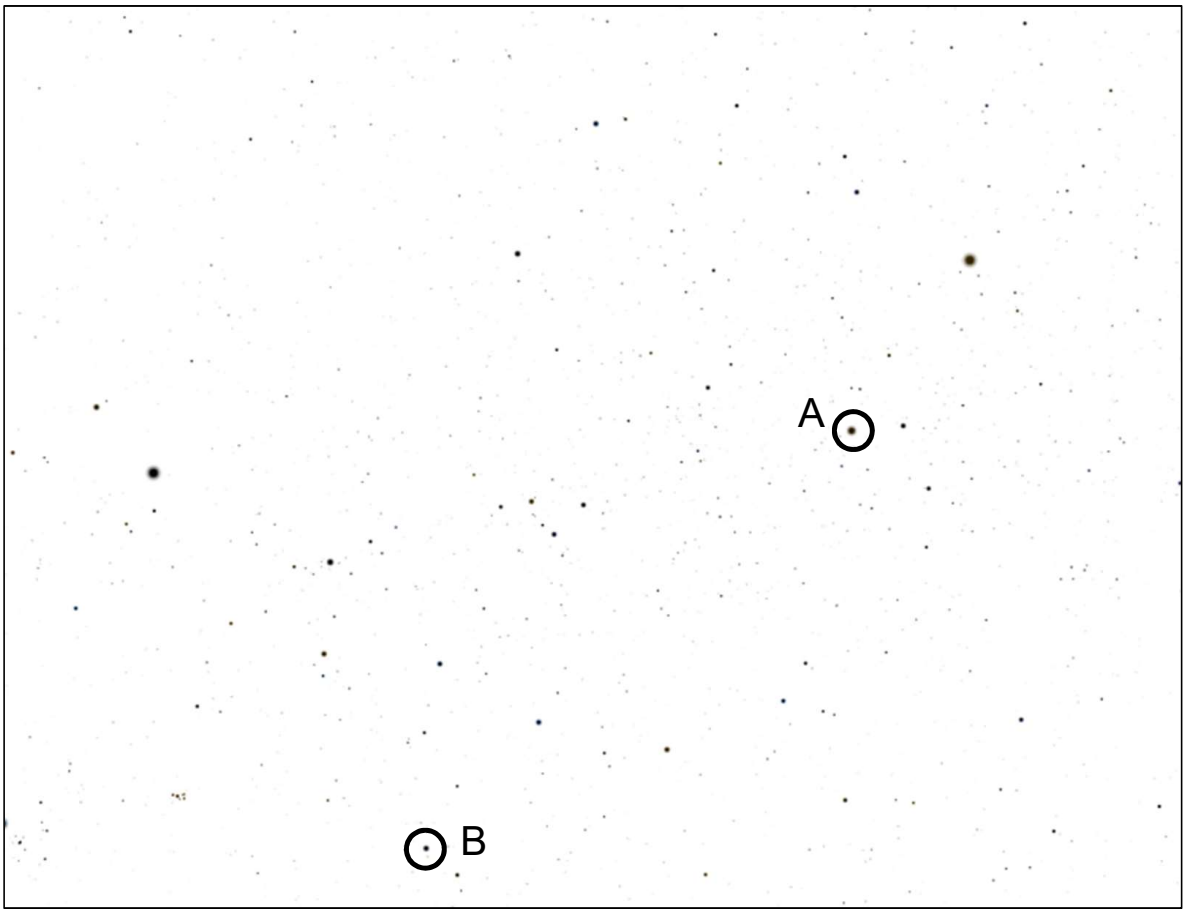


Figure 16: Part of the night sky

(a) Name stars A and B. Either the correct common name or the correct Bayer designation would suffice. **[2]**

Solution:

A: Deneb, B: Hamal

(b) Trace out two constellations and label them. **[2]**

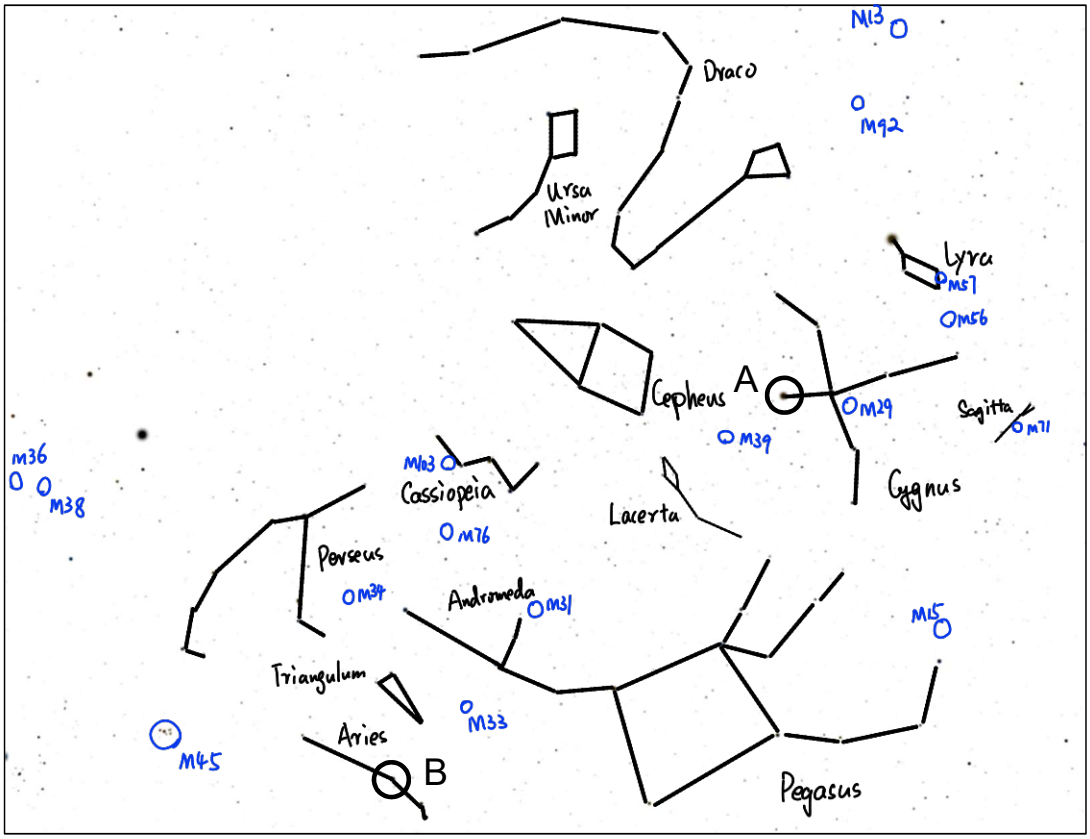
Solution:

Cassiopeia, Cygnus, Lyra, etc (accept any constellations FULLY in the image)

(c) Circle any two DSOs in this image and label them. **[2]**

Solution:

M13, M15, M29, M31, M33, M34, M36/M37/M38, M39, M45, M56, M57, M71, M76, M92, M103
(accept any DSO that is CORRECTLY labelled in the image)



DETACH THIS PAGE AND SUBMIT IT WITH THE REST OF YOUR ANSWERS.

DETACH THIS PAGE AND SUBMIT IT WITH THE REST OF YOUR ANSWERS.

He later finds the Moon, but when he looks through his eyepiece, he notices that something is wrong and takes a picture as shown in Figure I-2 on the Insert.

(d) What is wrong with Pablo's picture? **[1]**

Solution:

(Slight) chromatic aberration as seen from the characteristic coloured fringes.

(e) List and explain one advantage and one disadvantage of a refractor compared to a reflector. **[2]**

Solution:

Note that the following list of advantages and disadvantages are not exhaustive.

Advantages:

- More convenient as there is no need to collimate it unlike a Newtonian with multiple autonomous mirror elements.
- Closed-tube design makes it easier to clean/get dust off unlike long open-tube designs of Newtonians.
- Less sensitive to thermal currents due to the simpler optical path compared to Newtonians, which makes temperature acclimation unnecessary.
- Slightly sharper images vis-à-vis the same aperture as some of the incident light is blocked by a Newtonian's secondary mirror unlike in a refractor.

Disadvantages:

- Occurrence of chromatic aberration due to the dispersion of photons of different wavelengths, which is unwanted in imaging and visual observations. Newtonians do not refract light and so do not suffer from chromatic aberration.
- Increasingly difficult to manufacture ones of larger aperture as convex lenses are used.
- Lens might deform due to gravity, while the primary mirror in Newtonians can have reinforcements at its back to support its weight.
- Small defects on a lens affect sharpness more noticeably than on mirrors.
- Refractors can be more expensive due to the higher cost of lenses compared to mirrors.

You point out the problem to Pablo. He, in turn, shares the specifications of his telescope with you: a 2-element achromatic refractor with an aperture of 80mm and a focal ratio of $f/7$.

The focal ratio is a measure of the light-gathering ability of an optical system, which is defined by Equation 13.

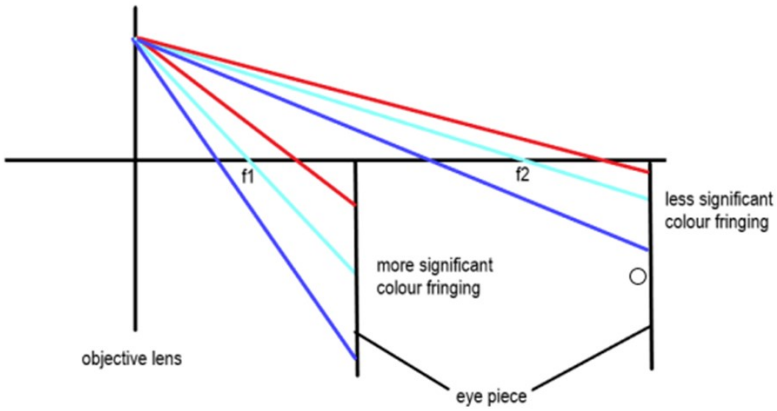
$$N = \frac{f}{D} \tag{13}$$

where N is the focal ratio, f is the focal length and D is the diameter of the aperture.

(f) He wants to reduce the problem in (d). Should he replace his refractor with one of the same aperture but a focal ratio of $f/5$ instead? Explain briefly with the aid of a ray diagram. Label the colour of your light rays clearly. **[2]**

Solution:

No since with the same aperture, a faster focal ratio would mean a shorter focal length and hence more chromatic aberration as demonstrated in this diagram.



Marking points for the diagram:

- Red having a longer focal length than blue
- Illustrate dispersion
- One eyepiece setup is sufficient

Pablo happens to have a 0.8× focal reducer on hand and attaches it to the original refractor.

(g) What is the effective focal length of his refractor? **[1]**

Solution:

$$N = \frac{f}{D}$$

$$0.8 \times 80\,\text{mm} = \frac{7}{D}$$

$$D = 448\,\text{mm}$$

Pablo then points you towards his current field of view, as shown in Figure 17.

Figure 17: Pablo's current field of view

(h) Roughly draw, inside a 8 cm by 8 cm box, and delineate the boundary, how it might have looked before he attached the focal reducer. Draw the very prominent objects only. **[1]**

Solution:

Criteria:

- Roughly a box 8 cm by 8 cm in size
- Orientation should not change
- Prominent objects need to become noticeably nearer to the borders.

DETACH THIS PAGE AND SUBMIT IT WITH THE REST OF YOUR ANSWERS.

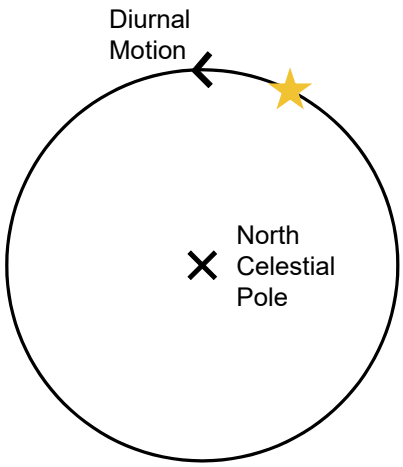
Pablo then sets his rig up for an *Electronically Assisted Astronomy* (EAA) session. An equatorial mount follows the diurnal motion of celestial objects, which centres the object of interest for an attached camera to expose itself for an arbitrary duration of time. The exposed camera sensor is continuously bombarded by photons, forming a progressively more detailed image called an *exposure*.

Such exposures will then be stacked to create an image with even more details. However, if the duration of the exposure is too long, there may be star trails, as the object of interest falls off-centre, while if the duration of the exposure is too short, there will be a lot of undesirable noise in the final image.

(j) Explain what diurnal motion (of celestial objects) is. You may draw a diagram if necessary. **[1]**

Solution:

Diurnal motion is the **apparent** motion of celestial objects with respect to an observer on Earth due to the axial rotation of the Earth.



He decides on a duration of 5 second for each exposure. He claims that tracking, which helps the equatorial mount to "follow" the diurnal motion of the celestial objects, may not always be necessary. His colleague Anna, however, asserts that tracking would be absolutely necessary in this case, to avoid star trails.

(k) Who is correct? Explain. **[2]**

Solution:

Anna is correct. In 5 s, the diurnal motion would be significant to generate star trail when looking through a telescope.

Impressed by your knowledge, Pablo decides to interview you on the spot for his agency, even though he technically does not have the authority to make such a decision. He suddenly starts asking you very complicated questions...

Before we start this section, let us first preface what Pablo is saying because he is using some big words.

Sidereal time is a bit different from solar time that we use every day. Sidereal time measures the Earth's rotation relative to the stars rather than the sun. As everyone knows, a solar day is 24 hours, which is the time it takes for the Sun to return to the same (apparent) position in the sky. However, the time it takes for stars to return to the same (apparent) position in our sky is, in fact, not the same.

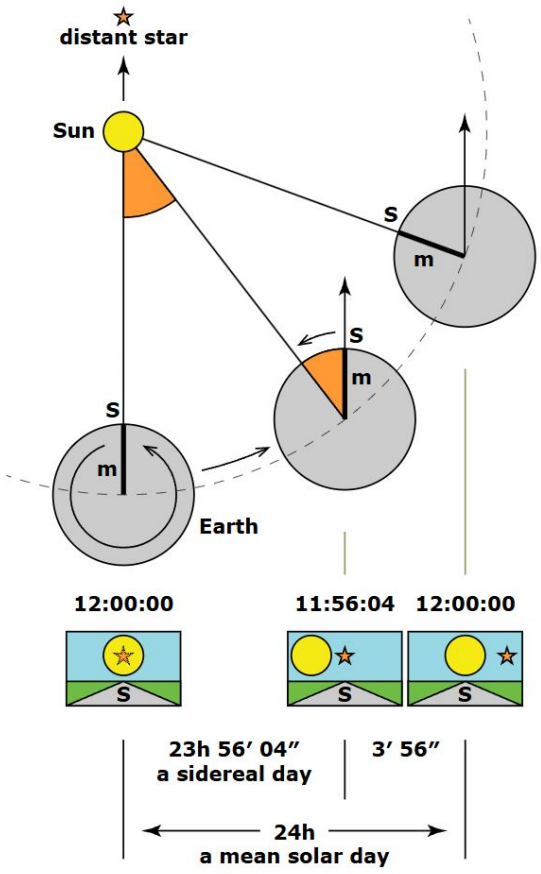


Figure 18: Comparison between sidereal time and solar time [5]

As shown in Figure 18, in the time that it takes for the Earth to spin around once (complete one sidereal rotation), it moves along its orbit a little bit. The Sun isn't actually in the same spot at the end of the rotation. Earth must rotate a bit more to bring the Sun to the same spot it originally was.

It is given that one sidereal day is 23 hours, 56 minutes and 4 seconds long.

(m) Based on Figure 18, do stars rise earlier, later or at the same time every day? Explain you answer. **[2]**

Solution:

Earlier. A star that is currently rising will be at the same position 23 hours 56 minutes and 4 seconds later. That is, about 4 minutes earlier on the clock of the second day.

Now onto Pablo's question...

(n) Let's say today is 29th May 2025. At 9:07PM, star A is currently at the local meridian. What time will star A cross the meridian on 6th June 2025? **[2]**

Round your answer off to the nearest minutes.

Solution:

6th June is 8 days later than 29th May. Since stars reach the same place in the night sky about 4 miuntes earlier every day, it will be at the meridian on 6th June at 2107-0032=2035hrs. That is, 8:35PM.

We also accept 8:36PM since it would be 8:35:32PM if it were to be calculated precise to the seconds.