Cave in the Tundra [20 marks]

One of the best blackbodies known in physics is a black hole, emitting a form of blackbody radiation known as Hawking radiation. Over time, if no incident matter or energy is absorbed by the black hole, this radiative emission results in the black hole losing mass in a process known as evaporation.

Therefore, a black hole can be treated as a blackbody of temperature T, where T is given by

$$T = \frac{\hbar c^3}{8\pi k G M}$$

Where
ħ is the reduced Planck constant $\hbar = h/2\pi$
k is the Boltzmann constant
G is the Universal Gravitational Constant
and M is the mass of the black hole

i) Find the blackbody temperature of Sagittarius A*, the supermassive black hole at the centre of the Milky Way Galaxy and comment on your result. Sagittarius A* has a mass of $4 \times 10^6\ M_\odot$. [2 marks]

$$T = \hbar c^3/8\pi kGM = 1.55 \times 10^{-14}\ K$$

This temperature is extremely low and close to absolute 0.

The escape velocity from an object of mass M is given by

$$v_{escape} = (2GM/R)^{0.5}$$

ii) Using the equation for escape velocity, obtain an equation relating the mass of a black hole to the radius of its event horizon. [2 marks]

Ans:

$$v_{escape} = (2GM/R)^{0.5}$$
$$M = R(c^2/2G)$$
$$M = 6.892 \times 10^{26}\ R$$

iii) Using the Stefan-Boltzmann law, the result from (ii) and the equation for black hole temperature above, derive an equation relating the total power emitted from Hawking radiation to the radius of a black hole. Hence, comment on the relationship between the total emissive power from Hawking radiation and the radius of the black hole event horizon. [3 marks]

Ans:

$$T = \hbar c^3/8\pi kGM = \hbar c^3/8\pi kGR(c^2/2G)$$
$$T = \hbar c/4\pi Rk$$
$$P = 4\pi R^2\sigma T^4 = (4\pi R^2\sigma) \times (\hbar c/4\pi Rk)^4 = \sigma(\hbar c/k)^4/\ 64\pi^3R^2$$

The radiative power of Hawking Radiation is inversely related to the square of the black hole radius/inversely related to the surface area of the black hole's event horizon.

iv) Based on your result in question (iii), explain why black holes are thought to radiate in a "flash" at the end of the evaporation process. [1 mark]

At the end of the evaporation process, R is extremely small, resulting in maximal hawking radiation P.

v) Find the solar radiative flux at a distance of 1AU [1 mark]

Solar Flux = $3.846 \times 10^{26} / 4\pi(1.5 \times 10^{11})^2 = 1360\ W/m^2$

Given that at 1 AU, there is a solar radiative flux of 1.4 kW/m^2.

vi) Estimate the total amount of solar radiation absorbed per second by a black hole with an event horizon of radius R, treating it as a solid blackbody sphere of **1.5R (photon sphere radius)** located at a distance of 1AU from the sun. [1 mark]

$$P = (1.5^2)1400\ \pi R^2 = 3150\ \pi R^2$$

Possible Common mistake: Students using the area of a sphere instead of a disc

In equilibrium, the incident radiative energy will be equal to the radiant energy from the black hole.

vii) Consider a black hole located at a distance of 1AU from the Sun. Find the equilibrium radius for such a black hole. [3 marks]

$$3150\ \pi R^2 = \sigma(\hbar c/k)^4 / 64\pi^3 R^2$$
$$3150\ R^4 = \sigma(\hbar c/k)^4 / 64\pi^2$$
$$R^4 = \sigma(\hbar c/k)^4 / 201600\pi^2$$

From formula booklet, $\sigma = 5.67 \times 10^{-8}$; $k = 1.38 \times 10^{-23}$; $\hbar = 1.05457 \times 10^{-34}$

$$\hbar c/k = 2.29 \times 10^{-3}$$

Therefore,

$$R^4 = (5.67 \times 10^{-8})\ (\hbar c/k)^4 / 201600\pi^2$$
$$R = (5.67/201600\pi^2)^{0.25} \times (2.2926 \times 10^{-5})$$
$$R = 9.42 \times 10^{-7}\ m$$

viii) Calculate the equivalent blackbody temperature of this black hole [1 mark]

$$T = \hbar c/4\pi Rk = (2.29 \times 10^{-3})/(4\pi \times 9.42 \times 10^{-7}) = 193.45K$$

ix) Comment about the significance of this equilibrium radius in terms of how the mass of a black hole changes over time [1 mark].

At this equilibrium radius, the black hole's mass does not change with time/steady state mass.

Consider instead a black hole located deep in intergalactic space, with no significant sources of radiation aside from the Cosmic Microwave Background Radiation (CMBR).

x) Given that the temperature of the CMBR is 2.725K, find the equilibrium radius and mass for such a black hole. [3 marks]

Comment on solution:
Since both the black hole and CMBR are nearly perfect blackbodies, in equilibrium, the temperature of such a black hole would equal the temperature of the CMBR. The same result can be obtained with a more tedious calculation involving calculating the emission from the CMBR as an isotropic blackbody emission and forming a similar equation to that used for a black hole irradiated by solar radiation.

1 mark is given for being able to tell that T of CMBR = T of black hole OR doing the longer calculation
1 mark is given for calculating R
1 mark is given for calculating M

Since T = 2.725K,
$R = \hbar c/4\pi kT = (2.29 \times 10^{-3})/(4\pi * 2.725) = \underline{6.695 \times 10^{-5}\ m}$

$M = 6.892 \times 10^{26} \times 6.695 \times 10^{-5} = \underline{4.614 \times 10^{22}\ kg}$

xi) Explain the significance of this result on any black hole with a radius larger than a black hole with Hawking radiation in equilibrium with the CMBR [2 mark].

Any black hole with a radius larger than 0.067mm <u>will not evaporate at the present time</u>.

DRQ: Earth in another Turf [20 marks]

Part I: Binary Star Systems [10 marks]

According to stellar surveys, more than half of all Sun-like stars are part of multiple star systems. This means that in considering the search for extra-terrestrial life, there is a significant chance that life may evolve on planetary systems in such multiple star-systems. In the first part of this question, we analyse a binary star system to understand how a planet orbiting in such a configuration will experience temperature variations.

	Star A	Star B
Mass (M)	0.9 $M_\odot$	1.1 $M_\odot$
Radius (Solar Radii)	0.83 $R_\odot$	2.1 $R_\odot$

Distance between star A and star B = 12 AU

i) Estimate the star surface temperatures [3 marks]

Star A

Luminosity = 3.846×10^{26} x 0.9^3.5= 2.66×10^{26} W

$L \propto R^2T^4$

$0.9^{3.5} = (0.83^2)(T/T_{sun})^4$

$0.9^{3.5} = (0.83^2)(T/5778)^4$

T = 5784 K

Star B

Luminosity = 3.846×10^{26} x 1.1^3.5 = 5.37×10^{26} W

$L \propto R^2T^4$

$1.1^{3.5} = (2.1^2)(T/T_{sun})^4$

$1.1^{3.5} = (2.1^2)(T/5778)^4$

T = 4334 K

The stars A and B are separated by a distance of 12 AU. A planet with characteristics identical to the Earth orbits a point X in a circular orbit with a radius of 2 AU (see figure 1).

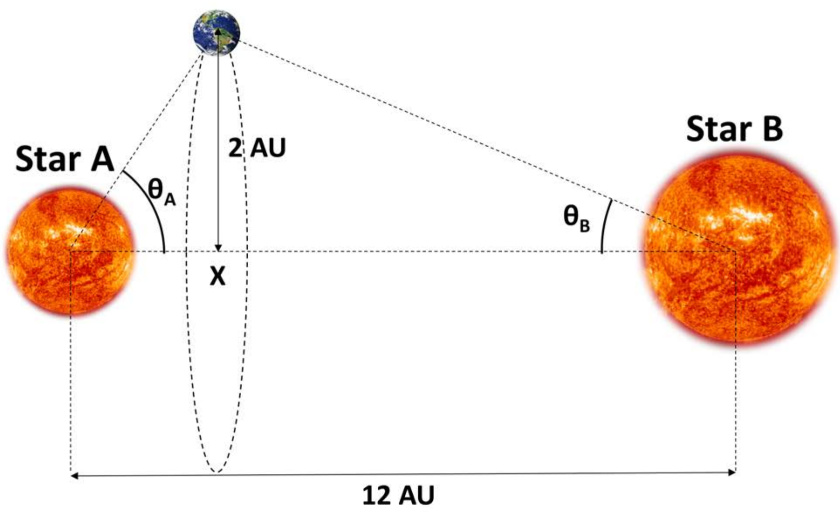


Figure 1 *Diagram of the A-B binary system with the Earthlike planet orbiting Star A. This diagram is not drawn to scale.*

The gravitational field strength around by a star is given as

$$g_{star} = GM_{star}/r^2$$

Where
G is Newton's gravitational constant (see formula booklet)
M_{star} is the mass of the star
And r is the distance from the centre of the star.

At point X, which lies between stars A and B, the gravitational fields of both stars cancel each other out completely (i.e. the magnitude of the gravitational field strength of star A and Star B are equal).

ii) Using the equation above or otherwise, calculate the distances from the centres of stars A and B to point X respectively [2 marks].

At point X, the gravitational field vectors of both stars cancel. Therefore,

$$GM_A/d_{AX}^2 = GM_B/d_{BX}^2$$
$$M_A/d_{AX}^2 = M_B/d_{BX}^2$$
$$M_A/M_B = d_{AX}^2/d_{BX}^2$$
$$d_{AX}/d_{BX} + 1 = (d_{AX} + d_{BX})/d_{BX} = d_{AB}/d_{BX}$$

Solving,

$$d_{BX} = (1/1.66942) \times 12 \text{ AU} = \underline{6.3008 \text{ AU}}$$
$$d_{AX} = 12 - 6.3008 = \underline{5.6992 \text{ AU}}$$

iii) Is such an orbit stable? Explain. [2 marks]

No, such an orbit is not stable as any slight movement (perturbation) of the planet away from the orbital plane around X will cause it to leave this orbital path.

The average surface temperature of a planet orbiting a single star can be estimated to be

$$[I_0/(16\pi d^2)](1-\alpha) = \sigma T_p^4$$

Where I_0 is the star's luminosity constant (3.827×10^{26} J/s for the Sun);
T_p is the average temperature of the planet;
α is the planetary albedo, which has a value of 0.3 for the Earth;
d is the distance from the star;
and σ is the Stefan-Boltzmann constant, which has a value of $5.67 \times 10^{-8} W/(m^2 \cdot K^4)$.

iv) Modify the above equation for the case of a binary star system. [1 mark]
(Hint: Linear addition can be performed on the left hand side of the equation)

Assuming that linear addition can be performed on the LHS of the equation,

$$[(I_{0,A}/(16\pi d_A^2)) + (I_{0,B}/(16\pi d_B^2))] \times (1-\alpha) = \sigma T_p^4$$

v) Find the steady state temperature of the planet as described in figure 1. [2 marks]

$d_A = (5.6992^2 + 2^2)^{0.5}$ AU) x (1.5×10^{11} m/AU) = 9.0599×10^{11} m
$d_B = (6.3008^2 + 2^2)^{0.5}$ AU) x (1.5×10^{11} m/AU) = 9.9159×10^{11} m

$[(2.66 \times 10^{26})/(9.0599 \times 10^{11})^2 + (5.37 \times 10^{26})/(9.9159 \times 10^{11})^2] \times (0.7)/16\pi$
$= 5.67 \times 10^{-8} T_p^4$

T_p = <u>121.7 K</u>

<u>Part II: Tidal Locks and Love Numbers [10 marks]</u>

When searching for potentially habitable worlds, there is generally both a lower and upper limit for stellar masses for parent stars. The upper limit of stellar masses exists due to the fact that stars of higher mass tend to exhaust their nuclear fuel at much greater rates, dying at timescales too short for life to evolve.

On the other hand, there is also a lower limit of stellar masses which arises from a less intuitive phenomenon. As the mass of a star decreases, its luminosity similarly decreases more than proportionately. This means that planets orbiting lower mass stars need to orbit at radii much closer to their host stars. At a certain point, the planet would become tidally locked to the star, with one side being plunged in constant daylight and another that never sees the light.

Such planets with incredibly asymmetric temperature profiles could therefore impact the habitability of life even in supposed habitable zones. Therefore, it would be of particular interest to get a ballpark estimate of the lower limit for such stars. This can be done by making several reasonable assumptions, and applying the equation for tidal locking and stellar luminosity.

The time for a planet to become tidally locked to its parent star t_{lock} can be estimated using the equation (in SI units) as follows:

$$t_{lock} = (\omega a^6 IQ)/(3GM_{star}^2 k_2 R^5)$$

where ω is the initial spin rate in radians per second
a is the semi-major axis of the motion of the planet around the star
R is the radius of the planet
$I = 0.4M_{planet}R^2$ is the moment of inertia of the planet
G is the gravitational constant
M_{star} is the mass of the star
k_2 is the Love number of the satellite, which measures a body's rigidity
Q is the dissipation function of the satellite

In practice, Q and k_2 are not easily determined values. In this case, for our estimation we consider the case where $k_2/Q = 0.0011$ which is equal to the known value for the Moon

Assuming an Earth-like planet in a circular orbit, we will calculate the smallest mass of star in which the planet can orbit in the habitable zone without becoming tidally locked within 1 billion years.

vi) Show that for this planet, $a^3/M_{star} = 101.52$ [3 marks]

$t_{lock} = (1 \times 10^9) \times 365 \times (24 \times 3600) = 3.1536 \times 10^{16}$ seconds
$\omega = 2\pi/(86164.1) = 7.292 \times 10^{-5}$ rad/s (Note: 86164.1s is the length of the sidereal day)
$R = 6.370 \times 10^6$ m (From formula book)
$I = 0.4 \times (5.972 \times 10^{24}) \times (6.370 \times 10^6)^2 = 9.693 \times 10^{37}$ kgm^2
$G = 6.67384 \times 10^{-11}$ Nm^{2}kg^{-2} (From formula book)
$k_2/Q = 0.0011$ (given)
a and M_{star} are unknown quantities.
Rearranging,
$a^6/M_{star}^2 = 3Gk_2R^5t_{lock}/\omega IQ = [3(GR^5t_{lock}) \times (k_2/Q)] / [\omega I]$
$= [3(6.67384 \times 10^{-11}) \times (6.370 \times 10^6)^5 \times (3.1536 \times 10^{16}) \times 0.0011]/[(7.292 \times 10^{-5}) \times (9.693 \times 10^{37})]$
$= [3 \times 6.67384 \times (6.370)^5 \times 3.1536 \times 0.0011]/[7.292 \times 9.693] \times [10^{-11} \times 10^{30} \times 10^{16}]/[10^{-5} \times 10^{37})]$
$= 1.0306 \times 10^4$
$a^3/M_{star} = 101.52$ (shown)

The inner and outer radii of the habitable zone of a star can be approximated by the following equations:

$$r_{inner} = (L_{star}/1.1L_{sun})^{0.5}$$
$$r_{outer} = (L_{star}/0.53L_{sun})^{0.5}$$

Where r is measured in Astronomical Units

vii) Using the above information, provide an expression for a (in appropriate units), the semi-major axis of the planet for our calculations, in terms of L_{star} and L_{sun} [1]. Briefly explain your answer in qualitative terms [1 mark].

Since the goal of our calculations is to find the minimum stellar mass that will not result in tidal locking within our specified timespan, we want to keep the planet as far away from the star as possible. Therefore, the appropriate value of a is simply r_{outer}.

Therefore, $a = 150 \times 10^9 \times ((L_{star}/L_{sun})/0.53)^{0.5}$
Examiner's comments: Nobody seemed to remember that it is a circular orbit.

viii) Using the answers obtained in Questions 4 and 5, find the minimum stellar mass for an Earthlike planet to orbit around it without being tidally locked for at least 1 billion years. [3 marks]

Based on the Mass-Luminosity Relation for Main Sequence stars, we know that
$L_{star}/L_{sun} = (M_{star}/M_{sun})^{3.5}$

Substituting,
$a/M_{star} = (150 \times 10^9)^3 \times ((M_{star}/M_{sun})^{3.5}/0.53)^{1.5}/ M_{sun} = 101.52\ M_{star}/M_{sun}$
$(M_{star}/M_{sun})^{4.25} \times ((1.50 \times 10^{11})^3 \times 0.53^{-1.5})/ M_{sun} = 101.52$
$(M_{star}/M_{sun})^{4.25} \times (8.75 \times 10^{33}) / (1.989 \times 10^{30}) = 101.52$
$(M_{star}/M_{sun})^{4.25} \times 4399.2 = 101.52$
$M_{star}/M_{sun} = \underline{0.412}$

TRAPPIST-1 is an ultra-cool dwarf star with 7 planets orbiting around it, of which five are approximately Earth-sized and 3 of which lie within the habitable zone. TRAPPIST-1 has a mass of 0.0802±0.0073 solar masses.

ix) Based on your result in question 6, explain qualitatively if there is any insight it provides on the habitability of these planets. [2 marks]

Possible answers:

The standard formula for calculating the habitable zone may not be appropriate for calculation of the habitable zones when tidal locking is present.

Since there is tidal locking, life may not be able to arise as one side of the planet will be in perpetual daylight, resulting in scorching temperatures, while temperatures on the night side is likely too low to support life.

Since there is tidal locking, life may only arise in the twilight zone/terminator region.

With tidal locking, life may still exist if there are winds/convection driven currents to balance temperatures on the day and night time side.

Other answers acceptable if they make sense. The purpose of this final part is just to see if they understand the question (especially since much of it is explained in the opening paragraph)

Life Around a Chubby Sun [20+1 marks]

Suppose that a group of amateur astronomers has discovered a new star system that is similar to our Solar System (consists of a main-sequence star and several planets). They used a telescope that has a limiting magnitude of +10. From careful analysis of the data obtained, it is known that the star has a mass of $20\ M_{\odot}$.

i) The distance of the star from Earth is roughly 1000 pc. Determine the visual magnitude, m_V, of the newly discovered star, if the luminosity of the star is $100L_{\odot}$. [1 mark]

ii) This star, like all other main-sequence star, produces energy in its core via hydrogen-burning process or also known as the proton-proton fusion chain reaction. In this reaction, 4 protons are fused to produce one He atom. If only 10% of the mass is available for burning, determine the main-sequence lifetime of this star **in years**. You may assume that the star is made up of hydrogen only where applicable. [4 marks]

iii) A boy who is eager to learn astronomy knows about the discovery of this star. He wants to observe it but using a weaker telescope. The diameter of his telescope is one-half of that of the telescope used by the astronomers. Is he able to observe it? Hint: calculate and compare the limiting magnitudes of the two telescopes. [2 marks]

iv) A fast-rotating planet of radius $R_{\oplus}$ is discovered to be revolving around this star with a circular orbit at distance d = 1.523 AU. The surface albedo of this planet is 0.25. Determine the average blackbody temperature at the surface of the planet. [5 marks]

v) Judging from the surface temperature only, is it possible for humans to live on that planet? If not, propose the best temperature range and the suitable distance between the star and planet so that humans are able to live on it. [5 marks]

vi) This star, after ending its lifetime in the main-sequence stage, will eventually move up along the giant branch, during which its temperature drops by a factor of 3 and its radius increases 100-fold. Determine its new visual magnitude. [4 marks]

vii) Bonus: State what's unrealistic with the scenario presented in this question. [1 mark]

SOLUTION

i) By using Pogson's equation:

$$m_V - m_{\odot} = -2.5\log\frac{E}{E_{\odot}}$$

$$m_V - (-26.7) = -2.5\log\left(\frac{100L_{\odot}}{L_{\odot}} \times \frac{(1AU)^2}{(1000\ pc)^2}\right)$$

$$m_V = 9.87$$

ii) Each chain reaction gives 26.73 MeV, 4.28×10^{-12} Joules of energy

The stellar luminosity is $100\ L_{\odot}$, thus you would need 8.99×10^{40} chain reactions/s.

Mass available for burning is 10% or 4×10^{30} kg, assuming these are matter made up of hydrogen, there would be 2.48×10^{57} hydrogen available, which then makes 5.99×10^{56} chain reactions possible.

This means, the main sequence lifetime would be 6.66×10^{15}s or 0.21 billion years.

iii) $m_{limit} = m_{eyes} + 2.5 \log \frac{D}{D_{pupil}}$

Hence, to compare two limiting magnitude of two telescopes:

$$m_{limit1} - m_{limit2} = 2.5 \log \frac{D_1}{D_2}$$

We know that $m_{limit1} = 10$ and $D_1 = 2D_2$. Hence we can calculate m_{limit2}. Putting in all the numbers into the equation will yield $m_{limit2} = 9.25$. Hence, it is impossible for the boy to observe the star.

iv) Let F be the radiation flux of the star at the planet's surface.

$$F = \frac{L}{4\pi d^2}$$

d is the distance from the star to the planet and L is the luminosity of the star. We assume that out of the total flux incident, fraction α is reflected and the rest is absorbed. Hence the absorption rate is given by

$$A = (1 - \alpha)\pi R^2 F$$

R is the radius of the planet.

Here, we will neglect the internal energy source of the planet. Let T be the black-body temperature of the planet's surface. Since it is rotating fast we can assume that the planet is being heated up uniformly to the same temperature T. At equilibrium, the total amount of black-body radiation emitted must be equal to the absorption rate.

$$4\pi R^2 \sigma T^4 = (1 - \alpha)\pi R^2 \frac{L}{4\pi d^2}$$

$$\therefore T = \sqrt[4]{\frac{(1 - \alpha)L}{16\pi\sigma d^2}}$$

We have derived the formula to determine the planet's surface temperature. We know that $\alpha = 0.25$, $d = 1.523$ AU and $L = 100L_{\odot}$. Putting in the numbers into the formula (and in S.I. units) will yield

$$T = 664.66\ K$$

v) Definitely not possible. This temperature is too high for living things to survive. A good range of temperature is from 273 K – 313 K.

For $T = 273K, d = 9.03\ AU$

For $T = 313K, d = 6.87\ AU$

Hence, the best distance is from 6.87 AU – 9.03 AU.

vi) By using Pogson's equation:

$$m_{V1} - m_{v2} = -2.5 \log \frac{E_1}{E_2}$$

m_{V1} is the magnitude in main-sequence lifetime

m_{V2} is the magnitude when moving up the giant branch.

$$9.87 - m_{v2} = -2.5\log\frac{L_1}{L_2}$$

$$9.87 - m_{v2} = -2.5\log\frac{R_1^2 T_1^4}{R_2^2 T_2^4}$$

$$9.87 - m_{v2} = -2.5\log\frac{R_1^2 \times 81 T_2^4}{10000 R_1^2 T_2^4}$$

$$m_{v2} = 4.64$$

vii) Just want to point out that it's quite unnatural for the star with $20\,M_\odot$ to have $100L_\odot$. But for the sake of the question, let's just accept it.

Space Telescope [20 marks]

The Hubble Space Telescope, which has been in service since 1990, is going to be succeeded by a new telescope, the James Webb Space Telescope (JWST). We will investigate the properties of the space telescope.

Part I: The Cosmological redshift [5 marks]

Due to the nature of the universe, very distant objects are subject to redshift. As a result, most features that would usually be observed in the visible spectrum will become redshifted. For a homogeneous and isotropic universe,

$$1+z = a_{now}/a_{then}$$

Where a is the cosmic scale factor, and z is redshift. A_{now} is assumed to be 1.

The James Webb Space Telescope aims to observe distant objects, and as such, will be able to analyze the redshifted spectra more readily. It has a detection range from 600 to 28,500 nm. Notice that this only covers a small portion of the visible spectrum.

i) For the following lines, which ones are visible for a scale factor of 0.12? [3 marks]

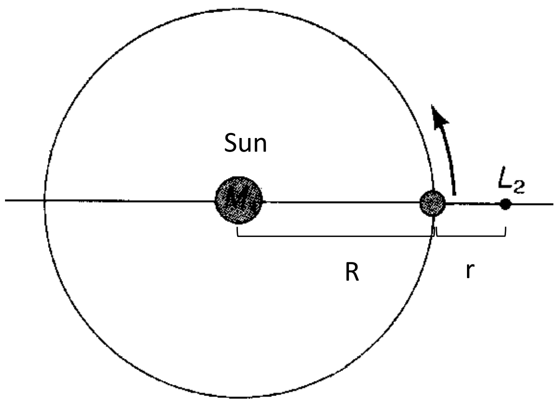
Line	Hα	Hβ	Lα	Lβ	Ne II	He II	H_2O
Wavelength (nm)	656.3	486.1	121.6	102.6	44.8	30.4	19230

ii) The most prominent feature of extremely distant galaxies is their Lyman lines. What is the maximum redshift that can be analyzed? [2 marks]

To calculate redshift, $z = (a_{now}/a_{then}) - 1$,
Since $z = \Delta\lambda/\lambda_0$, $\lambda = (z+1)\lambda_0$
Taking $a_{now}=1$,
$\Lambda=\lambda_0/a_{then}$
The lines which are visible are thus Hα , Hβ, Lα, Lβ. NE II and HE II are now visible in the visual spectrum. However, they are not within JWST's range. H_2O is shifted out of the range.
By shifting the Lβ line to 28500 nm, we get z = 276.8

Part II: Orbit [5 marks]
[Assume a spherical orbit, and disregard the moon in the following questions]

Due to the combined gravitational pull of both the sun and the earth, the James Webb Space Telescope orbits the sun under what is known as a "halo orbit" in the L2 Lagrange point. In this orbit, the relative position of the sun, earth and satellite is always fixed (refer to figure below):



iii) Derive an expression involving only m_{earth} , M_{sun} , R , and r. Confirm this for $r = 1.5\times10^9\ m$. [4 marks]

Since L2 is a point that follows the Earth, and the fact that the Sun, Earth, and L2 form one line:

$$F_{sun} + F_{earth} = \omega^2(R+r) \quad [1\ mark]$$

Inserting F_s and F_e,

$$GM/(R+r)^2 + Gm/r^2 = \omega^2(R + r)$$

Since $\omega = \omega_{earth}$,

$$GM/(R+r)^2 + Gm/r^2 = GM(R+r)/R^3 \ [1\ mark]$$

Eliminating G, we get

$$M/(R+r)^2 + m/r^2 = M(R+r)/R^3 \ [1\ mark]$$

Substitute values of R, m and M to and show that LHS = RHS. [1 mark]

iv) Compute the ratio of r to the earth-moon distance and comment on the feasibility of conducting repairs on the JWST. [1 mark]

$$\frac{r}{d_{earth-moon}} = 3.9$$

The large distance between moon (and hence earth) and the JWST renders it unfeasible to conduct repairs.

Part III: Orbital Redshift [5 marks]

In addition to the galaxy's expansion, the JWST's orbital motion around the sun leads to received electromagnetic waves undergoing a further Doppler shift as given by

$$z = \sqrt{\frac{c+v}{c-v}} - 1$$

v) Calculate the JWST's orbital velocity in the L2 point. [1 mark]

$$v = r\omega = (1.5 * 10^{11})(2\pi/1yr)$$
$$= 29886\ ms^{-1}$$

vi) Calculate the resulting Doppler shift, z. By how much, in nm, is the H-alpha line shifted due to this? [2 marks]

$$z = \sqrt{\frac{c+v}{c-v}} - 1$$
$$\approx \frac{v}{c}$$
$$= 9.96 * 10^{-5}$$
$$\Delta\lambda = z * \lambda_0$$
$$= (9.96 * 10^{-5})(656.3 * 10^{-9})$$
$$= 0.0654\ nm$$

vii) Compare your value of z with that obtained in part I, and comment on your result. [2 marks]

$$\frac{\lambda_{orbit}}{\lambda_{galactic}} = 73630$$

Redshift due to expansion is much more apparent than redshift due to orbital motion.

Part IV: Optics [5 marks]

Now, we analyze the mirror of the JWST. It is larger than the one in HST. Once again, the nature of the mirror necessitates the use of multiple segments as opposed to one large segment. As shown in the diagram, the telescope consists of 18 hexagonal segments, each 1.32m in diameter (flat edge to flat edge).

viii) Derive the total area of the mirror. What would be the equivalent diameter for a standard circular mirror? [2 marks]

ix) Compared to the Hubble Space Telescope (D=2.4m), how much more collecting power does the telescope have? [1 mark]

x) At what wavelength will the resolving power of the JWST match the one of the HST at visible light? Assume that is the JWST mirror is approximately circular. [2 marks]

For a hexagon, the area can be expressed as A=2r^2*sqrt(3), which gives us
Thus, the area is

18*A
=18*2*r^2*sqrt(3)
=27.2 m^2

The collecting power is directly proportional to the collecting area of the telescopes. Thus,

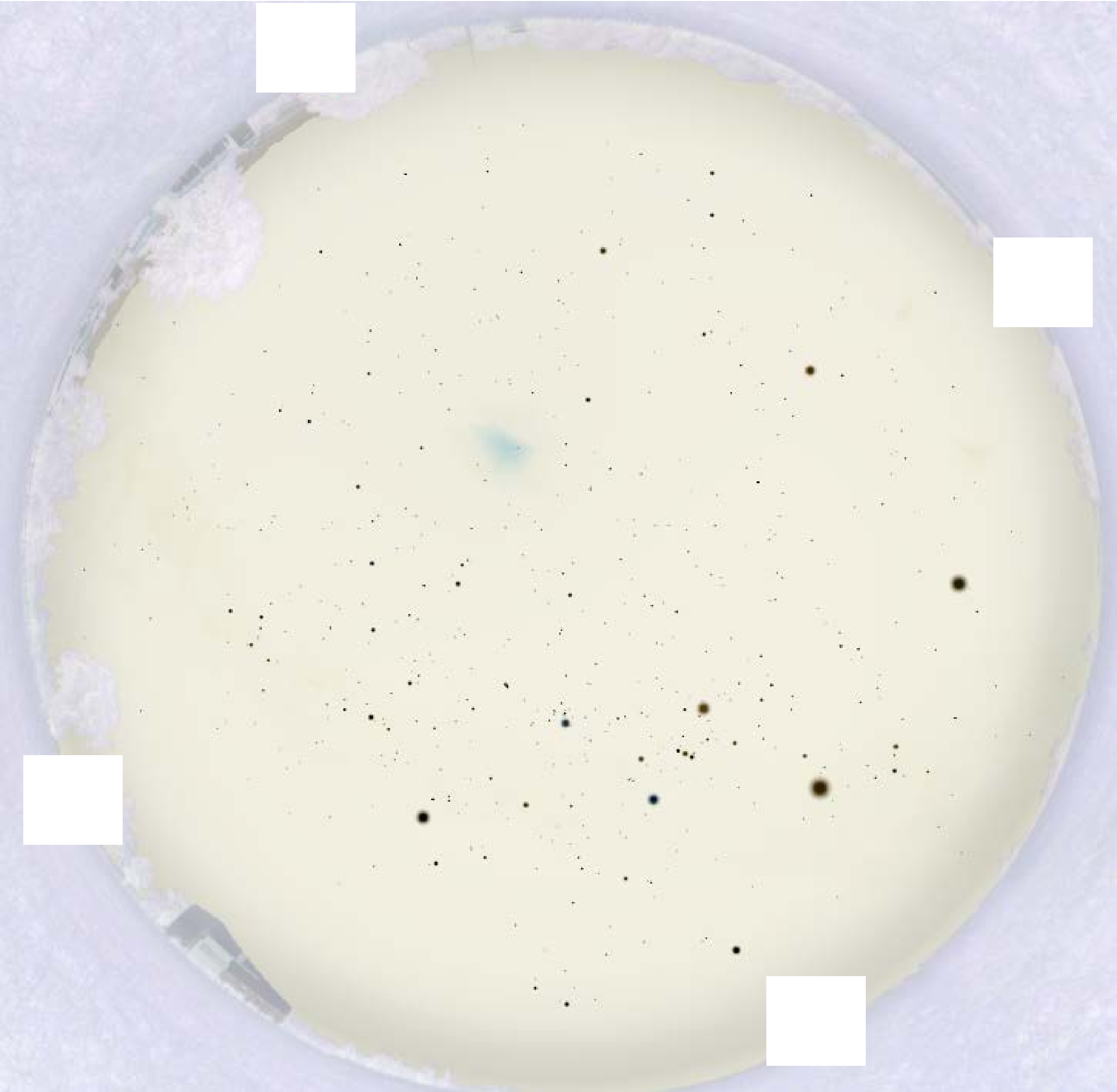
$P_S/P_h=A_s/A_H$
$=A_s/(\pi*(D_h/2)^2)$
=6x

The resolving power is defined for a wavelength by the Rayleigh criterion, which states that sin(α)=1.22λ/D, where α is in radians. Thus, equating for both

$1.22\lambda_s/D_s=1.22(\lambda_h/D_h)$
$\lambda_s = \lambda_h D_s/D_h$
=1348 nm

Practical Astronomy [20 marks]

Part I: The Night Sky [10 marks]



Identify the following on the diagram given:

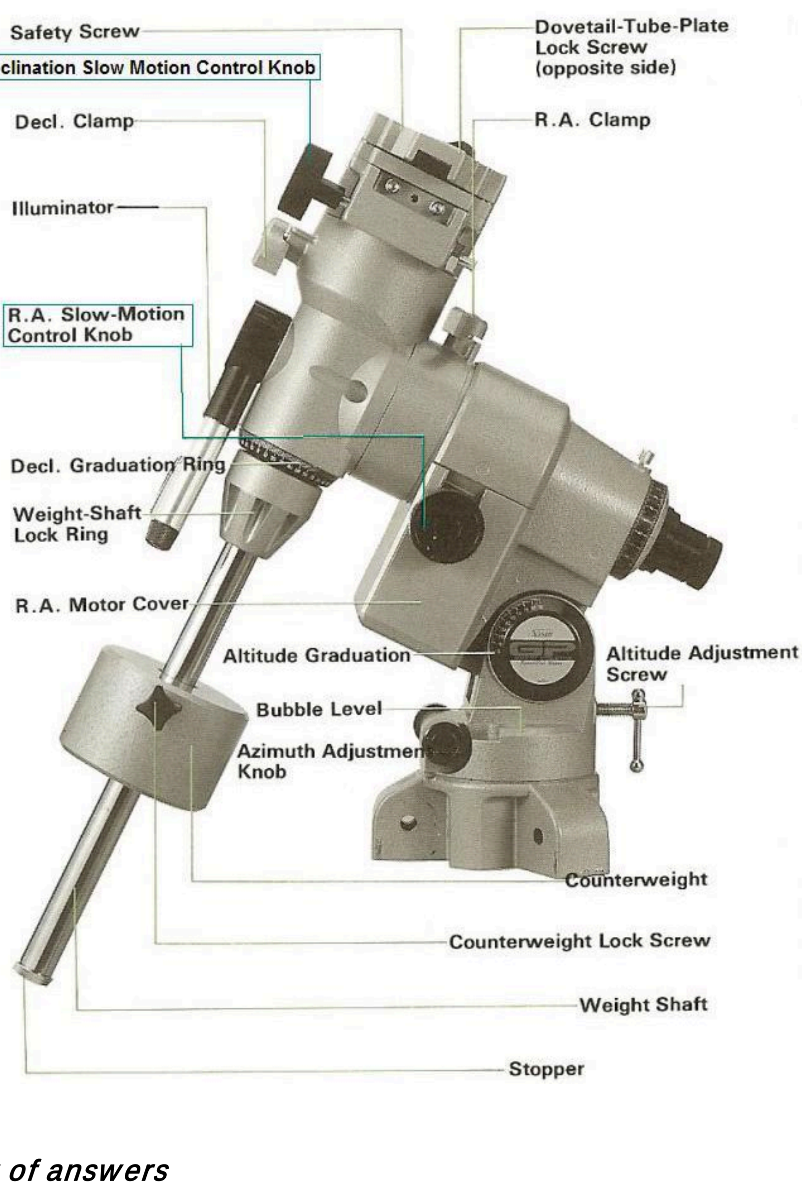
i) Circle Bellatrix (γ Ori). [1 mark]
ii) Circle Achernar (α Eri). [1 mark]
iii) Trace out the 'Winter Triangle' and label it. [1 mark]
iv) Trace out the 'Great Square of Pegasus'. [1 mark]
v) Trace out the constellation lines of 'Auriga'. [1 mark]
vi) List down 2 prominent galaxies and mark their approximate positions on the diagram. [2 marks]
vii) List down 1 prominent nebula aside from M42 and mark their positions on the diagram. [1 marks]
viii) List down 2 prominent open star clusters and mark/circle their approximate positions on the diagram. [2 marks]

Answers: Refer to Stellarium - 6 October 2017 2am

Part II: Setting up a German Equatorial Mount [10 marks]

You want to set up an equatorial mount to observe the night sky in Singapore. A detailed procedure of the setup process is written on the following page. Identify as many errors as possible, and explain how it should be corrected. Write your corrections on the blank space provided on the right side of the page.

A photograph of a standard equatorial mount is shown below for reference, with the parts labelled.
(Note: There are no errors in the photograph. It is for your reference only)



Sample format of answers

Procedure	Your answers here
1. Install the <u>tripod on top of the mount head</u> and secure it using the locking screws	*The mount head should be installed on the top of the tripod, not the other way round*

Note: You will be awarded 0.5 marks for every error found, and 1 mark for a correct explanation for a total of 1.5 marks per error

Procedure to Set up a German Equatorial Mount

Initial Steps

Your answers here

1. Transport the mount to the observation site. Ensure that both the RA and DEC clamps are locked and engaged so that the axes do not freely rotate when transported.

2. To begin the setup process, spread the tripod legs and place the tripod on flat ground. If the ground is soft or muddy, drive the tripod legs as deep into the ground as possible to ensure stability. Orient the tripod roughly in such a position that when the mount head is installed, the front of the mount (where the counterweight bar protrudes) will point due south.

3. Install the equatorial mount head onto the top of the tripod, and tighten the main central screw, leaving a small amount of slack so that the azimuth axis can be adjusted for more precise polar alignment.

4. Using a compass, ensure that the mount head is roughly polar aligned with the mount head pointing due south. Level the mount using a bubble level, and use the altitude adjustment screw to ensure that the latitude reading corresponds correctly to your geographic location.

Installing the telescope and counterweights

Your answers here

5. In steps 6 and 7, ensure that the mount's axes are unlocked to ensure smooth movement.

6. Install the telescope tube by sliding the telescope's dovetail into the mount's dovetail saddle. Tighten the locking screws on the saddle to hold the telescope in place.

7. Install the counterweight shaft onto the equatorial mount. Slide the counterweights into the counterweight shaft and tighten the counterweight's locking screw to hold it in place. Install the stopper to prevent the counterweights from falling off the shaft.

Balancing the setup **Your answers here**

8. Balance the Right ascension axis: Rotate the RA axis such that the counterweight shaft is parallel to the ground. Lock the RA axis and leave the DEC axis unlocked to determine the balance point. Shift the counterweights along the counterweight shaft until the RA axis is balanced (i.e. no tendency to rotate in any direction around the RA axis).

9. Balancing the Declination axis: With the counterweight shaft still parallel to the ground, Lock the RA axis and leave the DEC axis unlocked to determine the balance point. Shift the telescope tube forwards or backwards to balance the DEC axis. When there is no tendency for the DEC axis to rotate, the mount is sufficiently balanced.

10. As far as possible, do not install all the accessories (diagonals, eyepieces and finderscopes) until the balancing is complete to ensure that the balance of the system is not disrupted.

Answer Scheme

Erroneous steps in red, corrections in green

1. Transport the mount to the observation site. Ensure that both the RA and DEC clamps are locked and engaged so that the axes do not freely rotate when transported.
   The mount should be transported with the axes disengaged so that the gears will not be damaged during handling. [1.5]
2. To begin the setup process, spread the tripod legs and place the tripod on flat ground. If the ground is soft or muddy, drive the tripod legs as deep into the ground as possible to ensure stability. Orient the tripod roughly in such a position that when the mount head is installed, the front of the mount (where the counterweight bar protrudes) will point due south.
   We are in Singapore; mount should point north [1.5]
3. Install the equatorial mount head onto the top of the tripod, and tighten the main central screw, leaving a small amount of slack so that the azimuth axis can be adjusted for more precise polar alignment.
4. Using a compass, ensure that the mount head is roughly polar aligned with the mount head pointing due south. Level the mount using a bubble level, and use the altitude adjustment screw to ensure that the latitude reading corresponds correctly to your geographic location. As per point 2. No additional credit for picking this up twice.
5. In steps 6 and 7, ensure that the mount's axes are unlocked to ensure smooth movement. No please don't do that. Ensure it is locked to prevent any sudden movements during mounting the telescope and counterweights. [1.5]
6. Install the telescope tube by sliding the telescope's dovetail into the mount's dovetail saddle. Tighten the locking screws on the saddle to hold the telescope in place.
7. Install the counterweight shaft onto the equatorial mount. Slide the counterweights into the counterweight shaft and tighten the counterweight's locking screw to hold it in place. Install the stopper to prevent the counterweights from falling off the shaft.

Steps 6 and 7 should be swapped. Counterweights must be installed first! [1.5]

8. Balance the Right ascension axis: Rotate the RA axis such that the counterweight shaft is parallel to the ground. Lock the RA axis and leave the DEC axis unlocked to determine the balance point. Shift the counterweights along the counterweight shaft until the RA axis is balanced (i.e. no tendency to rotate in any direction around the RA axis). Should be the other way round. [1.5]
9. Balancing the Declination axis: With the counterweight shaft still parallel to the ground, Lock the RA axis and leave the DEC axis unlocked to determine the balance point. Shift the telescope tube forwards or backwards to balance the DEC axis. When there is no tendency for the DEC axis to rotate, the mount is sufficiently balanced.
   9 & 10 should be swapped. The DEC should be balanced before RA. [1.5]
10. As far as possible, do not install all the accessories (diagonals, eyepieces and finderscopes) until the balancing is complete to ensure that the balance of the system is not disrupted. We should install the accessories before balancing, so that the mount is balanced in the final configuration. [1.5]

Marking scheme: 0.5 marks for identifying the error, additional 1 mark for correcting the error correctly.

Total possible score: 1.5 x 7 = 10.5 (Max 10).