AstroChallenge 2025
Junior Team Round

Wednesday 4th June 2025

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. This paper has a total of **22** printed pages, including any blank pages and this cover page.
2. Any materials other than the Question Paper, Formula Booklet, and **ONE** A4-sized cheat sheet per team, are strictly prohibited.
3. Do **NOT** turn over this page until instructed to do so.
4. You have **2 hours** to attempt **ALL** questions in this paper.
5. Write your answers on blank pieces of A4 paper or graph paper (if necessary).
6. Use a separate piece of paper for each question; no one piece of paper should contain solutions to more than one question.
7. The marks for each question are given in brackets in the right margin, like such: **[2]**.
8. The **alphabetical** parts (i) and (l) have been intentionally skipped, to avoid confusion with the Roman numeral (i).
9. At the end of the paper, submit your answer script with solutions ordered accordingly. You do not need to submit this booklet.
10. Ensure that your school and team number are clearly indicated in your answer script.
11. It is **your team's** responsibility to ensure that all pages of your answer script have been submitted, including pages to be detached from this booklet.

References

[1] B. P. Abbott et al. 'GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral'. In: *Phys. Rev. Lett.* 119 (16 Oct. 2017), p. 161101. DOI: 10.1103/PhysRevLett.119.161101. URL: https://link.aps.org/doi/10.1103/PhysRevLett.119.161101.

[2] European Space Agency. *Beauty from chaos*. 10th Apr. 2014. URL: https://www.esa.int/Science_Exploration/Space_Science/Mars_Express/Beauty_from_chaos.

[3] J H Gillanders et al. 'Modelling the spectra of the kilonova AT2017gfo – II. Beyond the photospheric epochs'. In: *Monthly Notices of the Royal Astronomical Society* 529.3 (Nov. 2023), pp. 2918–2945. ISSN: 0035-8711. DOI: 10.1093/mnras/stad3688. eprint: https://academic.oup.com/mnras/article-pdf/529/3/2918/57065463/stad3688.pdf. URL: https://doi.org/10.1093/mnras/stad3688.

[4] Edward L. Wright. *Doppler Shift*. Accessed: 2025-05-12. 2002. URL: https://astro.ucla.edu/~wright/doppler.htm.

[5] Xaonon. *Comparison of sidereal and solar time*. SVG image, licensed under CC BY-SA 4.0. 2017. URL: %7Bhttps://commons.wikimedia.org/wiki/File:Sidereal_time.svg%7D.

[6] MedMig YT. *Amelia Watson ground pounds desk-kun [HololiveEN Animation]*. YouTube video. Screenshot taken at 0:03. Jan. 2021. URL: https://www.youtube.com/watch?v=wiMdgOoCBjI.

Question 1 Our Solar System Family

(a) Galaxies are filled with giant gas clouds, and they are mostly stable. Suggest one possible event that can cause the gas clouds to collapse and form stars. **[1]**

(b) At first, the gas cloud is barely rotating. As the gas cloud collapses to form a star cluster, it rotates faster and faster. Why? **[1]**

(c) Explain why the Sun and planets of the Solar System are all seen near the ecliptic. **[2]**

(d) Mercury's surface temperature can vary from 430 °C to −180 °C. Explain why this happens. **[2]**

(e) All planets of the Solar System orbit the Sun and rotate on its axis in the anticlockwise direction when viewed from the top, except for Venus, which rotates clockwise. Suggest a reason why. **[1]**

(f) The oldest rocks on Earth are estimated to be 4.5 billion years old, and we believe Earth is the same age. Explain why and how we can tell the age of the Earth just by 'looking at rocks'. **[2]**

(g) Although the moon is around the same age as the Earth, its surface is very heavily cratered compared to Earth. Suggest 2 reasons why. **[2]**

Figure 1: The surface of Mars showing long, valley like structures. [2]

(h) Suggest what may have caused the long, valley-like structure on the Martian surface (see Figure 1). **[1]**

(j) Some people think that Jupiter protects Earth from asteroids, while some think that it sends more asteroids our way. Why does Jupiter have the ability to do such things? **[2]**

(k) Saturn is very well known for its rings. In March 2025, the rings were not visible for any observers on Earth. Suggest a reason why. **[1]**

(m) Ancient civilizations more than 2000 years ago already knew about the existence of the Sun and the planets Mercury to Saturn, but Uranus was only discovered in 1781. Why did people not know about Uranus before that? **[1]**

(n) Although Neptune has a mass 17 times the mass of the Earth, its surface gravity is actually similar to that of Earth. If you were standing on its surface (if it has a solid surface), the gravity you would experience is very similar to Earth's. Why is this so? **[1]**

(o) Does your team think Pluto should be considered a planet? Why or why not? **[1]**

(p) Why is it not possible to get a 'family portrait' of all the planets at local midnight, even if you could live long enough to see all the possible arrangements and positions of the planets? Which planets can you not see, and why? Assume that your technology is good enough to capture a decent portion of the sky and can see Uranus and Neptune. **[2]**

Question 2 Exploring Colliding Dead Stars

Part I Finding the ripples in space-time

What happens when massive bodies move in the four-dimensional (three-spatial and one-time dimension) spacetime? Are there any perturbations we can observe? 10 years ago, in 2015 we made our first observations of gravitational waves. However, this question will discuss another instance of detection of these waves. On the 17th of August, 2017, Ligo detected gravitational waves (GW170817) from two orbiting objects with the pattern as shown in Figure I-1.

(a) What does the middle graph (labelled LIGO-Livingston) in Figure I-1 tell about the two orbiting objects? **[1]**

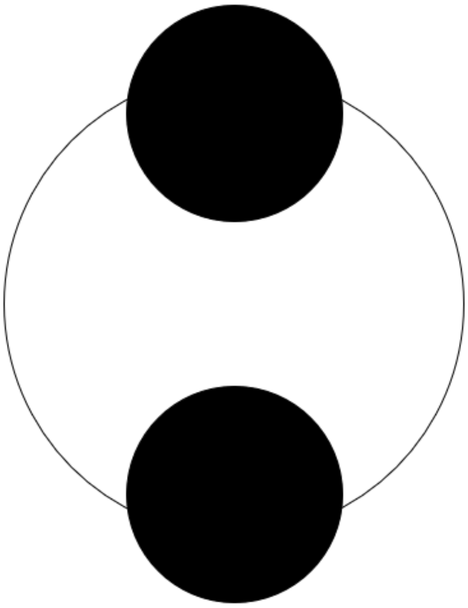

Figure 2: The two bodies of equal mass orbiting around the centre.

(b) Refer to Figure 2. Assuming the two objects are of equal mass, calculate the orbital period of the two objects at the gravitational wave frequency of 100Hz (This is a circular orbit). **[1]**

(c) Given $m_1 = (1.81 \pm 0.45)M_{\odot}$ and $m_2 = (1.11 \pm 0.25)M_{\odot}$, is this system more likely a neutron star-black hole pair or neutron star-neutron star pair? *(Hint: The Chandrasekhar limit is about 1.4$M_{\odot}$.)* **[1]**

Given that m_1=1.81$M_\odot$ and $m_2 = 1.11M_\odot$, and the total distance between the m_1 and the barycenter is 116.32 km, the total distance between m_1 and m_2 is about 300 km. the power loss from gravitational waves can be described using Equation 1:

$$-\left\langle \frac{dE}{dt} \right\rangle = \frac{32G^4 m_1^2 m_2^2 (m_1 + m_2)}{5c^5 a^5 (1 - e^2)^{7/2}} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right) \tag{1}$$

where e is the orbital eccentricity and a is the length of the semimajor axis.

It is known that 2.5% of the total energy of the system was lost as gravitational waves for a duration of 1.5 s in the collision.

Assuming that bodies involved in the collision are point mass and there was no other loss of energy, Milo tried to calculate the final mass of the remnant. This is his working:

Since the orbit is circular,

$$e = 0 \tag{2}$$

$$a = 300\,\text{km} \tag{3}$$

Using Equation 1,

$$P_{\text{loss}} = \frac{32G^4 m_1^2 m_2^2 (m_1 + m_2)}{5c^5 a^5 (1 - e^2)^{7/2}} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right) = \frac{32G^4 m_1^2 m_2^2 (m_1 + m_2)}{5c^5 a^5} \tag{4}$$

$$E_{\text{loss}} = P_{\text{loss}}\, t = \frac{32G^4 m_1^2 m_2^2 (m_1 + m_2) t}{5c^5 a^5} \tag{5}$$

$$E_{\text{remaining}} = \frac{0.975}{0.025} E_{\text{loss}} = \frac{0.975}{0.025} \times \frac{32G^4 m_1^2 m_2^2 (m_1 + m_2) t}{5c^5 a^5} \tag{6}$$

$$M_{\text{remaining}} = \frac{E_{\text{remaining}}}{c^2} = \frac{1}{c^2} \times \frac{0.975}{0.025} \times \frac{32G^4 m_1^2 m_2^2 (m_1 + m_2) t}{5c^5 a^5} = 5.12 \times 10^{27}\,\text{kg} \tag{7}$$

However, it turned out that the actual mass of the remnant is 5.45×10^{30} kg.

(d) One of the key steps above contained a mistake, which resulted in the disparity in the calculation outcome. Which step is it? Explain why it is a mistake. **[1]**

Note that there is no mistake in the numerical calculation. You may use the equation number to refer to the step.

(e) What is one technique that you can use to correct for this mistake? Qualitatively explain how this technique will be helpful. **[1]**

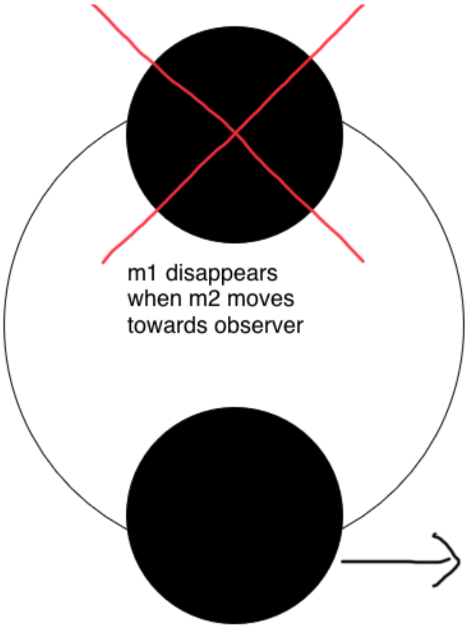

Figure 3: m_1 disappears, m_2 is directly coming towards you.

(f) Suppose before the collision, m_1 disappears when the orbital frequency is 100Hz. Figure 3 depicts the moment when m_1 disappears.

Find the angular displacement of m_2 after m_1 disappeared but before m_2 was flung out.

You may assume the speed of gravitational waves to be the speed of light. **[2]**

Part II The Kilonova

During the merger, atomic nuclei go through rapid neutron capture process (r-process), in which atoms capture neutrons quickly. Due to some of these nuclei being unstable, they decay quickly, releasing light - the kilonova! (The kilonova is accompanied by a short Gamma Ray Burst (GRB) of less than 2s and X-ray/radio synchrotron emission. Additionally there is a GRB afterglow in X rays and radio waves).

The discovery of the GRB together with the gravitational waves, confirmed our theory that gravity propagates at the same speed as light. Figure I-3 shows the emission spectrum of AT2017gfo, the electromagnetic counterpart detection of the merger. Studying the spectra tells the nature of the spherically expanding output material. We will investigate this in more detail in the coming questions.

(g) Explain why there are peaks at certain wavelengths in this light spectrum. **[1]**

(h) Explain why the light spectrum looks like a bell-shaped curve instead of sharp emission lines. **[1]**

(j) Take the 1.58 μm (micrometer) feature in the +8.4 d graph. Estimate the full width at half maximum intensity (FWHM)[1] of the curve. **[1]**

(k) Let f_o be the observed frequency, f_s be the source frequency, and v_s be the source radial velocity. The Doppler effect equation relating these quantities is Equation 8.

$$f_o = f_s \left(\frac{c}{c \pm v_s} \right) \tag{8}$$

Assuming spherical expansion of output and taking into account the Doppler broadening, find the expansion velocity of the output material in terms of the speed of light. **[2]**

[1] FWHM is the width of a spectrum curve measured between those points on the y-axis which are half the peak amplitude.

The emission spectrum of AT2017gfo shows the presence of tellurium, formed from the decay of heavy elements — these are formed via the rapid neutron capture process (r-process). Tellurium-128 is an isotope formed via the radioactive decay of antimony (Sb). This decay - negative beta decay - releases an electron (e^-) and an electron antineutrino ($\bar{\nu}_e$). The decay scheme of Sb-128 is given as follows:

$$^{128}_{51}\text{Sb} \longrightarrow ^{128}_{52}\text{Te} + e^- + \bar{\nu}_e$$

You are also given the molar mass of Sb-128 and Te-128:

$$\text{molar mass of } ^{128}_{51}\text{Sb} = 127.9091673\,\text{g/mol}$$

$$\text{molar mass of } ^{128}_{51}\text{Te} = 127.9044614\,\text{g/mol}$$

(m) Using $E = mc^2$, calculate the energy released (total energy of electrons and antineutrinos released) in the decay of one atom to 3 significant figures. **[1]**

(n) Given that the heliocentric velocity (receding away from the sun) of NGC 4993, the host galaxy of AT2017gfo, is 2930km/s, find the distance to NGC 4993 in Mpc. **[1]**

(o) Given that AT2017gfo is observed to have a peak apparent magnitude of 17.22 in the Y0 band, find the peak absolute magnitude of AT2017gfo in the Y0 band. **[1]**

(p) Given that the AT2017gfo peak magnitude of 17.22 is observed over 1 second, estimate the number of Sb-128 atoms that decayed during that second. **[2]**

Hint: Refer to the absolute bolometric magnitude formula in the formula book.

Part III Galactic Influence

(q) Galactic quenching is a process in which star formation is cut off in a galaxy. How may supernovae cause quenching? How do Kilonovae compare? **[2]**

(r) A popular astronomer, Carl Sagan, had said "We are made of star-stuff". With our newfound understanding, do you agree with the statement? Explain your answer. **[1]**

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Question 3 Problem Caused by *Skill Issue*

NOTE: THIS IS A QUESTION DESCRIBING A FICTIONAL SITUATION THAT OCCURRED IN A FICTIONAL UNIVERSE. ANY COINCIDENCE IN NAMES, EVENTS, ORGANISATION AND OTHER ENTITIES ARE PURELY COINCIDENTAL.

Tails, an inhabitant of Earth, was playing a game of *Valorant* and was about to hit the "Radiant" rank, where an Internet outage suddenly happened and made her lose the game! Seeing her rank points drop, Tails got so tilted that she slammed her desk, which to her (and everyone's) surprise, caused the Earth's position to shift and its orbit to become tilted!

Figure 4: A rough illustration of the desk slam that caused this whole incident [6]

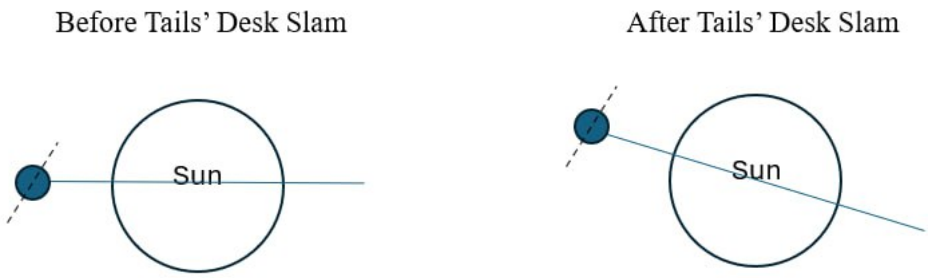

Figure 5: Tails' illustration of the solar system (before and after her tilt)

Note: As this question is set in a fictional universe, we shall make the following assumptions to make out lives easier. You are reminded of the need to take these assumptions into account when answering the questions.

1. The Solar System consists of only the Earth and the Sun, as shown in Figure 5.
2. The Earth is in a circular orbit around the Sun.
3. The physical properties of all objects in the Solar System, including the distance between the Earth and Sun, are as found in the Formula Booklet. The Solar System itself is assumed to not be moving.
4. Tail's Valorant tilt has only caused the Earth's position and the inclination of its orbit to change, as shown in Figure 5. The orientation of the rotational axis of the Earth, distance between the Earth and the Sun and other orbital parameters did not experience any changes.

Naturally, being small beings on Earth, it would be relatively difficult to physically feel the shift itself. However, certain objects can still serve as a strong indicator that the shift has happened, such as the Sun!

(a) With the aid of a diagram, describe and explain how one may immediately realise that the Earth's orbit has been perturbed? **[1]**

Realising that the Earth's orbit has been altered, Tails hurriedly contacted the only astronomers that she knows - us, **who are stationed at a space station far away from the Solar System**. Tails asks us whether the situation is an emergency, since this is the first time such an event has ever happened.

(b) Using your knowledge of astronomy, explain whether the change to Earth's orbit constitute an emergency to the inhabitants of Earth. **[1]**

Regardless of whether it is an emergency or not, it was decided that it would be the best to try and rectify the issue. After a bit of deliberation, Tails realised that a simple solution may be able to help her in making things back to how they were again.

> *"If I play enough* Valorant*, get tilted and then slam the table enough times, everything will probably go back to normal."*
> - Tails, probably

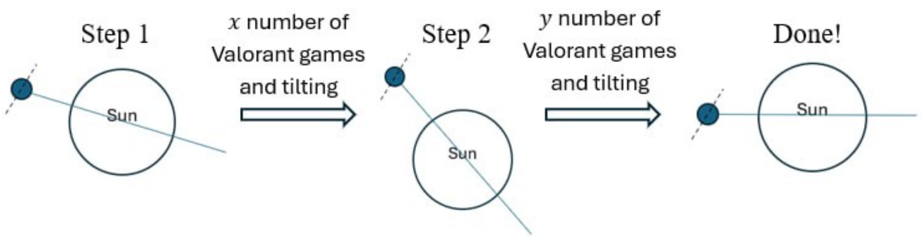

Figure 6: A brief visualisation of Tails' Plan

However, knowing that Tails suffers the chronic condition known by most as *Skill Issue*, she may accidentally cause the destruction of the Earth while conducting her plan. As such, it would be the best for us to track the presence of Earth, for us to see whether we will have to be self-sufficient for the rest of out lives in the space station. So, let's consider using some of the most common ways of detecting the presence of a planet like Earth (other than going to the planet and seeing whether it's there or not) and see what we can expect to see as Tails continues to tile and change the inclination of the Earth!

But before that, we should go through something that not many of us has heard before...

Part I Hold up - Orbits... Can Tilt?!

Just like any objects in 3-dimensional space, orbits can indeed be tilted! Such tilt takes a special name in astronomy (just like in any other sciences) - inclination. Formally, the definition of inclination is as such:

> *"The inclination of an orbit, i, is the angle between the reference plane and the orbital plane."*
> - Encyclopaedia Kushagra[2]

As Tails' legendary *Valorant* tilt caused a sudden tilting of the Earth's orbit, it hence would be of our interest to think about the change in the inclination. Relative to the space station that we are in, orbits with an inclination of 90° are edge on, while orbits with an inclination of 0° are face on. Figure 7 shows an example of what this means.

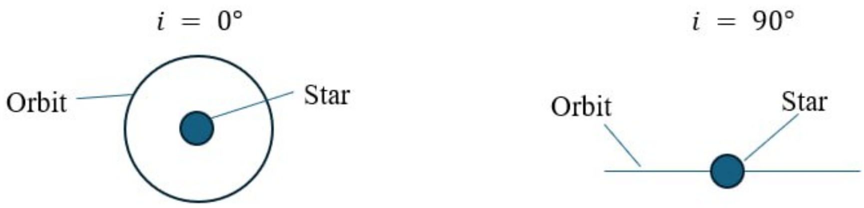

Figure 7: What inclination means relative to us

(c) What is the reference plane that we, the people on the space station, are using to determine the inclination of Earth? **[1]**

Alright, now that we have explored a little bit about inclination, let's get started on understanding the few methods of determining the presence of the Earth!

[2]This encyclopedia does not exist. Please do not actually search for it under any circumstances.

Part II Tails Angy, Tails Smash

One of the more common methods is what is known as Transit Photometry, where the presence of planets is detected through the dip in the brightness of stars during a transit. As planets pass in front of the star, they block some of the light emitted by the star they are orbiting about.

(d) Calculate the maximum percentage change in the brightness of the Sun with respect to the spaceship when the Earth transits the Sun, if possible. State any assumptions made in your calculations, if any. **[2]**

With this data, we can make a rough model of what we can expect to observe in terms of luminosity through a **light curve**, which is simply a graph which shows the brightness of an object throughout time. The light curve should be able to give us a rough expectation of what we should see if Tail's *Skill Issue* does not cause the destruction of Earth.

(e) Sketch the light curve of the Sun over the period of a year before the Earth's inclination was altered by Tail's initial tilt. **[2]**

Naturally, with the change in inclination caused by Tail's tilt, both initially and in her attempt to correct the orbit of the Earth, the light curve of the Sun would change - but exactly how so? To a certain extent, the decrease in inclination would cause the apparent transit path of Earth to shorten relative to an external observer, as shown in Figure 8.

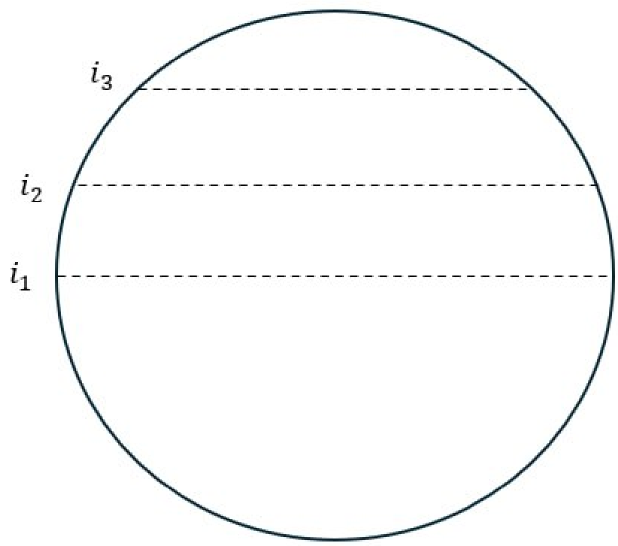

Figure 8: The change in transit path distance with decreasing inclination, from i_1 to i_3

Being the nerds that all of us are, is knowing that the duration of the transit would decrease with decreasing inclination good enough? Of course not! As people of cul- ahem I mean science, we seek to know exactly how the inclination would change the duration of the transit - would it change linearly (like a straight line graph), or will it change in some other weird manner? That is what we probably should (and we are going to) try and figure out.

Let's start by considering how a transit between the Sun and a planet of an orbital inclination i would look like from the space station's perspective. Since a transit is just a planet appearing to move across the Sun, the transit would probably look something like Figure 9 from the perspective of an observer.

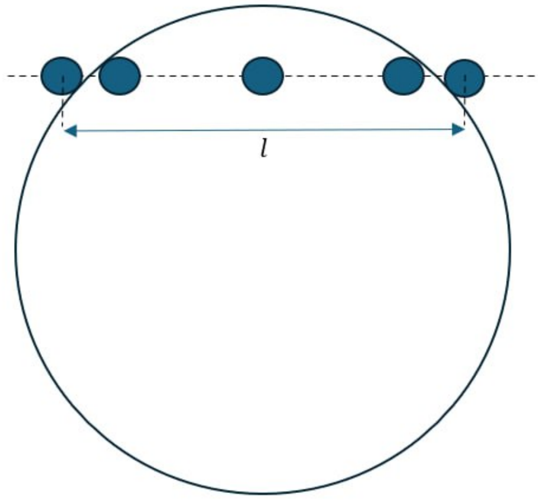

Figure 9: The Earth's transit across the Sun when the orbital inclination is not 90°

The apparent distance, l, shown in Figure 9, refers to the linear distance that the Earth has moved during the transit relative to us.

While it is simple for us to figure out the apparent distance when the inclination of the Earth is 90° (since it is just the diameter of the Sun), it becomes slightly harder to figure out for other inclinations. Thinking of the issue from another perspective could help us make things slightly simpler.

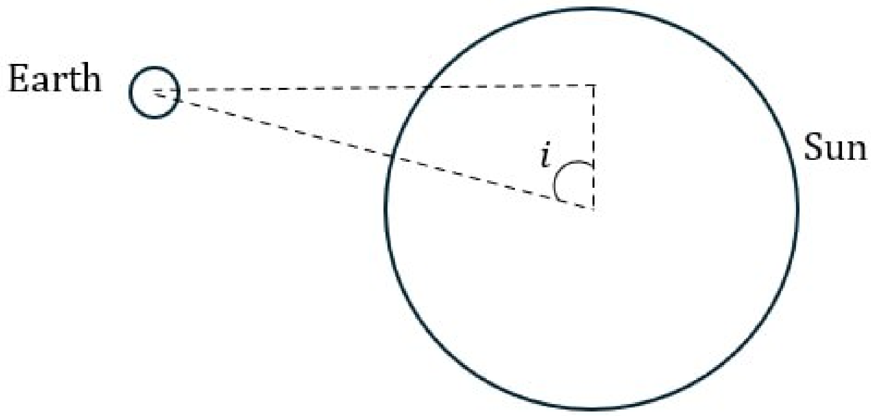

Figure 10: Side view of the Earth's transit when the orbital inclination is i°

(f) Given that the radius of Earth and Sun are R_E and R_S respectively, the inclination of the Earth's orbit at any point of time is i, and the radius of the Earth's orbit is d, show that the apparent distance travelled by the Earth, l during each transit is given by Equation 9.

$$l = 2\sqrt{(R_S + R_E)^2 - (d\cos i)^2} \tag{9}$$

You are highly encouraged to use diagrams wherever possible to aid in your derivation and explanation. **[2]**

Now that we have figured out the apparent distance the Earth has travelled, now what? One important thing to note is that we are living in a three-dimensional space, not some ideal two-dimensional world where every family consists of a handsome spy, a pretty assassin and a child who is able to read people's minds. Taking that into account, a planetary transit would look something like this:

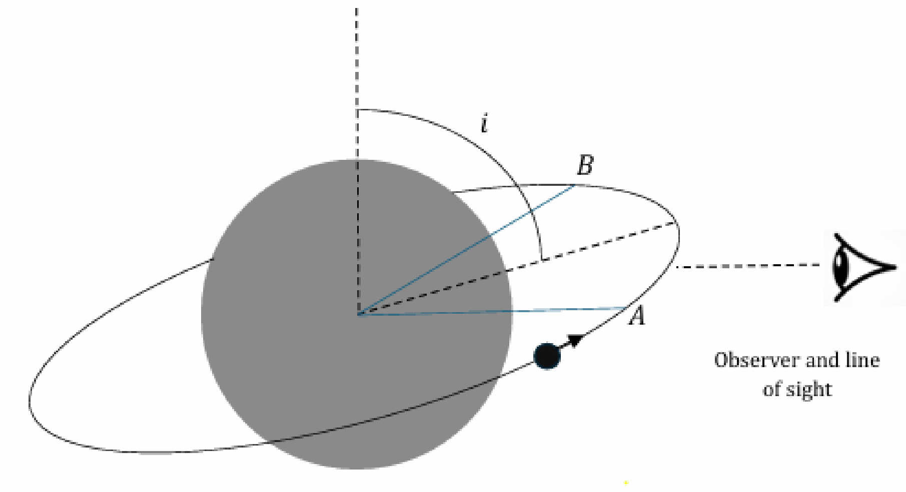

Figure 11: A 3-dimensional consideration of how a transit looks like

Note that in Figure 11, A and B are the position of the Earth at the start and the end of the transit

(g) Using the information provided in the previous parts and your own knowledge, express the duration of Earth's transit in terms of the orbital inclination of the Earth, i. **[2]**

And that's it! We have managed to figure out a method to keep track of the changing inclination of the Earth as a result of Tails repeatedly slamming the table. However, when we run this through our simulations, a new problem has shown up! No matter what, there is a point of time where the Earth's inclination has changed so much that it no longer appears to move in front of the Sun, making it impossible for us to detect any transits from there on!

Is that it? Are we just done from that point onwards - not knowing whether we have just become an accessory to the murder of 8 billion humans by accidentally allowing Tails to destory the Earth?

Find out in the next episode of *Dragon Ba* - wait! Before we start screaming in incoherent languages, let's consider another way we can potentially see this! Just... relax and hear me out...

Part III Tails tilts harder

An interesting thing to note about orbiting systems is that most objects that may appear to be stationary are, in fact, moving. This may sound weird based on our intuition, which tells us that the Sun should be the one that is causing things to move, not the thing that is moving in an orbit!

(h) Is the idea that the Sun experiences motion within the Earth-Sun system consistent with our understanding of physics? Explain why or why not. **[1]**

Regardless whether we should continue *cooking*, the reality **is** that the Sun experiences some form of motion within the Earth-Sun system. So let's just live with that and continue our journey of figuring out where exactly we are going with this.

As it turns out, the presence of the Earth itself could shift the centre of mass of the Earth-Sun system slightly from the centre of the Sun. This is similar to how carrying a bag could shift one's centre of gravity slightly, making it sometimes easier for us to fall down.The Earth and the Sun moves around this centre of mass, which in astronomy, is called the barycentre.

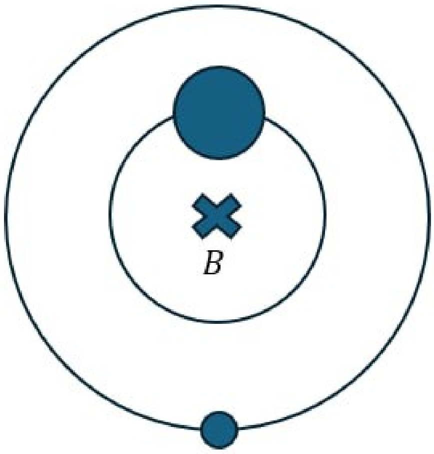

Figure 12: The orbits of a planet and star in a hypothetical stay system similar to the Solar System, with a barycentre marked as B

For simple two body problems such as the one shown in Figure 12 and the Solar System that Tails is in, the distance between one of the objects and the barycentre, r_1, can be easily calculated using the following formula:

$$r_1 = a \times \frac{m_2}{m_1 + m_2} \tag{10}$$

where

- a is the distance between the centres of the two objects in the system
- m_1 and m_2 are the masses of the two bodies

The position of the barycentre would determine the motion of the objects within the system. In the case of a simple 2-body system like the Solar System, objects tend to travel in circular motion. Such motion can be split into two different categories: wobbles (a periodic motion where the path of motion is entirely within the celestial body) and orbits (a periodic motion which follows an external path of motion).

(j) Using Equation 10 to explain your answer, deduce whether the Sun experiences motion in the form of an orbit or wobble. **[1]**

How exactly does this motion help us in determining whether the Earth still exists? The key is that as long as the Sun is travelling towards or away from an observer, the frequency of the light from the Sun would increase or decrease respectively. Such a phenomenon is also known as blue-shift/red-shift, which is a result of the Doppler effect.

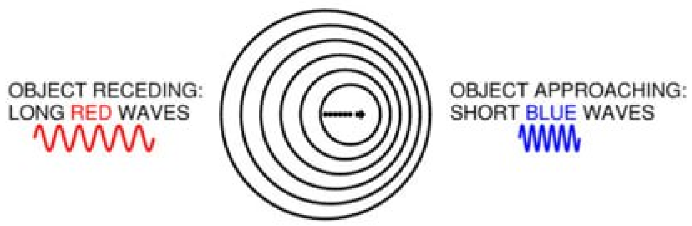

Figure 13: A brief visualisation of the Doppler effect [4]

Since the Sun is undergoing some form of circular motion with the Earth, the Sun would also appear to be travelling towards and away from the space station, resulting in the Doppler effect being observed by the observer.

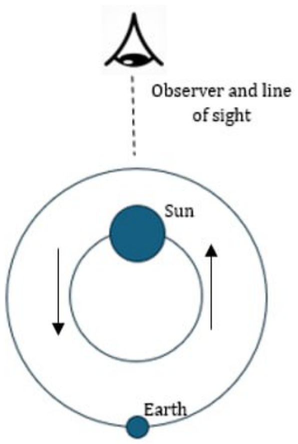

Figure 14: A 2-dimensional top-down view of the circular motion of the Sun in the Earth-Sun system

As Tails continues to change the inclination of the Earth, the inclination of the Sun's motion will change as well. To be more specific, the inclination of the Sun's motion will change by the same amount in the opposite direction of the Earth!

With the change in the inclination of the Sun's movement, the Doppler shift of the Sun's light would change. But again, as people of cul- ahem I mean science, we are more interested in figuring out exactly how this change will occur! We have a rough equation on the space station that claims it can help us, but we need to see whether it is correct!

Something good to note in the process of verifying the equation is that we will have to take barycentre to be the "primary body" of the system since the Sun and the Earth are orbiting around it. This results in slight changes in the various formula that we know and love, such as the Kepler's Third Law, which in this case would take the form of Equation 11.

$$T^2 = \frac{4\pi^2}{G(M_1 + M_2)}(R_1 + R_2)^3 \tag{11}$$

In addition to this, the Conservation of Linear Momentum, which states that the Linear Momentum within any closed system is constant in the absence of any external forces still applies. In other words, the momenta of the Earth and Sun should be equal to each other in value. The linear momentum, p, of an object is the product of the mass and velocity of the object.

$$p = mv$$

(k) Show that the maximum Doppler shift of the Sun's light as observed by us, z_{max}, is related to the inclination of the Earth's orbit, i, in the form of Equation 12.

$$z_{max} = \frac{\frac{M_{\text{Earth}}}{M_{\text{Sun}}} \frac{r_{\text{Earth}}\sqrt{G(M_{\text{Sun}}+M_{\text{Earth}})}}{a^{3/2}}}{c} \sin i \tag{12}$$

where a is the distance between the Earth and the Sun. **[3]**

Naturally, with most people being allergic to mathematics, it would be much better to describe what changes we expect to see in the data that we would be working with by demonstrating what the data would look like for a certain inclination. Doing so would require some initial data, as shown in Figure 15.

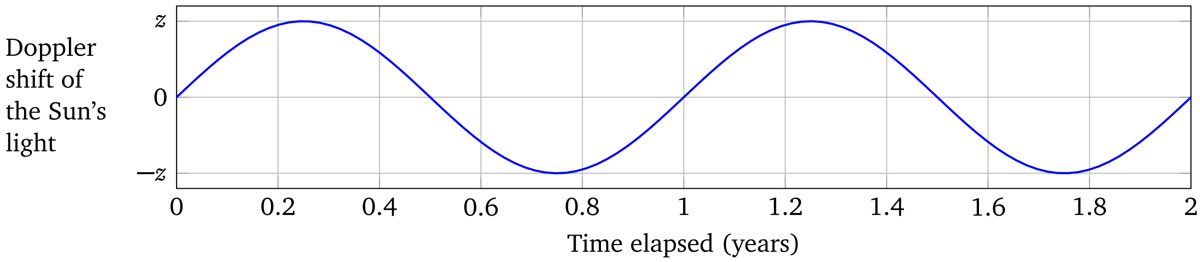

Figure 15: Doppler shift of the Sun's light before the Earth's inclination changed

(m) Taking the maximum Doppler shift of the Sun's light before the inclination of the Earth was changed, that is when $i = 90°$, to be the constant z, sketch the graph of the Doppler shift of the Sun's light against the time elapsed in years when Tails changes the Earth's inclination to 30°.

You are required to label the maximum and minimum value of the Doppler shift in terms of z, and the period of the graph, if any. **[1]**

And that's about it for the Doppler shift method! But before we get all smug and let Tails work her magic, we should consider an important question - is there an inclination where both the transit and Doppler shift method will not help us in figuring out whether the Earth exist? This is especially important to consider, so that we will not get caught off guard.

(n) State an inclination where both transit photometry and Doppler shift will not help us to detect the presence of the Earth. Explain why at such inclination, both methods are ineffective. **[1]**

(o) Suggest and explain in detail, another method that we can use to detect the Earth's presence when the Earth's orbit has an inclination of what was stated in n. You are encouraged to use diagrams to aid in your explanation. **[2]**

With that, we are about done with what we can do! Tails has just been notified that she can begin with the plan. As she begins to play her first *Valorant* game to get her tilted enough, there is suddenly one additional question that came to her mind: if Tails slammed her table so hard that it changed the Earth's orbit... and the table did not break, what exactly is her table made of?

BONUS: Suggest what Tails' table is made of. (Interesting answers may be featured)

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Question 4 *El Primo*

Pablo El Sancho is a self-proclaimed astronomer working at the European Space Agency (ESA). He is best known for his natural aptitude in astrophysics and astrobiology, particularly his studies on Astronomical Mass Observable Gravitational Units (AMOGUs), which are hypothetical objects found outside space stations. Another part of his work is ESA's new deep sky survey project, Intelligence Mission Programme for Organisation of Star and Telescopic Arithmetic Research (IMPOSTAR). He is probably Spanish... but he won't let me look at his personal file, so who knows.

Today he is giving all of us a little lecture on the night sky, which he is quite new to. Using his latest obsession Stellarium, he starts yapping about the Milky Way and its vast selection of associated deep sky objects (DSOs). Unfortunately, Pablo fails to read the room and spends two hours talking about why Mensa is a bad constellation.

After finally sensing that the crowd tonight is a bit tough and frankly uninterested in hearing an incredibly detailed explanation of why he hates a specific constellation in a "night sky talk", he relents and brings the crowd outside to go stargazing. He asks for any suggestions to start off a cracking night of observation...

Figure 16 shows a part of the night sky as it appears in front of you.

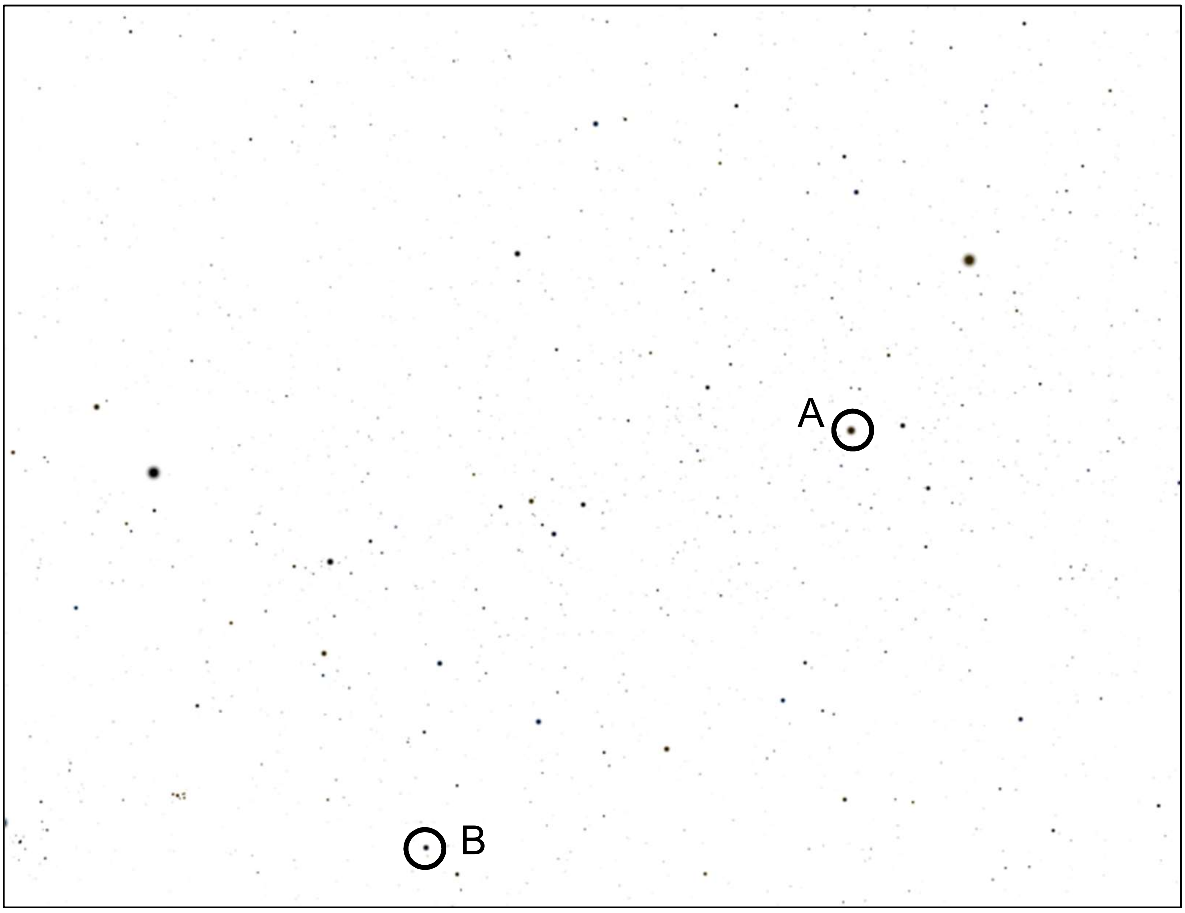

Figure 16: Part of the night sky

(a) Name stars A and B. Either the correct common name or the correct Bayer designation would suffice. **[2]**

(b) Trace out two constellations and label them. **[2]**

(c) Circle any two DSOs in this image and label them. **[2]**

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He later finds the Moon, but when he looks through his eyepiece, he notices that something is wrong and takes a picture as shown in Figure I-2 on the Insert.

(d) What is wrong with Pablo's picture? **[1]**

(e) List and explain one advantage and one disadvantage of a refractor compared to a reflector. **[2]**

You point out the problem to Pablo. He, in turn, shares the specifications of his telescope with you: a 2-element achromatic refractor with an aperture of 80mm and a focal ratio of $f/7$.

The focal ratio is a measure of the light-gathering ability of an optical system, which is defined by Equation 13.

$$N = \frac{f}{D} \tag{13}$$

where N is the focal ratio, f is the focal length and D is the diameter of the aperture.

(f) He wants to reduce the problem in (d). Should he replace his refractor with one of the same aperture but a focal ratio of $f/5$ instead? Explain briefly with the aid of a ray diagram. Label the colour of your light rays clearly. **[2]**

Pablo happens to have a 0.8× focal reducer on hand and attaches it to the original refractor.

(g) What is the effective focal length of his refractor? **[1]**

Pablo then points you towards his current field of view, as shown in Figure 17.

Figure 17: Pablo's current field of view

(h) Roughly draw, inside a 8 cm by 8 cm box, and delineate the boundary, how it might have looked before he attached the focal reducer. Draw the very prominent objects only. **[1]**

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Pablo then sets his rig up for an *Electronically Assisted Astronomy* (EAA) session. An equatorial mount follows the diurnal motion of celestial objects, which centres the object of interest for an attached camera to expose itself for an arbitrary duration of time. The exposed camera sensor is continuously bombarded by photons, forming a progressively more detailed image called an *exposure*.

Such exposures will then be stacked to create an image with even more details. However, if the duration of the exposure is too long, there may be star trails, as the object of interest falls off-centre, while if the duration of the exposure is too short, there will be a lot of undesirable noise in the final image.

(j) Explain what diurnal motion (of celestial objects) is. You may draw a diagram if necessary. **[1]**

He decides on a duration of 5 second for each exposure. He claims that tracking, which helps the equatorial mount to "follow" the diurnal motion of the celestial objects, may not always be necessary. His colleague Anna, however, asserts that tracking would be absolutely necessary in this case, to avoid star trails.

(k) Who is correct? Explain. **[2]**

Impressed by your knowledge, Pablo decides to interview you on the spot for his agency, even though he technically does not have the authority to make such a decision. He suddenly starts asking you very complicated questions...

Before we start this section, let us first preface what Pablo is saying because he is using some big words.

Sidereal time is a bit different from solar time that we use every day. Sidereal time measures the Earth's rotation relative to the stars rather than the sun. As everyone knows, a solar day is 24 hours, which is the time it takes for the Sun to return to the same (apparent) position in the sky. However, the time it takes for stars to return to the same (apparent) position in our sky is, in fact, not the same.

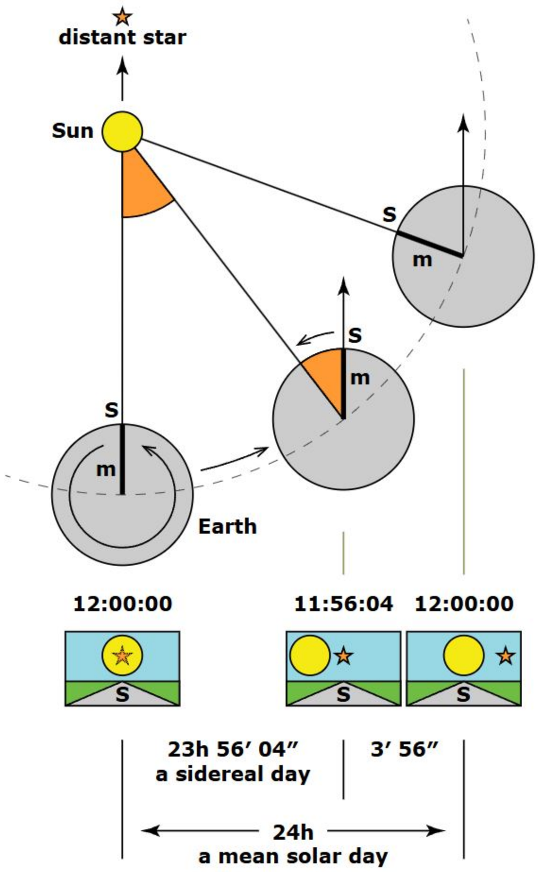

Figure 18: Comparison between sidereal time and solar time [5]

As shown in Figure 18, in the time that it takes for the Earth to spin around once (complete one sidereal rotation), it moves along its orbit a little bit. The Sun isn't actually in the same spot at the end of the rotation. Earth must rotate a bit more to bring the Sun to the same spot it originally was.

It is given that one sidereal day is 23 hours, 56 minutes and 4 seconds long.

(m) Based on Figure 18, do stars rise earlier, later or at the same time every day? Explain you answer. **[2]**

Now onto Pablo's question...

(n) Let's say today is 29th May 2025. At 9:07PM, star A is currently at the local meridian. What time will star A cross the meridian on 6th June 2025? **[2]**

Round your answer off to the nearest minutes.