



ASTROCHALLENGE FORMULA BOOKLET

revised 9th May 2025

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. THIS BOOKLET CONSISTS OF 5 PRINTED PAGES, EXCLUDING THIS COVER PAGE.
2. Do **NOT** MAKE ANY MARKINGS ON THIS BOOKLET.
3. RETURN THIS BOOKLET TO THE INVIGILATOR AT THE END OF THIS ROUND OF COMPETITION TOGETHER WITH YOUR ANSWER SCRIPT.

1 Useful Constants

Table 1: Physical and orbital characteristics of selected bodies in the Solar System

Property	Sun ☉	Mercury ☿	Venus ♀	Earth ♂	Moon ☾	Mars ♂	Jupiter ♃	Saturn ♄	Uranus ♅	Neptune ♆
Mass m / kg	1.989×10^{30}	3.302×10^{23}	4.868×10^{24}	5.972×10^{24}	7.348×10^{22}	6.419×10^{23}	1.899×10^{27}	5.685×10^{26}	8.681×10^{25}	1.024×10^{26}
Radius R / m	6.963×10^8	2.439×10^6	6.051×10^6	6.370×10^6	1.738×10^6	3.396×10^6	7.149×10^7	6.027×10^7	2.556×10^7	2.476×10^7
Orbital semi-major axis a / m	-	5.791×10^{10}	1.082×10^{11}	1.496×10^{11}	3.843×10^8	2.279×10^{11}	7.785×10^{11}	1.433×10^{12}	2.877×10^{12}	4.503×10^{12}
Orbital period T	-	87.97 days	224.70 days	365.24 days	27.322 days (sidereal) 29.531 days (synodic)	686.97 days	11.86 years	29.46 years	84.32 years	164.79 years
Orbital eccentricity ϵ	-	0.205	0.0067	0.0167	0.0549	0.0933	0.0488	0.0557	0.0444	0.0112

Table 2: Commonly used fundamental constants and unit definitions

Units and Physical Quantities	Universal Constants
1 astronomical unit (AU) = $1.49597870700 \times 10^{11} \text{ m}$	Planck's constant $h = 6.62606957 \times 10^{-34} \text{ m}^2 \text{ kg s}^{-1}$
1 light year (ly) = $c \times 1 \text{ year} \sim 9.4605284 \times 10^{15} \text{ m}$	Reduced Planck's constant $\hbar = \frac{h}{2\pi}$
1 parsec (pc) $\approx 3.26163344 \text{ ly}$	Gravitational constant $G = 6.67384 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
1 electron-volt (eV) $\approx 1.60217657 \times 10^{-19} \text{ J}$	Speed of light $c = 2.99792458 \times 10^8 \text{ m s}^{-1}$
Avogadro's number, $N_A = 6.0221413 \times 10^{23}$	Boltzmann's constant $k_B = 1.3806488 \times 10^{-23} \text{ J K}^{-1}$
Average solar luminosity, $L_\odot \approx 3.846 \times 10^{26} \text{ W}$	Coulomb constant $k_e = \frac{1}{4\pi\epsilon_0} = 8.98755179 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$
Average solar temperature, $T_\odot \approx 5778 \text{ K}$	Stefan-Boltzmann constant $\sigma \approx 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Atomic mass unit $u \approx 1.660539 \times 10^{-27} \text{ kg}$	Electronic charge $q_e \approx 1.602 \times 10^{-19} \text{ C}$
Proton mass, $m_p \approx 1.672622 \times 10^{-27} \text{ kg} = 1.007276u$	Fine structure constant $\alpha = \frac{k_e(q_e)^2}{\hbar c} \approx \frac{1}{137}$
Neutron mass, $m_n \approx 1.674927 \times 10^{-27} \text{ kg} = 1.008665u$	Wien's displacement constant $b \approx 2.89776829 \times 10^{-3} \text{ m K}$
Electron mass, $m_e \approx 9.10938 \times 10^{-31} \text{ kg}$	Hubble constant $H_0 = 67.80 \pm 0.77 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (as of March 2013)



2 Useful Formulae

Table 3: Mathematical formulae

Description	Formula
Arc length on a circle is proportional to circular angle in radians	$s = r\theta$ (Gives the circumference when $\theta = 2\pi$)
Law of sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = 2R \text{ (on a plane)}$ $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} \text{ (on a sphere)}$
Law of cosines	$c^2 = a^2 + b^2 - 2ab \cos C \text{ (on a plane)}$ $\cos c = \cos a \cos b + \sin a \sin b \cos C \text{ (on a sphere)}$
Small-angle approximations ($x \ll 1$, x in radians)	$\sin x \approx x$ $\cos x \approx 1 - \frac{x^2}{2}$ $\tan x \approx x$
First-order binomial expansion	$(1 + x)^y \approx 1 + xy$

Table 4: Classical Astrophysics

Description	Formula
Kinetic energy	$E_{\text{kin}} = \frac{1}{2}mv^2$
Newton's universal law of gravitation	$\vec{\mathbf{F}} = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$
Gravitational potential energy	$E_{\text{pot}} = -\frac{Gm_1m_2}{r}$
Gravitational binding energy of a uniform sphere	$U = -\frac{3}{5}\frac{GM^2}{R}$
Roche limit for a small, rigid body of density ρ_2 approaching a larger body of density ρ_1 and radius R	$d_{\text{Roche}} = 1.26R \times \left(\frac{\rho_1}{\rho_2}\right)^{\frac{1}{3}}$
Centripetal acceleration and force	$a_c = \omega^2 r = \frac{v^2}{r}; \quad F_c = ma_c$
Kepler's 3 rd law	$T^2 = \frac{4\pi^2}{G(m_1 + m_2)}a^3$
Hydrostatic equilibrium	$\frac{dP}{dR} = -\rho_r \frac{GM_r}{R^2}$
Quantisation of energy–momentum	$E = hf = \hbar\omega; \quad p = \frac{h}{\lambda} = \hbar k$
Planck's law for intensity per unit frequency	$I_f = \frac{2\pi hf^3}{c^2} \frac{1}{e^{\frac{hf}{kT}} - 1}$
Stefan–Boltzmann law	$L = 4\pi R^2 \sigma T^4$
Wien's displacement law	$\lambda_{\text{max}} = \frac{b}{T}$
Jeans length	$R_J = \sqrt{\frac{15k_B T}{4\pi G \langle m \rangle \cdot \langle \rho \rangle}}$

Table 5: Relativistic Expressions

Description	Formula
Lorentz factor	$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$
Velocity addition	$u' = \frac{u + v}{1 + \frac{uv}{c^2}}$
Time dilation and length contraction	$\Delta t' = \gamma \Delta t \text{ and } L' = \frac{L}{\gamma}$
Relativistic Doppler effect	$f_{\text{observed}} = f_{\text{source}} \cdot \sqrt{\frac{c - v}{c + v}}$
Relativistic redshift	$z = \sqrt{\frac{c + v}{c - v}} - 1 \approx \frac{v}{c}$
Schwarzschild radius	$r_s = \frac{2GM}{c^2}$
Redshift	$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$

Table 6: Practical Astronomy

Description	Formula
Keplerian orbital ellipse as a function of angular deviation from periapsis	$r = \frac{a(1 - \epsilon^2)}{1 + \epsilon \cos \phi}$
Orbital eccentricity in terms of other parameters	$\epsilon = \frac{a - r_{\text{periapsis}}}{a} = \frac{r_{\text{apoapsis}} - a}{a} = \frac{r_a - r_p}{r_a + r_p}$
Rayleigh resolution criterion with aperture diameter D	$\sin \Delta \phi_{\min} = 1.220 \frac{\lambda}{D}$
Beam divergence angle with initial beam width D	$\delta = \frac{4\lambda}{\pi D}$
Rocket equation	$\Delta v = v_{\text{ex}} \log_e \frac{m_i}{m_f}$

Table 7: Distance Determination and Some Empirical Results

Description	Formula
Absolute bolometric magnitude	$M_{\text{bol}} = -2.5 \log_{10} \frac{L}{L_{\odot}} + 4.7554$
Distance modulus: difference between apparent and absolute magnitude	$m - M = 5 \log_{10} \frac{d}{10 \text{ pc}}$
Relationship between luminosity and absolute magnitude	$\frac{L_1}{L_2} = 10^{(M_2 - M_1)/2.5}$
Determining distance d in parsecs using an observed parallax p in arcseconds	$d \approx \frac{1}{p}$
Period–luminosity relationship for Cepheid variable stars, with period P in days	$M = -2.76 \log_{10} P - 1.4$
Absolute magnitude of RR Lyrae stars	$M \sim 0.75$
Absolute magnitude of Type Ia supernovae (at peak)	$M \sim -19.3$
Tully–Fisher relation	$L \propto V^4$
Mass–luminosity relation for main sequence stars	$L \propto M^{3.5}$
Hubble’s law	$v = H_0 d$