AstroChallenge 2014
Data Response (Senior)

INSTRUCTIONS

- This booklet contains 5 questions and consists of 10 printed pages, excluding this cover page.
- Do **NOT** turn over this page until instructed to do so.
- You have 2 hours to finish all questions in this booklet.
- At the end of the paper, submit this booklet together with your answer script and Formula Book.
- Start every question on a fresh sheet of paper. Clearly indicate your school and team number on each sheet.
- It is your team's responsibility to ensure that all pages of your answer script have been submitted.

1 Lifespans of Globular Clusters [20.00 marks total]

1.1 Questions

Over long periods of time, open clusters gradually dissipate.

1.1.1 Briefly explain why this is the case. [2.00]

While globular clusters have much longer lifespans compared to open clusters, most members of these globular clusters eventually escape from the cluster. This process is known as evaporation. Define the evaporation time of a globular cluster as the time taken for the globular cluster to lose most of its members (aka de facto dissipation).

Due to the large number of stars in a typical cluster, the evaporation time of any cluster must be estimated through numerical simulations. In order to simplify the computations involved, these simulations only consider the internal dynamics of the cluster. By doing multiple runs, the cluster evaporation time can be estimated.

Using this methodology, a team of researchers have compiled the cluster evaporation time, t_{evap}, for the following globular clusters, along with other information. You may or may not find the following information useful.

Table 1: Evaporation Time and other parameters for various Globular Clusters

Cluster Name	$t_{\text{evap}}/10^{11}$ years	m_v[1]	Distance from galactic centre/kpc[2]	Distance to Sun/kpc
Omega Centauri	12.30	-10.26	6.4	5.2
M2	2.51	-9.03	10.4	11.5
M3	6.16	-8.88	12	10.2
M4	0.85	-7.19	5.9	2.2
M5	2.57	-8.81	6.2	7.5
M10	0.79	-7.48	4.6	4.4
M12	0.74	-7.31	4.5	4.8
M13	2.00	-8.55	8.4	7.1
M14	2.45	-9.1	4	9.3
M15	2.09	-9.19	10.4	10.4
M19	2.39	-9.13	1.7	8.8
M22	1.70	-8.5	4.9	3.2
M28	1.48	-8.16	2.7	5.5
M30	0.76	-7.45	7.1	8.1

It is hypothesized that the evaporation time for each i^{th} cluster is related to cluster mass as

$$t_{\text{evap},i} = k_1 {M_i}^{\gamma} \varepsilon_i, \tag{1}$$

where k_1 is an unknown constant, γ is a parameter of interest to be estimated, M_i is the total mass of the cluster, and ε_i is an error term specific to the i^{th} cluster. This error term is essentially an adjustment for each individual cluster due to differences in physical characteristics (i.e. factors other than mass). Since these error terms are assumed to be uncorrelated with cluster mass, **you can treat them as constants for the rest of the question**.

[1]The stated absolute visual magnitudes have been corrected for interstellar extinction.
[2]Proxy for semi-major axis of these globular clusters

1.1.2 An astronomer suspects that γ is larger than 0. Using your knowledge of astrophysics, explain intuitively why this is likely to be the case [3.00]

However, estimates for a globular cluster's mass often contain significant measurement error. As an alternative, an astronomer suggests testing the following hypothesis instead:

$$t_{\text{evap},i} = k_2 {L_i}^{\gamma} \varepsilon_i, \tag{2}$$

where k_2 is another unknown constant, and L_i is the globular cluster's luminosity.

1.1.3 What critical assumption is required for these two equations (and therefore hypotheses) to be equivalent? [2.00]

1.1.4 Assume that your critical assumption holds. Through a suitable transformation, propose how you would test the latter hypothesis. Your answer should include an equation involving quantities provided in Table 1. [3.00]

1.1.5 Using the data provided, plot your estimated equation on a suitable graph. Comment on the validity of the above hypothesis. [6.00]

Equivalence notwithstanding, models inevitably diverge from reality.

1.1.6 Which globular cluster(s) are most likely to evaporate before their predicted evaporation time? Give some examples from the sample above and briefly explain your reasoning. [4.00]

2 Orbital Mechanics [20.00 marks total]

2.1 Elliptical Orbit [11.00 marks]

Satellite A, with mass m_1, moves in a circular orbit around Earth with radius R_1 and orbital period T. A small rocket on the satellite is turned on to change its direction so that it becomes an ellipse with semimajor axis a. These changes cause the satellite to lose half its angular momentum, in such a way as to keep its orbital period and total energy constant (Figure 1).

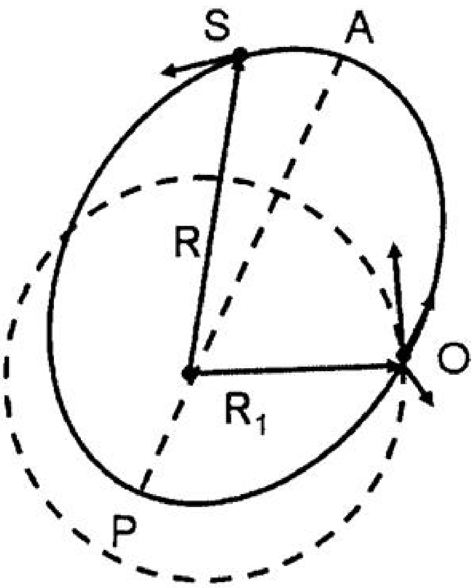

Figure 1

2.1.1 Show that the orbital velocity of the satellite at any point r in the new orbit is given by: [4.00]

$$v = \sqrt{GM_\oplus \left(\frac{2}{r} - \frac{1}{a}\right)} \tag{3}$$

2.1.2 What is the distance from the satellite to the center of the Earth, expressed in terms of R_1, at apogee and perigee? [5.00]

2.1.3 Hence, or otherwise, determine the eccentricity of the elliptical orbit.

2.2 Orbital Decay [9.00 marks]

Now, suppose that the satellite remains in this specific orbit at its end of its useful life. Due to atmospheric drag, the magnitude of its total energy increases by 1% over the course of a year.

2.2.1 Calculate the resultant change in the semi-major axis and its new perigee, supposing no change in eccentricity. [3.00]

Instead of letting satellite A's orbit decay uncontrollably, the owner decides to fire the satellite's rocket at point P. This causes the satellite to lose all of its kinetic energy and engage in free-fall.

2.2.2 How long does it take for the satellite to fall to Earth? Express your answer in terms of T. [6.00]

For simplicity, you may treat the Earth as a point.

Hint: A straight line is technically a very eccentric ellipse.

3 A Clash of Two Stars [20.00 marks total]

3.1 An Unexpected Parting [9.00 marks]

The Death Star has been chasing several capital ships from the retreating Rebel Alliance. All that travel has depleted its fuel cells however, and the Death Star has had to call off the chase and replenish its energy supplies near Antares.

As an astroseismologist, you (obviously) take the opportunity to scan Antares with your instruments during your approach. Astroseismology involves the study of a star's natural oscillation frequencies in order to probe its internal structure; as different modes of oscillation penetrate into different layers of the star, the presence of certain normal modes and their periods can shed some light about the density profile of the star and its internal structure.

After asking your research assistant to load all the data you have gathered into your personal supercomputer, you take some time to prepare a cup of tea. When you return a few minutes later, the supercomputer has completed its work and is printing out its results.

You scan through the printouts, but chance upon this line:

```
Silicon Fusion shell:  PRESENT
```

You unleash a "hrkkk" of surprise; your tea splatters in the general direction of your hapless assistant. Displeased, he looks over at the printouts. "What's so special about this 'silicon fusion shell' that requires you to spit tea all over ME?"

3.1.1 Explain to your nonplussed assistant the significance of your results. In particular, what's coming up in the very near future? [1.00]

Given the mutinous look on his face, you are advised to limit yourself to a few sentences.

"There isn't much time. Inform the Emperor that we must change course immediately."

As your assistant scurries away, you recall some details about silicon fusion.

Silicon fusion commences at a temperature of 2.7 gigakelvin in a tiny shell of approximately 0.003 solar radii. While the whole silicon fusion shell is a seething frothing mess that undergoes complex reactions, we may, for simplicity, approximate these fusion processes by the overall reaction:

$$ {}^{28}_{14}\mathrm{Si} + {}^{28}_{14}\mathrm{Si} \rightarrow {}^{56}_{28}\mathrm{Ni} + \gamma \qquad (Q = 10.914\ \mathrm{MeV}) \tag{4} $$

After further simulations with your personal supercomputer, you find that a net total of 10^{36} J of EM radiation plus heat is transferred to the shell's surroundings every second.

3.1.2 Hence, estimate the time left before core collapse. [4.00]

You may assume:

- All of this energy is supplied through the previous overall reaction with 100% efficiency.
- Heat and EM radiation are the only significant means of energy transfer: there are no other means in which thermal energy can be lost from the shell.

- $^{56}_{28}$Ni has a mass of 56 amu.
- Initially, Antares does not possess any iron/nickel core.
- All nickel produced in the silicon fusion shell immediately sinks to form an iron/nickel core.
- Core collapse occurs when the core of the star reaches 1.5 solar masses

When it is finally formed, simulations suggest that the radius of the 1.5 solar mass iron-nickel core reaches 6000 km. Shortly after reaching this critical point, the entire core collapses into a neutron star of radius 30 km within seconds.

3.1.3 Calculate the energy released by core collapse. [1.00]

It occurs to you that the power of the onboard Death Star laser exceeds 10^{33} W, enough to pulverise Alderaan into flying dust in less than a second. Yet the energy released in Antares' death throes makes the Death Star looks positively weak in comparison.

Recalling your work, you know that most of this energy is converted into neutrinos, particles that rarely interact with matter. Hence, the greatest threat from Antares lies from its EM blast.

Only 1/10 000 of this stupendous amount of energy is converted into EM radiation, of which 1/50 000 is isotropically emitted in the initial blast wave. While the Death Star is implausibly well-prepared with an army of engineers, shields and heat dissipating radiators, the Death Star will suffer catastrophic hull failure when the power of incident radiation exceeds 5 MJ m^{-2}.

3.1.4 What is the minimum radius of the safe zone? Express your answer in terms of light hours. [3.00]

In other words, how far must the nigh-invulnerable Death Star be from Antares for it to survive utter destruction?

3.2 The Great Escape [11.00 marks]

Glancing over at the monitor, you note that the Death Star is already 500 light hours away from Antares. For perspective, that's more than 3600 AU!

"Whew! Soon, we'll be safely be out of reach." Or so you thought. The silence of your office is suddenly interrupted by a cacophony of pings, each one signifying a neutrino detection event. Amidst the chaos, you immediately alert the flight deck to accelerate ASAP.

The Emperor is highly displeased and seems ready to kill you with Force lightning at any moment. However, he decides that this can wait until he's saved his own skin.

Your estimate of the time of core collapse has proven to be way too optimistic. You really should have checked if your answer felt right!

3.2.1 Name and briefly explain one process that we have neglected in our calculations, leading to this overestimation. [2.00]

The engine shudders with an almighty noise and you are tossed backward by a tremendous force as the ship accelerates. You almost blank out under the pain. Could this be but a terrible dream?

When you recover, you see on your monitor that the Death Star is now moving at its top speed of $0.9c$. Thankfully, your simulations suggest that the EM blast wave emerges 3 hours after the initial neutrino pulse. You frantically hurry to determine your fate...

3.2.2 To an observer at rest in the safe zone, how much time (in hours) does it take for the Death Star to reach the safe zone? [1.00]

3.2.3 How about for the blast wave? [1.00]

Your assistant helpfully points out that you forgot to consider the effects of special relativity, which are significant at these speeds.

3.2.4 Determine the amount of time (in hours) it takes for the ship to travel to the safe zone in the ship's point of view. [3.00]

Your assistant comments: "It seems like if you apply classical physics, we are doomed. But if we apply Special Relativity, we seem to end up outpacing the blast wave (as calculated in 3.2.2 and 3.2.3). So which should I believe? Or have we done something wrong?"

Why do these outcomes differ so wildly? Should we reject one answer, both, or neither?

3.2.5 Determine whether the Death Star lives to die another day. [4.00]

State your reasoning carefully; your answer determines the fate of many. May the Force be with you!

4 780 Days of Work [20 marks total]

An astrophotographer has been taking astronomical images of several objects over the past 780 days with each observation session separated by 20 days.

During these sessions, he has meticulously kept track of the positions of two planets relative to the Sun (⊙); these records are shown in Table 2. Unfortunately, he has lost the accompanying notes and now cannot determine which planet corresponds to the planet in his observation log.

Table 2: Right Ascension values for Planets 1 and 2 over 780-day observation period

Day	$RA_1 - RA_\odot/h$	$RA_2 - RA_\odot/h$	Day	$RA_1 - RA_\odot/h$	$RA_2 - RA_\odot/h$
0	-1.35389	1.52806	400	-2.65472	-8.11694
20	-0.95917	1.15556	420	-3.00139	-9.29250
40	-0.57028	0.82472	440	-2.94778	-10.7317
60	-0.23056	0.53278	460	-2.78056	-12.40722
80	0.00753	0.25861	480	-2.62861	-14.06644
100	0.39139	-0.02417	500	-2.50944	-15.49467
120	0.73750	-0.32167	520	-2.36944	-16.67611
140	1.15194	-0.67444	540	-2.13028	-17.64889
160	1.55694	-1.05694	560	-1.75861	-18.42644
180	1.87583	-1.43944	580	-1.31000	-19.01389
200	2.09750	-1.79556	600	-0.87722	-19.42944
220	2.27639	-2.12194	620	-0.50750	-19.70922
240	2.47750	-2.44222	640	-0.18917	-19.90856
260	2.74194	-2.79361	660	0.11917	-20.09754
280	3.05333	-3.22000	680	0.45667	-20.34467
300	3.29028	-3.76000	700	0.82500	-20.68967
320	3.21556	-4.43694	720	1.16889	-21.11689
340	2.50194	-5.24556	740	1.43722	-21.56500
360	0.76944	-6.14472	760	1.64500	-21.97611
380	-1.43000	-7.09917	780	1.85194	-22.32811

4.1 Questions

4.1.1 A fellow astronomer suggests the use of a Dobsonian. Explain why this is bad advice in astrophotography, and suggest an alternative type of telescope that would be suitable for astrophotography. [2.00]

Please be specific about your proposed setup.

4.1.2 A student claims that $\mathbf{RA} - \mathbf{RA}_{\odot}$, in hours, measures how many hours the planet can be seen after sunset. Is this true? [3.00]

4.1.3 What does the term synodic period mean? Hence or otherwise, derive a formula for the synodic period of a planet relative to Earth [5.00]

4.1.4 For each planet, plot $\mathbf{RA} - \mathbf{RA}_{\odot}$ over time. Hence or otherwise, estimate the synodic period for each planet. Also, find the observation period(s) when each planet reaches greatest elongation(s) [8.00]

4.1.5 Which planets in our solar system are likely to correspond to Planet 1 and Planet 2? Justify your answer. [2.00]

5 M3 [20.00 marks total]

Figure 2 shows the color-magnitude diagram for M3. Located 22,500 light years away in the constellation Canes Venatici, it is one of the brightest globulars in the night sky.

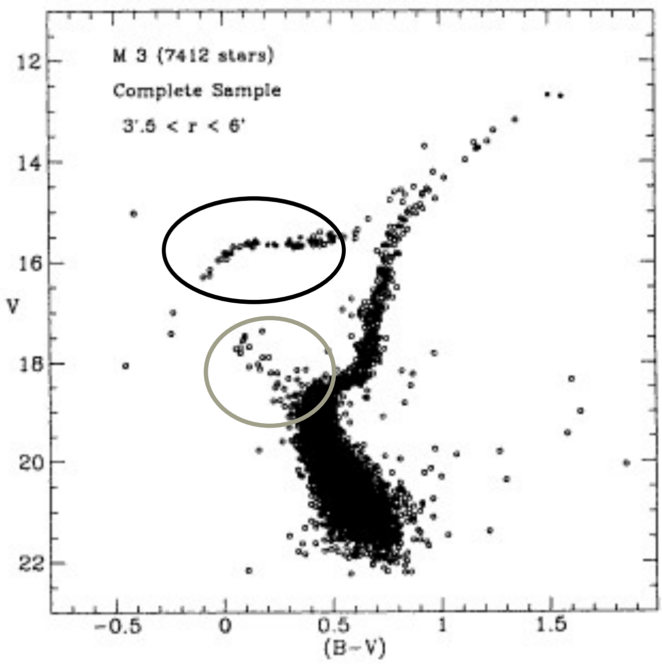

Figure 2

5.1 Questions

5.1.1 What are the stars in the lower circle categorised as? [1.00]

5.1.2 Explain how they are believed to have formed and how it leads to their unusual position on this diagram. [2.00]

5.1.3 What are the stars in the upper circle categorised as? [1.00]

5.1.4 Explain how they are believed to have formed and how it leads to their unusual position on this diagram. [2.00]

5.1.5 What is the apparent magnitude of stars at the main sequence turn-off point? [1.00]

5.1.6 How about their absolute magnitude and luminosity? [3.00]

5.1.7 If the mass-luminosity relation holds true, what is the approximate relationship between a star's mass and its lifetime on the main sequence? Briefly explain your reasoning and express your answer in the form of $T \propto M^x$ where x is a value to be derived with suitable mathematical workings. [4.00]

5.1.8 Hence, estimate the age of M3, assuming that all the stars in M3 formed at the same time. For reference, the main sequence lifetime of our sun is 10 billion years. [2.00]

Due to the density of globular clusters, many stars in M3 are actually binary star systems. Many of these binary star systems consist of a main sequence star and a white dwarf/neutron star. Presumably, these binary star systems were initially both main sequence stars (with masses m and M). The heavier star (with mass M) subsequently evolved faster, losing significant amounts of mass and leaving behind a stellar remnant.

5.1.9 How much mass can the heavier star lose before the system becomes unbound? Express your answer in terms of the total initial mass of the system. To simplify your calculations, you may assume a circular orbit. [4.00]