AstroChallenge 2014
Data Response (Junior)

INSTRUCTIONS

- This booklet contains 5 questions and consists of 12 printed pages, excluding this cover page.
- Do **NOT** turn over this page until instructed to do so.
- You have 2 hours to finish all questions in this booklet.
- At the end of the paper, submit this booklet together with your answer script and Formula Book.
- Start every question on a fresh sheet of paper. Clearly indicate your school and team number on each sheet.
- It is your team's responsibility to ensure that all pages of your answer script have been submitted.

1 Sundial [20.00 marks total]

Prior to the invention of mechanical timekeeping devices (starting with Huygens's pendulum clock in the 17th century), sundials were the only accurate way to measure time during the day, using an astronomical standard (i.e. the Sun). After their invention, however, discrepancies were discovered between the mean time (that is to say, the time kept by a properly calibrated clock) and the time shown by a sundial at various times of the year. This question will investigate these discrepancies and some of their implications.

1.1 Sundial Construction [10.00 marks]

A *sundial* is a device that makes use of the shadow of an object to keep track of time. Let us explore the behaviour of such an object under various limiting cases to build up towards its real-world behaviour.

First, let us consider the behaviour of a sundial on a planet orbiting the Sun in a circle, whose rotational axis is perpendicular to the ecliptic. Such a planet would experience no seasons.

On the equator, the most primitive sundial is a vertical stick in the ground, and as the planet rotates, the length of the shadow of the stick changes: at dawn the shadow lies entirely due West; at noon the shadow of a vertical stick vanishes; at dusk the shadow lies entirely due East. The primary shadow-casting object in a sundial is called a "gnomon"; in this sundial, it is the vertical stick.

1.1.1 What would the shadows look like throughout the day in such a situation if the gnomon were *not* perfectly vertical? [2.00]

At the equator, instead of using a vertical stick, one could (with a little more effort) suspend a horizontal one some distance above the ground in a North-South orientation, and use that as a gnomon instead. By doing this, we mark time using the *position*, instead of the *length*, of the shadow of the horizontal stick.

In more sophisticated sundials, however, it is somewhat common to suspend this horizontal bar not directly above the ground, but rather as the axis of a hemispherical frame (Figure 1 is a rather pretty example).

1.1.2 Why is this done? [1.00]

Naturally, away from the equator, the equipment must be modified somewhat: at higher latitudes, the gnomon is no longer exactly horizontal, but points in a particular direction.

1.1.3 Seen from above, along what direction does the gnomon point? [0.50]

1.1.4 In the Northern hemisphere, what angle does this gnomon make with the ground? What about at the North Pole? [2.00]

(Hint: At what angle must the gnomon's shadow fall relative to the gnomon at noon?)

Figure 1: A sundial consisting of a bar suspended across a hemispherical frame

1.1.5 Putting the last two questions together, in what direction does the gnomon point in the Northern hemisphere? What about in the Southern hemisphere? [1.50]

1.1.6 Compare the gnomon from a sundial placed at an arbitrary latitude and one at the equator, along the same meridian. What property do these two bars have? [1.00]

On a planet with axial tilt, however, we would expect somewhat different observations. As the declination of the sun changes throughout the year, the noontime shadow of the vertical stick at the equator will now not vanish, but rather point north-south, and change in length throughout the year also. Hemispherical sundials were invented partly to have their shadows point north-south, and so circumvent this difficulty.

1.1.7 At the equator, a vertical stick casts a shadow at noon, whose length and direction depends on the time of year. What is the ratio of the length of the shadow and the length of the stick? [2.00]

1.2 Sundial Correction [10.00 marks]

All of the above discussion was predicated on the assumption that the length of the day remains constant throughout the year. For the ancients, this was a rather reasonable assumption to make; after all, daylight and starlight were essentially their only available horological standards.

With a sufficiently accurate clock, however, we could instead subdivide a tropical year into days of equal length, and tell time by measuring how much of it has elapsed since the start of the year. This is called *mean solar time*, and a length of a day under this definition is called the *mean solar day*.

It was soon discovered that readings on sundials, which are based on the actual position of the sun, disagreed with the mean solar time. The discrepancy between the *true solar time* and the mean solar time was, however, a cyclical phenomenon: it varied in a rather predictable fashion throughout the year (see Figure 2). Empirically, it was a simple matter to develop some tables for this discrepancy in order to reconcile the true and mean solar times. Theoretically, however, it was a rather more tricky affair.

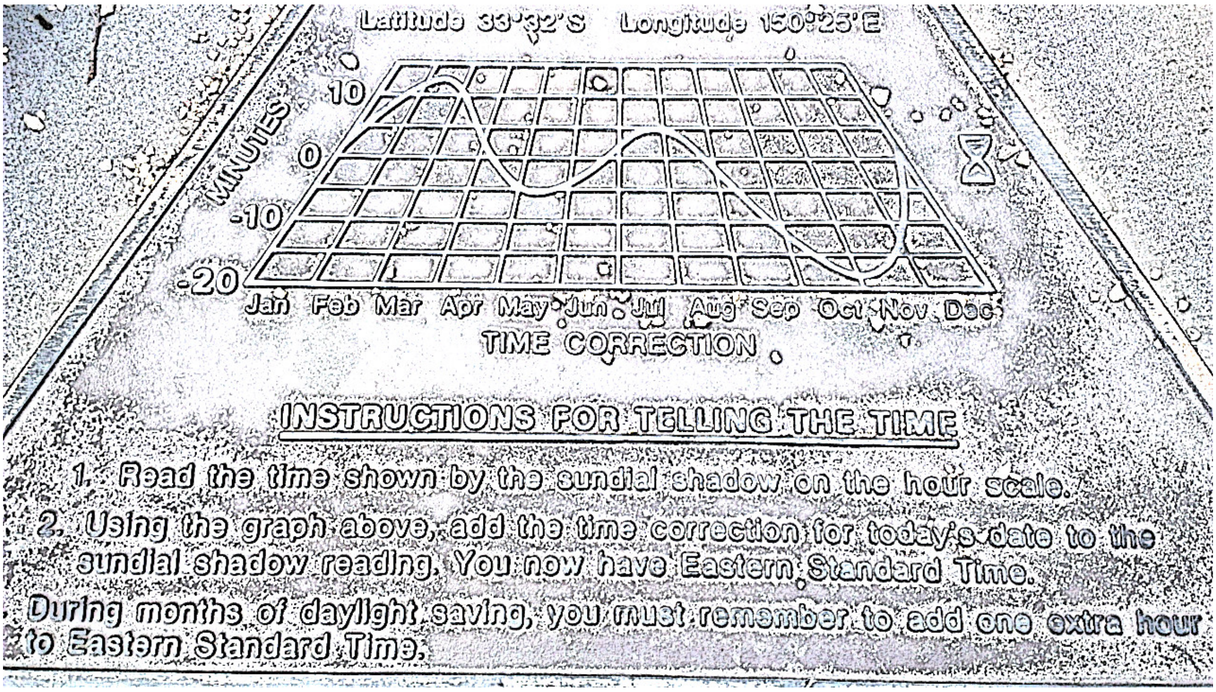


Figure 2: An inscription on a sundial showing discrepancy between mean and true solar time

1.2.1 Why is there a discrepancy between the mean and true solar time? [2.00]

(Hint: How do we break a tropical year down into solar days? Does the Earth orbit the Sun in a perfect circle? How else does the Sun's right ascension change over the course of a year?)

The time correction in Figure 2 is known as the "equation of time" (here "equation" is used in the archaic sense of a corrected discrepancy, and not in the modern sense of the term). As with any periodic function, we may rescale it to be a function of θ on the interval $0 \leqslant \theta < 2\pi$ by measuring θ in radians. Letting $\theta = 0$ correspond to the start of the calendar year (i.e. January 1), Figure 3 shows the equation of time using this parameterisation.

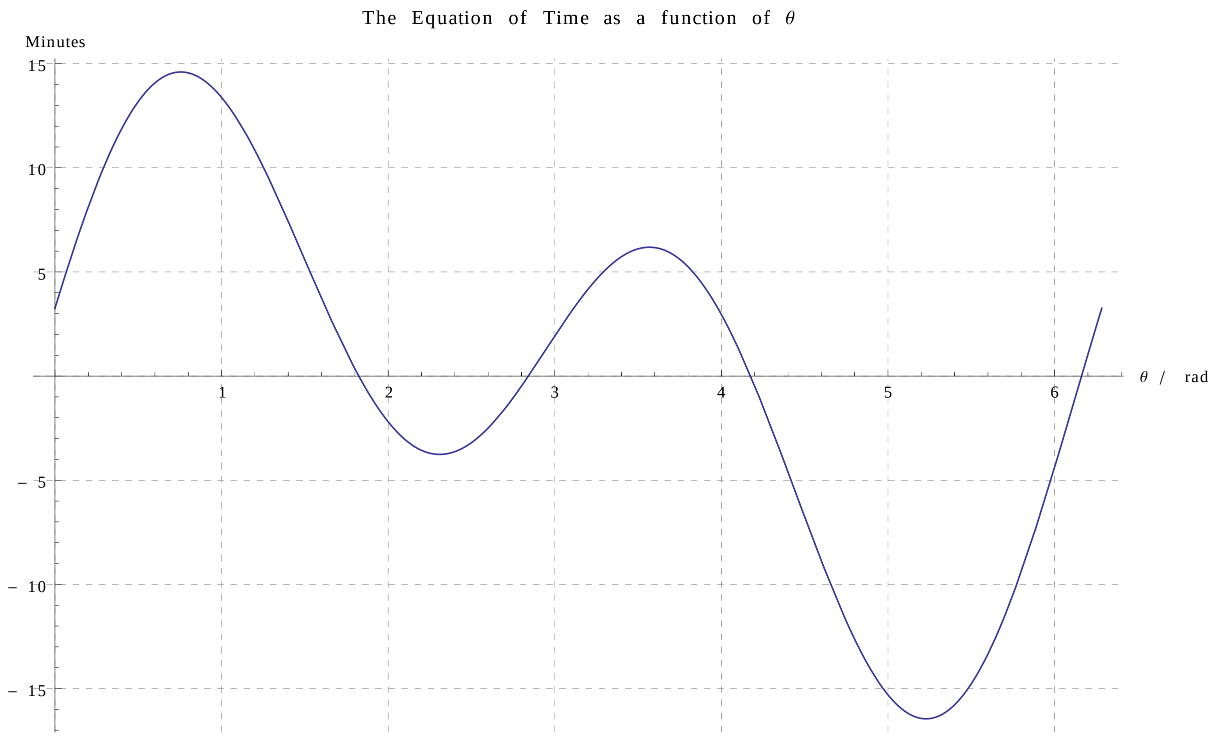
The equation of time measures the discrepancy between the actual right ascension of the Sun, and the right ascension required for the mean solar time to be accurate. Equivalently, the equation of time gives us the hour angle of the sun as it would be observed at noon, where noon is measured by mean solar time.

However, the obliquity of the Earth's rotational axis means that the declination δ of the sun also changes throughout the year. Using some spherical geometry, we can show that using the same parametrisation as above,

$$\sin\delta = \sin\varepsilon \sin(\theta - \phi), \tag{1}$$

where $\varepsilon = 23.5°$ is the obliquity of the Earth's rotational axis, and again θ goes from 0 to 2π, with $\theta = 0$ representing the start of the calendar year. In this equation, $\phi = \frac{162\pi}{365}$ is an offset term to reflect that the declination of the Sun is zero and increasing not at the start of the

Figure 3



calendar year, but at the Vernal Equinox. All quantities in Equation 1 should be measured in radians.

1.2.2 Using no less than 20 data points, plot a graph of δ against θ, with all quantities in radians. [4.00]

Now, Figure 3 gives the Sun's hour angle at mean solar noon as a function of θ, the time of year; in Question 1.2.2 you found the Sun's declination, also as a function of θ, the time of year. Together, these specify the location of the Sun at mean solar noon, at different times of year. When we observe the sun at mean solar noon every day over the course of the year, we see that its trajectory traces out a smooth curve, called an *analemma*.

1.2.3 Using no less than 20 data points, plot the analemma, with the declination of the sun as the y-axis and the equation of time as the x-axis. [3.50]

2 Stars and Cosmology [18.00 marks total]

2.1 Short Answer Questions [18.00 marks]

2.1.1 A star has an apparent magnitude of 0.43, a stellar parallax of 0.21", and a temperature of 6,500K. By what factor is the star fainter than the Sun in apparent brightness? [2.00]

2.1.2 Calculate the luminosity of the star in solar luminosities. [4.00]

2.1.3 Calculate the radius of the star in solar radii. [4.00]

2.1.4 At what wavelength would the star emit most radiation? [1.00]

2.1.5 What form of radiation has this wavelength? [1.00]

2.1.6 What is Wien's Law? [2.00]

2.1.7 What are the distinct properties of a black body? [2.00]

2.1.8 What is the currently accepted resolution of Olber's paradox? [2.00]

3 Astrobiology [25.00 marks total]

The search for extra-terrestrial life is mankind's answer to the existential question: "are we alone in our Universe? Is life on Earth unique, or does life exist elsewhere?" Many efforts have been made to categorize extrasolar planets and other objects, and the results so obtained have been interesting. In this question, you are to deduce if an alien system 6.2 parsecs away from Earth is feasible for sustaining extraterrestrial life, based on the following data set.

3.1 Data Tables

These tables contain statistics for celestial bodies orbiting an extrasolar system, and details about the star in this extrasolar system. Some of the information in the tables is missing. As you answer the questions below, **fill in the tables below whenever applicable**, leaving numerical values to 3 significant figures.

Table 1: Stellar Characteristics for Star X

Parameter	Value
Apparent magnitude	10.56
Absolute magnitude	
Distance/pc	6.2
Mass/$M_\odot$	0.310
Radius/$R_\odot$	0.294
Luminosity/$L\odot$	0.0126
Surface temperature/K	

Table 2: Planetary Characteristics in Star X System

Observed object	Body A	Body B	Body C
Semi-major axis/AU		0.0730	
Radius of orbit at periapsis/AU	0.0393		0.0685
Radius of orbit at apoapsis/AU	0.0411		
Eccentricity		0.0315	1.022
Orbital period	5.37 days	12.9 days	
Mass/$M_\oplus$	15.6	7.7	1.12×10^{-9}
Density/g cm^{-3}	0.988	3.74	2.67
Tidally-locked?	Yes	Yes	No
Atmosphere?	Yes	Yes	?
Primary composition of Atmosphere	Hydrogen, Helium, Methane, Ammonia	Hydrogen, Methane, Water	?
Average Albedo	0.43	0.61	0.05
Mean surface temperature/°C	112	7	-268

The Apparent Magnitude of the Sun is -26.7; 1 AU = 4.848×10^{-6} pc

3.2 Questions [25.00 marks]

3.2.1 **Calculate the Eccentricity and Semi-major axis of Body A if applicable; otherwise, explain why if you cannot obtain a reasonable answer. [2.00]**

3.2.2 **Calculate the Radius of orbit at apoapsis and periapsis of Body B if applicable; otherwise, comment on the significance of the eccentricity value. [2.00]**

3.2.3 **Calculate the Semi-major axis and Radius of orbit at apoapsis of Body C if applicable; otherwise, comment on the significance of the eccentricity value. [2.00]**

3.2.4 **Make a rough sketch of the system, showing the relative size and shapes of the orbits of the three celestial bodies around Star X. Assume all the orbits follow Body A's ecliptic. The diagram can be exaggerated and need not be drawn to exact scale, but label the celestial bodies in the diagram. [3.00]**

3.2.5 **Comment on the significance of the values you have obtained so far and those in the table, while making a well-informed guess of what types of celestial objects Body A, B and C are. [3.00]**

3.2.6 **Calculate the surface temperature and absolute magnitude of Star X, and comment on the significance of the values [3.00]**

3.2.7 **Suppose Star X is a main sequence star. What type of star would you expect it to be? Explain your answer briefly. [2.00]**

3.2.8 **Deduce which celestial body, if any, is most likely to sustain life. Your answer is expected to be as rigorous and convincing as possible using the given data. [5.00]**

3.2.9 **State the condition present for a planet that is present in the circumstellar habitable zone of a star. Must a celestial body be within this location to be capable of sustaining life? Explain your answer. You are to give a hypothetical or real example to support your answer, if any. [3.00]**

4 Polaris and the Cosmic Distance Ladder [17.00 marks total]

4.1 Astronomical Units [8.00 marks]

While the Astronomical Unit was first calculated (to reasonable accuracy) with transits of Venus, the advent of radar astronomy has allowed us to give a more accurate estimate of the Astronomical Unit. By bouncing radar pulses off Venus, we can directly compute the distance between Earth and Venus and hence the Astronomical Unit.

In April 1961, Venus was in inferior conjunction relative to Earth. During this time, a team of astronomers bounced radar pulses off Venus at a frequency of 440 MHz. Their results on April 8th are shown below.

Table 3

Time of transmission (UT)	Measured round-trip travel time/s	Standard deviation of measurement error/s	Measured Doppler shift/Hz
20 h 27 min 35 s	283.67188	0.00007	2247.76

4.1.1 Assuming that Venus was in inferior conjunction at this point in time, calculate the implied value of the Astronomical Unit. [4.00]

For simplicity, Venus and Earth may be treated as having circular orbits and negligible mass. Use the data given in the Formula Book.

4.1.2 b) Comparing this value against your Formula Book, you realise that there is a slight discrepancy between the two values. Using data where available, suggest two possible sources of error and briefly explain how they can account for this discrepancy. [4.00]

4.2 Cepheid Variables [9.00 marks]

Unknown to many, Polaris is actually the closest Cepheid variable: the Hipparcos satellite survey found that Polaris displayed an annual parallax of 7.54 milliarcseconds. Proper characterisations of stars like Polaris are vital for calibrating the period-luminosity relationship for Cepheids.

4.2.1 Given this value of parallax, find the distance to Polaris and express it in units of light years. [1.00]

Polaris itself is a multiple star system. Polaris Aa (the Cepheid) forms a tight binary with Polaris Ab, a main sequence star. Polaris B orbits around the center of mass of this binary star system, and is joined by Polaris C and D. Some details about Polaris Aa and Ab are shown in the table below. You may treat the apparent bolometric magnitude of both stars as equal to their visual magnitude.

Table 4

Property	Polaris Aa	Polaris Ab
Apparent magnitude m_v	1.98, variable	9.2
Spectral type	F7Ib	F6V
Semi-major axis, a/arcsec	0.133	
Period	29.59 years	

4.2.2 Find their semi-major axis in AU. Hence, determine the total mass of the system. [3.00]

4.2.3 By using the mass-luminosity relation where appropriate, estimate the individual masses of Polaris Aa and Polaris Ab [5.00]

5 Astronomy on Mars [20.00 marks total]

What would happen if humans colonized Mars? In this question, you will explore how astronauts in the northern hemisphere of Mars would experience the night sky.

5.1 Viewing the Earth [7.00 marks]

Light, and other electromagnetic radiation, travels in wave packets. One consequent property of optical imaging is that even with ideal equipment, some defocusing must still be observed; this is called *diffraction-limited seeing.* It is known that the human eye has a pupil diameter of approximately 6 mm during twilight.

5.1.1 What would be the angular resolution of the naked eye under such conditions, assuming that the peak emitted wavelength for most celestial objects lies around 480 nm? [2.00]

Express your answer in arcminutes.

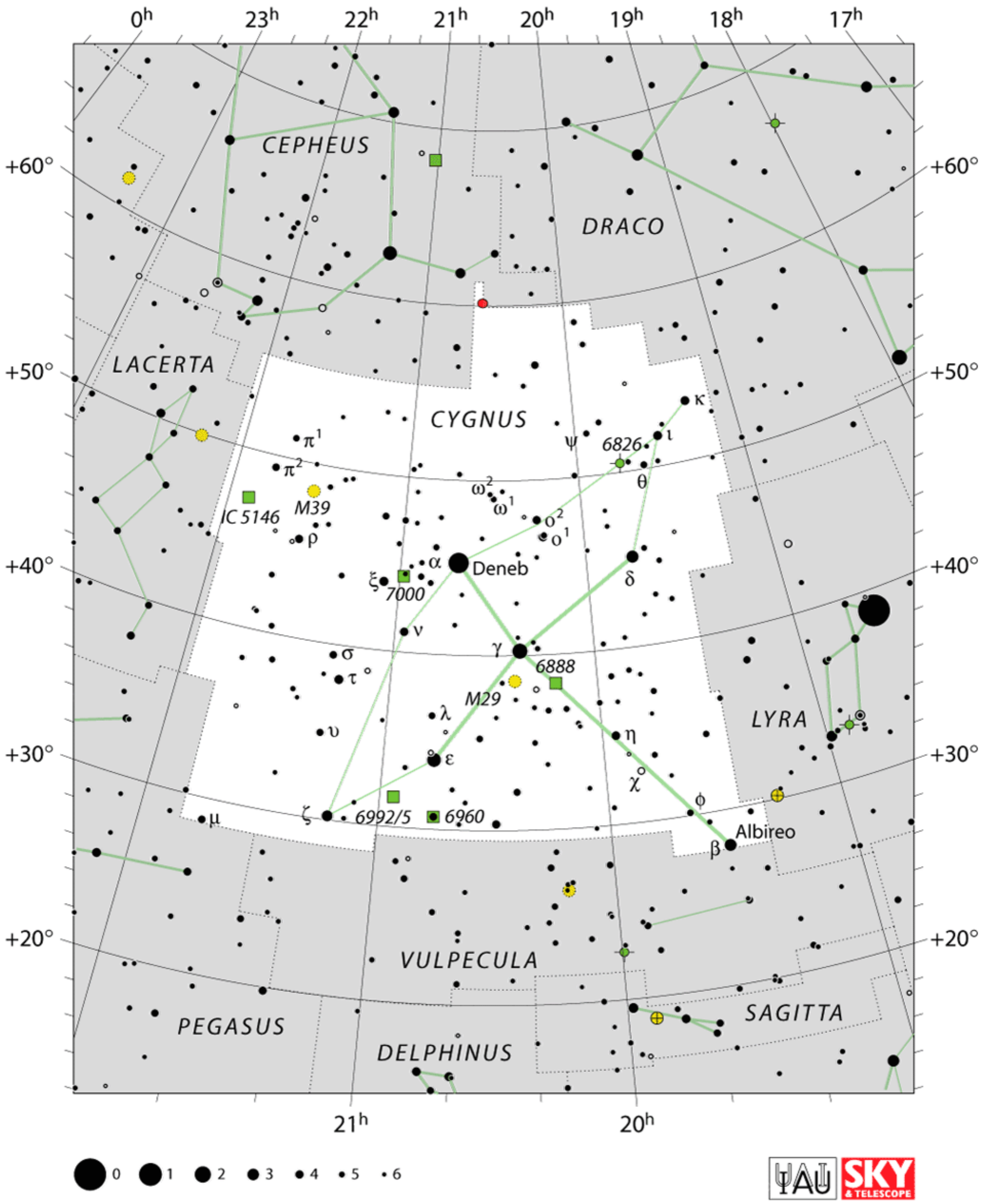
5.1.2 When Earth is at its greatest elongation from the Sun, what is the maximum possible angular separation of the Moon and Earth from Mars? Hence, is it possible to resolve the Earth and Moon into separate objects with the naked eye? [5.00]

Assume all objects are on the same plane, have circular orbits, and Earth is being viewed under twilight conditions.

5.2 Viewing the Stars [6.00 marks]

The Martian North Celestial Pole is located in Cygnus and has coordinates R.A.$21^{h}10^{m}42^{s}$/ Dec.$+52°53.0'$.

5.2.1 Mark the location of these coordinates on the star map provided. [1.00]



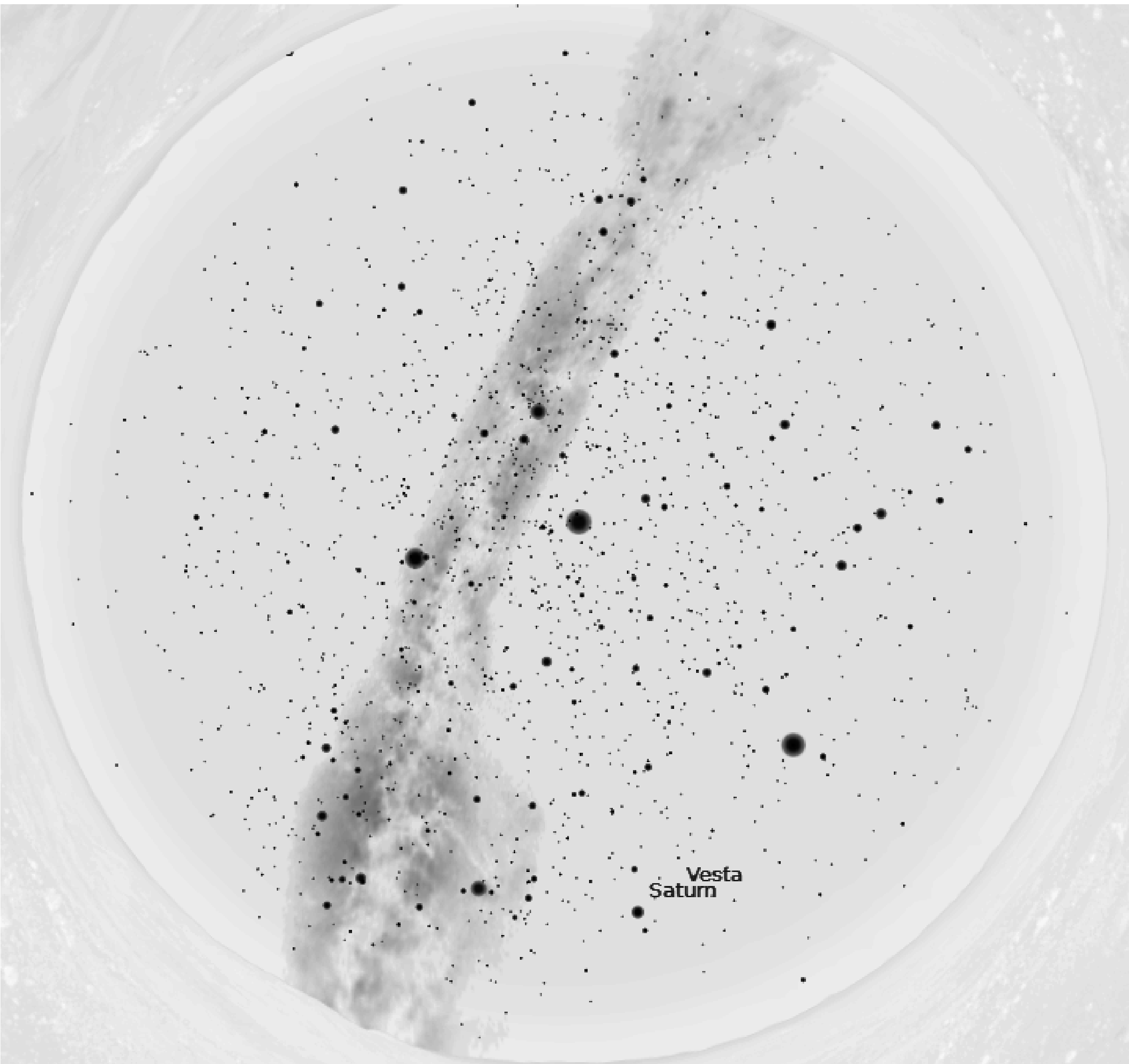
5.2.2 What is the best way to navigate to the Martian North Celestial Pole? [1.00]

5.2.3 How could one do polar alignment accurately during Martian night? [4.00]

Assume that you don't have any equipment to tell you where Martian north is.

5.3 The Observation Session [7.00 marks]

One night, you decide to hold an observation session outdoors. As you step outside, the following scene greets you. (Planetary bodies have been labelled to avoid confusion)



You have a 6 inch telescope in your hands (focal length 1500 mm), a 32 mm eyepiece and a 10 mm eyepiece. No light pollution is evident.

5.3.1 Given the sky right now, name 7 visually observable deep sky objects that are within reach of your telescope. [7.00]

In your answer, be sure to name the constellation as well as what you should expect to see (galaxy, open cluster, nebula etc.). Be specific when naming your targets! Oh, and you are sick of Cygnus, so please don't give any deep sky objects within said constellation.

PS: If you can't identify which constellation the object belongs to, mark its **exact** location on the star map **clearly**.