AstroChallenge Formula Book

INSTRUCTIONS

- This booklet consists of 5 printed pages, excluding this cover page.
- Do **NOT** make any markings on this booklet.
- Return this booklet to the invigilator at the end of this round of competition together with your answer script.

1 Useful Constants

Table 1: Physical and orbital characteristics of selected bodies in the Solar System

Property	Sun ☉	Mercury ☿	Venus ♀	Earth ⊕	Moon ☾	Mars ♂	Jupiter ♃	Saturn ♄	Uranus ⛢	Neptune ♆
Mass m/ kg	1.989×10^{30}	3.302×10^{23}	4.868×10^{24}	5.972×10^{24}	7.348×10^{22}	6.419×10^{23}	1.899×10^{27}	5.685×10^{26}	8.681×10^{25}	1.024×10^{26}
Radius R / m	6.963×10^{8}	2.439×10^{6}	6.051×10^{6}	6.370×10^{6}	1.738×10^{6}	3.396×10^{6}	7.149×10^{7}	6.027×10^{7}	2.556×10^{7}	2.476×10^{7}
Orbital Semi-major axis a / m	-	5.791×10^{10}	1.082×10^{11}	1.496×10^{11}	3.843×10^{8}	2.279×10^{11}	7.785×10^{11}	1.433×10^{12}	2.877×10^{12}	4.503×10^{12}
Orbital period T	-	87.97 days	224.70 days	365.24 days	27.322 days (sidereal) 29.531 days (synodic)	686.97 days	11.86 years	29.46 years	84.32 years	164.79 years
Orbital Eccentricity ϵ	-	0.205	0.0067	0.0167	0.0549	0.0933	0.0488	0.0557	0.0444	0.0112

Table 2: Commonly used fundamental constants and unit definitions

Units and Physical Quantities	Universal Constants
1 Astronomical Unit (AU) $= 1.49597870700 \times 10^{11}$ m	Planck's Constant $h = 6.62606957 \times 10^{-34}$ m^2 kg s^{-1}
1 light year (ly) $= c \times 1$ year $= 9.4605284 \times 10^{15}$ m	Reduced Planck's Constant $\hbar = \frac{h}{2\pi}$
1 parsec (pc) $= 3.26163344$ ly	Gravitational Constant $G = 6.67384 \times 10^{-11}$ N m^2 kg^{-2}
1 electron-volt (eV) $= 1.60217657 \times 10^{-19}$ J	Speed of Light $c = 2.99792458 \times 10^{8}$ m s^{-1}
Avogadro's Number $N_A = 6.0221413 \times 10^{23}$	Boltzmann's Constant $k_B = 1.3806488 \times 10^{-23}$ J K^{-1}
Average Solar Luminosity $= 3.846 \times 10^{26}$ W	Coulomb Constant $k_e = \frac{1}{4\pi\epsilon_0} = 8.98755179 \times 10^{9}$ N m^2 C^{-2}
Average Solar Temperature $= 5778$ K	Stefan-Boltzmann Constant $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
Atomic mass unit $u = 1.660539 \times 10^{-27}$ kg	Electronic charge $q_e = 1.602 \times 10^{-19}$ C
Proton Mass $= 1.672622 \times 10^{-27}$ kg $= 1.007276u$	Fine structure constant $\alpha = \frac{k_e(q_e)^2}{\hbar c} \approx \frac{1}{137}$
Neutron Mass $= 1.674927 \times 10^{-27}$ kg $= 1.008665u$	Wien's Displacement Constant $b = 2.89776829 \times 10^{-3}$ m K
Electron Mass $= 9.10938 \times 10^{-31}$ kg	Hubble Constant $H_0 = 67.80 \pm 0.77$ km s^{-1} Mpc^{-1} (as of 03/13)

2 Useful Formulae

Table 3: Mathematical formulae

Description	Formula
Arc length on a circle is proportional to circular angle in radians	$s = r\theta$ (Gives the circumference when $\theta = 2\pi$)
Law of sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = 2R$ (on a plane) $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$ (on a sphere)
Law of cosines	$c^2 = a^2 + b^2 - 2ab\cos C$ (on a plane) $\cos c = \cos a \cos b + \sin a \sin b \cos C$ (on a sphere)
Small-angle approximations ($x \ll 1$, x in radians)	$\sin x \approx x$ $\cos x \approx 1 - \frac{x^2}{2}$ $\tan x \approx x$
First-order binomial expansion	$(1+x)^y \approx 1 + xy$

Table 4: Classical Astrophysics

Description	Formula
Kinetic Energy	$E_{\text{kin}} = \frac{1}{2}mv^2$
Newton's Universal Law of Gravitation	$\vec{\mathbf{F}} = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$
Gravitational Potential Energy	$E_{\text{pot}} = -\frac{Gm_1m_2}{r}$
Gravitational binding energy of a uniform sphere	$U = -\frac{3}{5}\frac{GM^2}{R}$
Roche Limit for a small, rigid body of density ρ_2 approaching a larger body of density ρ_1 and radius R	$d_{\text{Roche}} = 1.26R \times \left(\frac{\rho_1}{\rho_2}\right)^{-\frac{1}{3}}$
Angular Velocity ω and angular momentum l	$v = r\omega;\ \omega = 2\pi f = \frac{2\pi}{T};\ l = I\omega = mr^2\omega$ (for orbiting bodies)
Centripetal acceleration and force	$a_c = \omega^2 r = \frac{v^2}{r};\ F_c = ma_c$
Kepler's 3rd Law	$T^2 = \frac{4\pi^2}{G\left(m_1 + m_2\right)}a^3$
Hydrostatic Equilibrium	$\frac{\mathrm{d}P}{\mathrm{d}R} = -\rho_r\frac{GM_r}{R^2}$
Quantisation of energy-momentum	$E = hf = \hbar\omega;\ p = \frac{h}{\lambda} = \hbar k$
Planck's Law for intensity per unit frequency	$I_f = \frac{2\pi hf^3}{c^2}\frac{1}{e^{\frac{hf}{kT}} - 1}$
Stefan-Boltzmann Law	$L = 4\pi R^2\sigma T^4$
Wien's Displacement Law	$\lambda_{max} = \frac{b}{T}$
Jeans Length	$R_J = \sqrt{\frac{15k_BT}{4\pi G\langle m\rangle \cdot \langle\rho\rangle}}$

Table 5: Relativistic Expressions

Description	Formula
Lorentz Factor	$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$
Velocity Addition	$u' = \frac{u + v}{1 + \frac{uv}{c^2}}$
Time Dilation and Length Contraction	$\Delta t' = \gamma \Delta t$ and $L' = \frac{L}{\gamma}$
Relativistic Doppler Effect	$f_{\text{observed}} = f_{\text{source}} \cdot \sqrt{\frac{c - v}{c + v}}$
Relativistic Redshift	$z = \sqrt{\frac{c + v}{c - v}} - 1 \approx \frac{v}{c}$
Schwarzschild Radius	$r_s = \frac{2GM}{c^2}$
Redshift	$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$

Table 6: Practical Astronomy

Description	Formula
Keplerian orbital ellipse as a function of angular deviation from periapsis	$r = \frac{a(1 - \epsilon^2)}{1 + \epsilon \cos \phi}$
Orbital Eccentricity in terms of other parameters	$\epsilon = \frac{a - r_{\text{periapsis}}}{a} = \frac{r_{\text{apoapsis}} - a}{a} = \frac{r_{\text{a}} - r_{\text{p}}}{r_{\text{a}} + r_{\text{p}}}$
Rayleigh resolution criterion with aperture diameter D	$\sin \Delta\phi_{min} = 1.220 \frac{\lambda}{D}$
Beam divergence angle with initial beam width D	$\delta = \frac{4\lambda}{\pi D}$
Rocket Equation	$\Delta v = v_{\text{exh}} \log_e \frac{m_i}{m_f}$

Table 7: Distance Determination and Some Empirical Results

Description	Formula
Absolute Bolometric Magnitude	$M_{\text{bol}} = -2.5\log_{10}\frac{L}{L_\odot} + 4.7554$
Distance modulus: difference between apparent and absolute magnitude	$m - M = 5\log_{10}\frac{d}{10\text{ pc}}$
Relationship between Luminosity and Absolute Magnitude	$\frac{L_1}{L_2} = 10^{\frac{M_2-M_1}{2.5}}$
Determining distance d in parsecs using an observed parallax p in arc seconds	$d \approx \frac{1}{p}$
Period-Luminosity relationship for Cepheid variable stars, with period P in days	$M = -2.76\log_{10}P - 1.4$
Absolute magnitude of RR Lyrae stars	$M \sim 0.75$
Absolute magnitude of Type Ia supernovae (at peak)	$M \sim -19.3$
Tully-Fisher Relation	$L \propto V^4$
Mass-Luminosity Relation for Main Sequence stars	$L \propto M^{3.5}$
Hubble's Law	$v = H_0 d$