



## THE ASTROCHALLENGE FORMULA BOOKLET

2023 REVISED EDITION

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- DO **NOT** MAKE ANY MARKINGS ON THIS BOOKLET.
- RETURN THIS BOOKLET TO THE INVIGILATOR AT THE END OF THIS ROUND OF COMPETITION TOGETHER WITH YOUR ANSWER SCRIPT.

# 1 Useful Constants

Table 1: Physical and orbital characteristics of selected bodies in the Solar System

Property	Sun ☉	Mercury ☿	Venus ♀	Earth ⊕	Moon ☾	Mars ♂	Jupiter ♃	Saturn ♄	Uranus ♅	Neptune ♆
Mass $m / \text{kg}$	$1.989 \times 10^{30}$	$3.302 \times 10^{23}$	$4.868 \times 10^{24}$	$5.972 \times 10^{24}$	$7.348 \times 10^{22}$	$6.419 \times 10^{23}$	$1.899 \times 10^{27}$	$5.685 \times 10^{26}$	$8.681 \times 10^{25}$	$1.024 \times 10^{26}$
Radius $R / \text{m}$	$6.963 \times 10^8$	$2.439 \times 10^6$	$6.051 \times 10^6$	$6.370 \times 10^6$	$1.738 \times 10^6$	$3.396 \times 10^6$	$7.149 \times 10^7$	$6.027 \times 10^7$	$2.556 \times 10^7$	$2.476 \times 10^7$
Orbital Semi-major axis $a / \text{m}$	-	$5.791 \times 10^{10}$	$1.082 \times 10^{11}$	$1.496 \times 10^{11}$	$3.843 \times 10^8$	$2.279 \times 10^{11}$	$7.785 \times 10^{11}$	$1.433 \times 10^{12}$	$2.877 \times 10^{12}$	$4.503 \times 10^{12}$
Orbital period $T$	-	87.97 days	224.70 days	365.24 days	27.322 days (sidereal) 29.531 days (synodic)	686.97 days	11.86 years	29.46 years	84.32 years	164.79 years
Orbital Eccentricity $\epsilon$	-	0.205	0.0067	0.0167	0.0549	0.0933	0.0488	0.0557	0.0444	0.0112

Table 2: Commonly used fundamental constants and unit definitions

Units and Physical Quantities	Universal Constants
1 Astronomical Unit (AU) = $1.49597870700 \times 10^{11}$ m	Planck's Constant $h = 6.62606957 \times 10^{-34}$ m <sup>2</sup> kg s <sup>-1</sup>
1 light year (ly) = $c \times 1$ year = $9.4605284 \times 10^{15}$ m	Reduced Planck's Constant $\hbar = \frac{h}{2\pi}$
1 parsec (pc) = $3.26163344$ ly	Gravitational Constant $G = 6.67384 \times 10^{-11}$ N m <sup>2</sup> kg <sup>-2</sup>
1 electron-volt (eV) = $1.60217657 \times 10^{-19}$ J	Speed of Light $c = 2.99792458 \times 10^8$ m s <sup>-1</sup>
Avogadro's Number $N_A = 6.0221413 \times 10^{23}$	Boltzmann's Constant $k_B = 1.3806488 \times 10^{-23}$ J K <sup>-1</sup>
Average Solar Luminosity = $3.846 \times 10^{26}$ W	Coulomb Constant $k_e = \frac{1}{4\pi\epsilon_0} = 8.98755179 \times 10^9$ N m <sup>2</sup> C <sup>-2</sup>
Average Solar Temperature = 5778 K	Stefan-Boltzmann Constant $\sigma = 5.67 \times 10^{-8}$ W m <sup>-2</sup> K <sup>-4</sup>
Atomic mass unit $u = 1.660539 \times 10^{-27}$ kg	Electronic charge $q_e = 1.602 \times 10^{-19}$ C
Proton Mass = $1.672622 \times 10^{-27}$ kg = $1.007276u$	Fine structure constant $\alpha = \frac{k_e(q_e)^2}{\hbar c} \approx \frac{1}{137}$
Neutron Mass = $1.674927 \times 10^{-27}$ kg = $1.008665u$	Wien's Displacement Constant $b = 2.89776829 \times 10^{-3}$ m K
Electron Mass = $9.10938 \times 10^{-31}$ kg	Hubble Constant $H_0 = 67.80 \pm 0.77$ km s <sup>-1</sup> Mpc <sup>-1</sup> (as of 03/13)



## 2 Useful Formulae

Table 3: Mathematical formulae

Description	Formula
Arc length on a circle is proportional to circular angle in radians	$s = r\theta$ (Gives the circumference when $\theta = 2\pi$ )
Law of sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = 2R \text{ (on a plane)}$ $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} \text{ (on a sphere)}$
0.9 Law of cosines	$c^2 = a^2 + b^2 - 2ab \cos C \text{ (on a plane)}$ $\cos c = \cos a \cos b + \sin a \sin b \cos C \text{ (on a sphere)}$
Small-angle approximations ( $x \ll 1$ , $x$ in radians)	$\sin x \approx x$ $\cos x \approx 1 - \frac{x^2}{2}$ $\tan x \approx x$
First-order binomial expansion	$(1 + x)^y \approx 1 + xy$

Table 4: Classical Astrophysics

Description	Formula
Kinetic Energy	$E_{\text{kin}} = \frac{1}{2}mv^2$
Newton's Universal Law of Gravitation	$\vec{\mathbf{F}} = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$
Gravitational Potential Energy	$E_{\text{pot}} = -\frac{Gm_1m_2}{r}$
Gravitational binding energy of a uniform sphere	$U = -\frac{3}{5}\frac{GM^2}{R}$
Roche Limit for a small, rigid body of density $\rho_2$ approaching a larger body of density $\rho_1$ and radius $R$	$d_{\text{Roche}} = 1.26R \times \left(\frac{\rho_1}{\rho_2}\right)^{-\frac{1}{3}}$
Angular Velocity $\omega$ and angular momentum $l$	$v = r\omega; \omega = 2\pi f = \frac{2\pi}{T}; l = I\omega = mr^2\omega$ (for orbiting bodies)
Centripetal acceleration and force	$a_c = \omega^2 r = \frac{v^2}{r}; F_c = ma_c$
Kepler's 3 <sup>rd</sup> Law	$T^2 = \frac{4\pi^2}{G(m_1 + m_2)}a^3$
Hydrostatic Equilibrium	$\frac{dP}{dR} = -\rho_r \frac{GM_r}{R^2}$
Quantisation of energy-momentum	$E = hf = \hbar\omega; p = \frac{h}{\lambda} = \hbar k$
Planck's Law for intensity per unit frequency	$I_f = \frac{2\pi hf^3}{c^2} \frac{1}{e^{\frac{hf}{kT}} - 1}$
Stefan-Boltzmann Law	$L = 4\pi R^2 \sigma T^4$
Wien's Displacement Law	$\lambda_{\text{max}} = \frac{b}{T}$
Jeans Length	$R_J = \sqrt{\frac{15k_B T}{4\pi G \langle m \rangle \cdot \langle \rho \rangle}}$

Table 5: Relativistic Expressions

Description	Formula
Lorentz Factor	$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$
Velocity Addition	$u' = \frac{u + v}{1 + \frac{uv}{c^2}}$
Time Dilation and Length Contraction	$\Delta t' = \gamma \Delta t \text{ and } L' = \frac{L}{\gamma}$
Relativistic Doppler Effect	$f_{\text{observed}} = f_{\text{source}} \cdot \sqrt{\frac{c - v}{c + v}}$
Relativistic Redshift	$z = \sqrt{\frac{c + v}{c - v}} - 1 \approx \frac{v}{c}$
Schwarzschild Radius	$r_s = \frac{2GM}{c^2}$
Redshift	$z = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}}$

Table 6: Practical Astronomy

Description	Formula
Keplerian orbital ellipse as a function of angular deviation from periapsis	$r = \frac{a(1 - \epsilon^2)}{1 + \epsilon \cos \phi}$
Orbital Eccentricity in terms of other parameters	$\epsilon = \frac{a - r_{\text{periapsis}}}{a} = \frac{r_{\text{apoapsis}} - a}{a} = \frac{r_a - r_p}{r_a + r_p}$
Rayleigh resolution criterion with aperture diameter $D$	$\sin \Delta \phi_{\text{min}} = 1.220 \frac{\lambda}{D}$
Beam divergence angle with initial beam width $D$	$\delta = \frac{4\lambda}{\pi D}$
Rocket Equation	$\Delta v = v_{\text{exh}} \log_e \frac{m_i}{m_f}$

Table 7: Distance Determination and Some Empirical Results

Description	Formula
Absolute Bolometric Magnitude	$M_{\text{bol}} = -2.5 \log_{10} \frac{L}{L_{\odot}} + 4.7554$
Distance modulus: difference between apparent and absolute magnitude	$m - M = 5 \log_{10} \frac{d}{10 \text{ pc}}$
Relationship between Luminosity and Absolute Magnitude	$\frac{L_1}{L_2} = 10^{\frac{M_2 - M_1}{2.5}}$
Determining distance $d$ in parsecs using an observed parallax $p$ in arc seconds	$d \approx \frac{1}{p}$
Period-Luminosity relationship for Cepheid variable stars, with period $P$ in days	$M = -2.76 \log_{10} P - 1.4$
Absolute magnitude of RR Lyrae stars	$M \sim 0.75$
Absolute magnitude of Type Ia supernovae (at peak)	$M \sim -19.3$
Tully-Fisher Relation	$L \propto V^4$
Mass-Luminosity Relation for Main Sequence stars	$L \propto M^{3.5}$
Hubble's Law	$v = H_0 d$