Editor's Note: This is the 2014 paper

AstroChallenge 2013
Data Response (Junior)

INSTRUCTIONS

- This booklet contains m questions and consists of 7 printed pages, excluding this cover page.
- Do **NOT** turn over this page until instructed to do so.
- You have 2 hours to finish all questions in this booklet.

1 Sundial [20.00 marks total]

Prior to the invention of mechanical timekeeping devices (starting with Huygens's pendulum clock in the 17th century), sundials were the only accurate way to measure time during the day, using an astronomical standard (i.e. the Sun). After their invention, however, discrepancies were discovered between the mean time (that is to say, the time kept by a properly calibrated clock) and the time shown by a sundial at various times of the year. This question will investigate these discrepancies and some of their implications.

1.1 Sundial Construction [10.00 marks]

A *sundial* is a device that makes use of the shadow of an object to keep track of time. Let us explore the behaviour of such an object under various limiting cases to build up towards its real-world behaviour.

First, let us consider the behaviour of a sundial on a planet orbiting the Sun in a circle, whose rotational axis is perpendicular to the ecliptic. Such a planet would experience no seasons.

On the equator, the most primitive sundial is a vertical stick in the ground, and as the planet rotates, the length of the shadow of the stick changes: at dawn the shadow lies entirely due West; at noon the shadow of a vertical stick vanishes; at dusk the shadow lies entirely due East. The primary shadow-casting object in a sundial is called a "gnomon"; in this sundial, it is the vertical stick.

1.1.1 What would the shadows look like throughout the day in such a situation if the gnomon were *not* perfectly vertical? [2.00]

Solution At noon, any shadow cast by the stick will be directly under it, so the shadow points in the same direction as the stick. Before noon, this shadow is sheared West-ward, and after noon East-ward. (Bonus points if they can mention shearing). For a stick pointing north-south, the length of projection of the shadow onto the East-West axis would be the same as if the stick were perfectly vertical, as a consequence of the shear operation.

At the equator, instead of using a vertical stick, one could (with a little more effort) suspend a horizontal one some distance above the ground in a North-South orientation, and use that as a gnomon instead. By doing this, we mark time using the *position*, instead of the *length*, of the shadow of the horizontal stick.

In more sophisticated sundials, however, it is somewhat common to suspend this horizontal bar not directly above the ground, but rather as the axis of a hemispherical frame (Figure 1 is a rather pretty example).

1.1.2 Why is this done? [1.00]

Solution This is so that the shadow of the horizontal bar always falls perpendicular to the sundial no matter what the time is.

Figure 1: A sundial consisting of a bar suspended across a hemispherical frame

Naturally, away from the equator, the equipment must be modified somewhat: at higher latitudes, the horizontal bar is no longer exactly horizontal, but points in a particular direction.

1.1.3 Seen from above, along what direction does the gnomon point? [0.50]

Solution Along the ground, its direction is unchanged by the latitude, and it remains pointing North-South.

1.1.4 In the Northern hemisphere, what angle does this gnomon make with the ground? What about at the North Pole? [2.00]

(Hint: At what angle must the bar's shadow fall relative to the bar at noon?)

Solution The bar's shadow falls perpendicular to the bar at noon. Therefore, the angle between the bar and the ground is exactly equal to he latitude. Consequently, the bar is vertical at the poles.

1.1.5 Putting the last two questions together, in what direction does the gnomon point in the Northern hemisphere? What about in the Southern hemisphere? [1.50]

Solution The bar is oriented such that one end points toward the North Celestial Pole and the other towards the South Celestial Pole, irrespective of latitude or choice of hemisphere.

1.1.6 Compare the gnomon from a sundial placed at an arbitrary latitude and one at the equator, along the same meridian. What property do these two bars have? [1.00]

Solution They are parallel to each other.

On a planet with axial tilt, however, we would expect somewhat different observations. As the declination of the sun changes throughout the year, the noontime shadow of the vertical stick at the equator will now not vanish, but rather point north-south, and change in length throughout the year also. Hemispherical sundials were invented partly to have their shadows point north-south, and so circumvent this difficulty.

1.1.7 At the equator, a vertical stick casts a shadow at noon, whose length and direction depends on the time of year. What is the ratio of the length of the shadow and the length of the stick? [2.00]

Solution The tangent of the declination of the Sun.

1.2 Sundial Correction [10.00 marks]

All of the above discussion was predicated on the assumption that the length of the day remains constant throughout the year. For the ancients, this was a rather reasonable assumption to make; after all, daylight and starlight were essentially their only available horological standards.

With a sufficiently accurate clock, however, we could instead subdivide a tropical year into days of equal length, and tell time by measuring how much of it has elapsed since the start of the year. This is called *mean solar time*, and a length of a day under this definition is called the *mean solar day*.

It was soon discovered that readings on sundials, which are based on the actual position of the sun, disagreed with the mean solar time. The discrepancy between the *true solar time* and the mean solar time was, however, a cyclical phenomenon: it varied in a rather predictable fashion throughout the year (see Figure 2). Empirically, it was a simple matter to develop some tables for this discrepancy in order to reconcile the true and mean solar times. Theoretically, however, it was a rather more tricky affair.

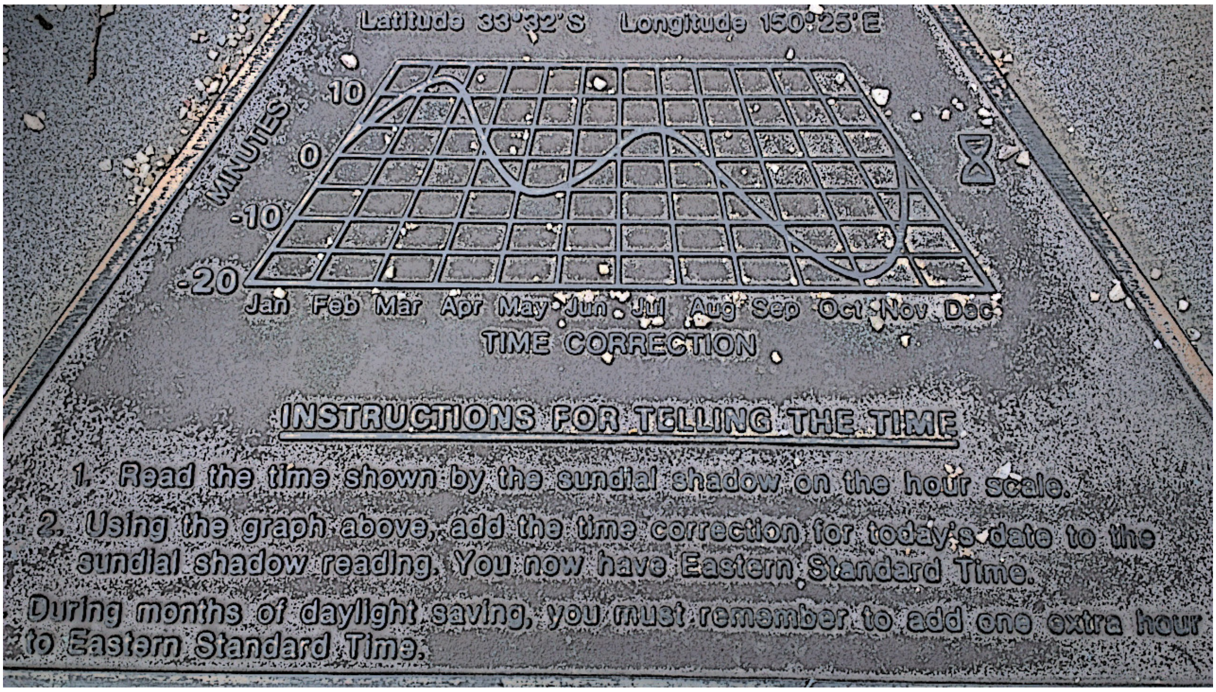


Figure 2: An inscription on a sundial showing discrepancy between mean and true solar time

1.2.1 Why is there a discrepancy between the mean and true solar time? [2.00]

(Hint: How do we break a tropical year down into solar days? Does the Earth orbit the Sun in a perfect circle? How else does the Sun's right ascension change over the course of a year?)

Solution The length of the mean solar day is simply a fixed fraction of the tropical year, which is determined by the rate of Earth's rotation, from which the Earth's orbital angular frequency (which depends on the length of the year) is subtracted. The length of the true solar day is determined also by the rate of Earth's rotation, but not directly by the length of the year; rather it is determined by the rate of change of the true anomaly (i.e. the Earth's angular velocity with respect to the Sun). By Kepler's Second Law, this is higher at perihelion and lower at aphelion. Therefore the days are longer near perihelion and shorter near aphelion.

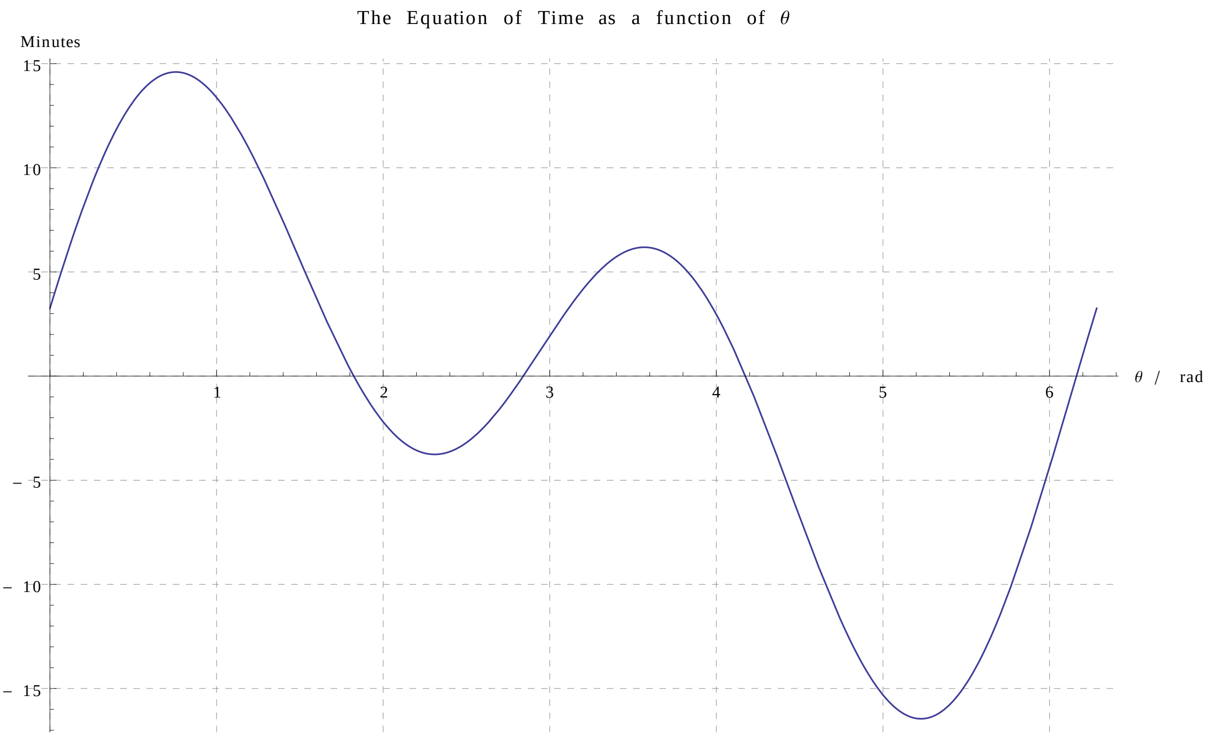
Also, were the Earth's orbit to be circular, the Sun would move along the ecliptic at a constant rate. At the solstices, its rate of change of declination is zero, whereas at the equinoxes its rate of change of declination is at maximum. Therefore at the solstices its rate of change of right ascension is larger than at the equinoxes.

These two effects combined result in the equation of time, as shown in the figures.

The time correction in Figure 2 is known as the "equation of time" (here "equation" is used in the archaic sense of a corrected discrepancy, and not in the modern sense of the term). As with any periodic function, we may rescale it to be a function of θ on the interval $0 \leqslant \theta < 2\pi$ by measuring θ in radians. Letting $\theta = 0$ correspond to the start of the calendar year (i.e. January 1), Figure 3 shows the equation of time using this parameterisation.

The equation of time measures the discrepancy between the actual right ascension of the Sun, and the right ascension required for the mean solar time to be accurate. Equivalently, the

Figure 3



equation of time gives us the hour angle of the sun as it would be observed at noon, where noon is measured by mean solar time.

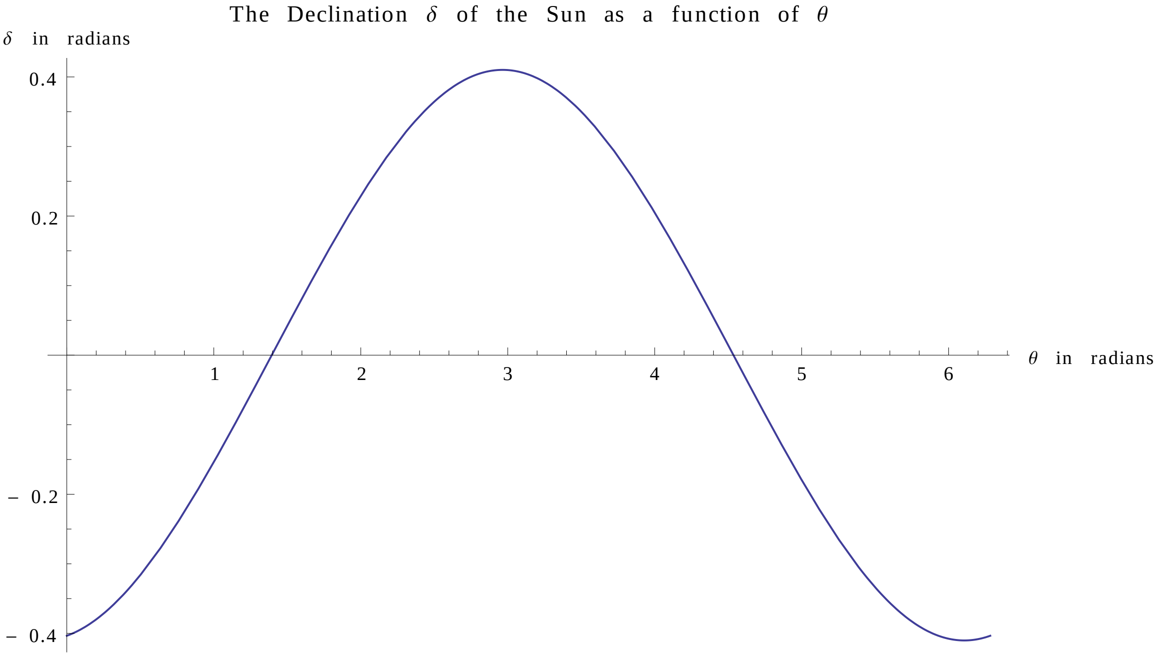
However, the obliquity of the Earth's rotational axis means that the declination δ of the sun also changes throughout the year. Using some spherical geometry, we can show that using the same parametrisation as above,

$$\sin\delta = \sin\varepsilon\sin(\theta - \phi), \tag{1}$$

where $\varepsilon = 23.5^\circ$ is the obliquity of the Earth's rotational axis, and again θ goes from 0 to 2π, with $\theta = 0$ representing the start of the calendar year. In this equation, $\phi = \frac{162\pi}{365}$ is an offset term to reflect that the declination of the Sun is zero and increasing not at the start of the calendar year, but at the Vernal Equinox. All quantities in Equation 1 should be measured in radians.

1.2.2 Using no less than 12 data points, plot a graph of δ against θ, with all quantities in radians. [4.00]

Solution The graph should look generally like this:

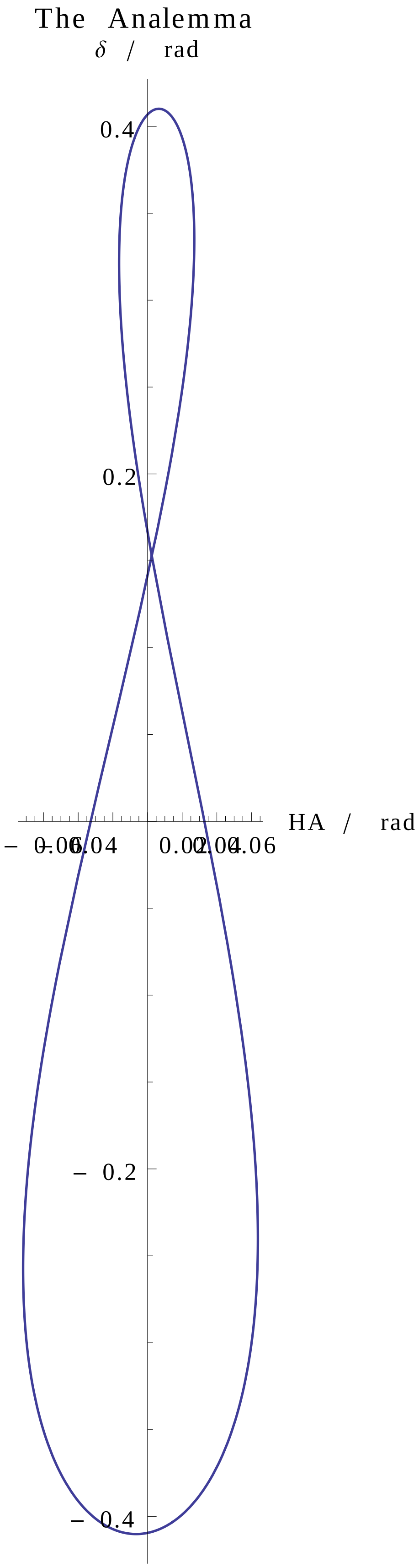


The minimum should correspond to the winter solstice, maximum to the summer solstice, zeroes to the equinoxes, and the amplitude should be as indicated. They should interpolate from many data points (no less than 12), so no marks for derivation.

Now, Figure 3 gives the Sun's hour angle at mean solar noon as a function of θ, the time of year; in Question 1.2.2 you found the Sun's declination, also as a function of θ, the time of year. Together, these specify the location of the Sun at mean solar noon, at different times of year. When we observe the sun at mean solar noon every day over the course of the year, we see that its trajectory traces out a smooth curve, called an *analemma*.

1.2.3 Using no less than 12 data points, plot the analemma, with the declination of the sun as the y-axis and the equation of time as the x-axis. [3.50]

Solution Again, we should have no less than 12 data points, yielding the following plot:



Stars and Cosmology: Short Answer Questions (18 marks)

a) A star has an apparent magnitude of 0.43, a stellar parallax of 0.21", and a temperature of 6,500K. By what factor is the star fainter than the Sun in apparent brightness? **[2 marks]**

Let the sun be annotated by 1, and the star, 2.

$x = m_2 - m_1$

$x = 0.43 - (-26.74) = 27.17$

$2.512^x = b_1/b_2$

$2.512^{27.17} = 7.38811 \times 10^{10}$

The star is $\underline{7.38811 \times 10^{10}}$ times fainter than the sun in apparent brightness.

OR the star is $\underline{1.3535 \times 10^{-11}}$ the brightness of the sun in apparent brightness.

b) Calculate the luminosity of the star in solar luminosities**. [4 marks]**

Distance to the star in AU = 206265 × 1 / 0.21 = 982,214.2857AU

$$\frac{L_{star}}{L_{sun}} = \frac{b_{star}}{b_{sun}}\left(\frac{r_{star}}{r_{sun}}\right)^2$$

$L_{star} = 1.3535 \times 10^{-11} \times (982{,}214.2857)^2 = \underline{13.06 \text{ solar luminosity}}$

c) Calculate the radius of the star in solar radii. **[4 marks]**

$$\frac{L_{star}}{L_{sun}} = \frac{R^2{}_{star}}{R^2{}_{sun}} \times \frac{T^4{}_{star}}{T^4{}_{sun}}$$

$$\frac{R_{star}}{R_{sun}} = \sqrt{\frac{L_{star}}{L_{sun}} \times \frac{T^4{}_{sun}}{T^4{}_{star}}} = \sqrt{13.06 \times \left(\frac{6000K}{6500K}\right)^4} = \underline{3.0793 \text{ solar radii}}$$

d) At what wavelength would the star emit most radiation? **[1 mark]**
What form of radiation has this wavelength? **[1 mark]**

Using Wien's Law, $\lambda_{max}T = b$

λ_{max} (6500) = 2.8977685×10^{-3}mK

$\lambda_{max} = 4.45811 \times 10^{-7}$m

Ultraviolet

e) What is Wien's Law? **[2 marks]**

Wien's displacement law states that the wavelength distribution of thermal radiation from a black body at any temperature has essentially the same shape as the distribution at any other temperature, except that each wavelength is displaced on the graph.

f) What are the distinct properties of a black body? **[2 marks]**

1. It is an ideal emitter: it emits as much or more energy at every frequency than any other body at the same temperature.
2. It is a diffuse emitter: the energy is radiated isotropically, independent of direction.

g) What is the currently accepted resolution of Olber's paradox? **[2 marks]**

In a universe that exists for a finite amount of time, only the light of finitely many stars has had a chance to reach us yet. Additionally, in an expanding universe distant objects recede from us, which causes the light emanating from them to be redshifted and diminished in brightness.

Astrobiology (25 marks)

The search for extra-terrestrial life served to answer mankind's burning question: "Are we alone in our Universe? Is life on Earth unique, or does life exist elsewhere?" Many efforts have thus been made to categorize extrasolar planets and other objects, and yielded many interesting results. As a young astronomer, you are given a set of data about these objects in an alien system 6.2 parsecs away from Earth within the Milky way, in order to deduce if it is feasible of sustaining extraterrestrial life. (Based on Gliese 581 system with hypothetical values)

The following tables show statistics for celestial bodies orbiting an extrasolar system and the details about the star in this extrasolar system. Some of the information in the tables is missing. As you answer the questions, fill in the tables below whenever applicable, leaving them in 3 significant figures. (Marks in individual sections; this table is for reference)

Observed object	Body A	Body B	Body C
Semi-major axis (AU)	0.0402	0.0730	-2.17 – hyperbolic!
Radius of orbit at periapsis (AU)	0.0393	0.0707	0.0685
Radius of orbit at apoapsis (AU)	0.0411	0.0753	Trick question! It exits the system!
Eccentricity	0.0224	0.0315	1.022
Orbital period	5.37 days	12.9 days	Unknown
Mass (M⊕)	15.6	7.7	1.12×10^{-9}
Density (g/cm^3)	0.988	3.74	2.67
Tidal-locked?	Yes	Yes	No
Atmosphere?	Yes	Yes	Unknown
Primary composition of Atmosphere	Hydrogen, Helium, Methane, Ammonia	Hydrogen, Methane, Water	-
Average Albedo	0.43	0.61	0.05
Mean surface temperature (°C)	112	7	-268

Parameters	Star X
Apparent magnitude	10.56
Absolute magnitude	12.95
Distance (parsecs)	6.2
Mass ($M_\odot$)	0.310
Radius ($R_\odot$)	0.294
Luminosity ($L_\odot$)	0.0126
Surface temperature (K)	3570

Apparent magnitude of Sun: -26.7 1 Astronomical Unit = 4.848×10^{-6} Parsecs

Calculate the Eccentricity and Semi-major axis of Body A if applicable; otherwise, explain why if you cannot obtain a reasonable answer.

e = $\frac{ra-rp}{ra+rp} = \frac{0.0411-0.0393}{0.0411+0.0393}$ = 0.0224

e = $\frac{a-rp}{a} = \frac{a-0.0393}{a} = 0.0224$; a = 0.0402

Calculate the Radius of orbit at apoapsis and periapsis of Body B if applicable; otherwise, comment on the significance of the eccentricity value.

e = $\frac{a-rp}{a} = \frac{0.0730-rp}{0.0730} = 0.0315$; r_p = 0.0707 (3.s.f.)

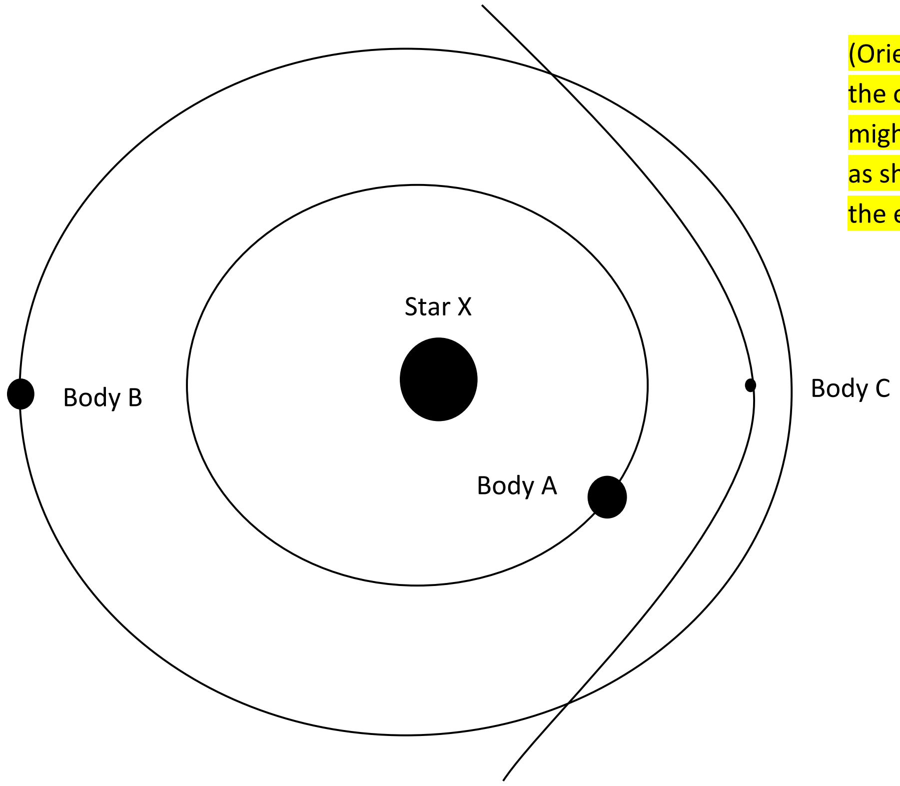
e = $\frac{ra-a}{a} = \frac{ra-0.0730}{0.0730} = 0.0315$; r_a = 0.0753 (3.s.f.)

Calculate the Semi-major axis and Radius of orbit at apoapsis of Body C if applicable; otherwise, comment on the significance of the eccentricity value.

e = $\frac{a-rp}{a} = \frac{a-0.0685}{a}$ = 1.022 ; a = -2.17, i.e. Hyperbolic orbit

Body C has an eccentricity of > 1. It will exit the system

Make a rough sketch of the system, showing the relative size and shapes of the orbits of the three celestial bodies around Star X. Assume all the orbits follow Body A's ecliptic. The diagram can be exaggerated and need not be drawn to exact scale, but label the celestial bodies in the diagram.



(Orientation of the orbits might not be as shown in the example)

Comment on the significance of the values you have obtained so far and those in the table, while making a well-informed guess of what types of celestial objects Body A, B and C are.

Body A: Gas giant planet

Average density of 0.988 g/cm^3 indicates it is less dense than water overall; unlikely to be a terrestrial planet and more likely to be a Gas giant similar to Saturn.

15.6 Earth Mass further indicates that it is a relatively large planet and less likely to be terrestrial in nature.

Low eccentricity of 0.0224 and semi-major axis of 0.0402 indicates it has a relatively circular orbit and is the closest body to Star X.

Body B: (Large) Terrestrial planet/ 'Super-Earth'

Density of 3.74 g/cm^3 indicates it is unlikely to be a gas giant, and more likely to contain iron, silicate or carbon compounds, though it is less dense than Earth.

7.7 Earth Mass however indicates that this planet is far heavier than planet Earth.

Low eccentricity of 0.0315 and semi-major axis of 0.0730 indicates its orbit is relatively more elliptical compared to Body A, and it is further than Body A to Star X.

Body C: Comet/ Asteroid

Density of 2.67 g/cm^3 indicates it is less dense than Body B, and again Iron, silicate, ice or carbon containing;

1.12 x 10^{-9} Earth Mass indicates it is a really small object compared to the others, about the order of magnitude of a large comet or a stray asteroid.

High eccentricity of 1.022, above 1, plus negative semi-major axis: it will exit the star system because it is travelling in a hyperbolic orbit, never to return. This accounts for its near absolute zeo surface temperature of -269 °C, further indicative of that Body C is highly unlikely to be a planetary body.

Using information from the data booklet, calculate:

The surface temperature of Star X and the Sun, and comment on the significance of the values

Using Stefan-Boltzmann law: $L = 4\pi R^2 \sigma T^4$

Sun: $3.846 \times 10^{26} = 4\pi \times (6.963 \times 10^8) \times 5.67 \times 10^{-8} \times T^4$

T = 5780 K (3.s.f.)

Star X: $0.0126\ (3.846 \times 10^{26}) = 4\pi \times 0.294\ (6.963 \times 10^8) \times 5.67 \times 10^{-8} \times T^4$

T = 3570 K (3.s.f.) i.e. Star X is cooler than the sun

The absolute magnitude of Star X and the Sun, and comment on the significance of the values

Using distance modulus: $m - M = 5 \log_{10} (\frac{d}{10})$

Sun: $-26.7 - M_{\odot} = 5 \log_{10} (\frac{4.848 \times 10^{-6}}{10})$

$-26.7 - M_{\odot} = -31.57$; $M_{\odot} = 4.87$ //

Star X: $10.56 - M_x = 5 \log_{10} (\frac{6.2}{10})$

$10.56 - M_x = -1.038$; $M_x = 11.6$ // i.e. Star X is less bright than the Sun

Suppose Star X is a main sequence star. What type of star would you expect it to be? Explain your answer briefly.

Red dwarf; (Award bonus ½ mark if spectral class K/**M/L**/T is mentioned, these are accepted)

Lower surface temperature of 3570 K, absolute magnitude of 11.6, mass and size both smaller than the Sun, i.e. below the Sun as a main sequence star in the Hertzsprung-Russell diagram;

Deduce which celestial body, if any, is most likely to sustain life. Your answer is expected to be as rigorous and convincing as possible using the given data.

Answer 1: Celestial Body B OR Answer 2: None of the above/ Not possible without further data

Answer 1's approach: Tidally-locked Super-Earth Terrestrial planet

Reasonably low Eccentricity of 0.0315, most likely planet to stay within the habitable zone, plus an average surface temperature of 7 °C suggests likelihood of liquid water;

Density of 3.74 g/cm^3 suggests it is a capable of being a terrestrial planet with a rocky core and even possibly an ocean;

Presence of Atmosphere / Surface Ocean to distribute heat despite tidal locking, in conjunction with the presence of water and carbon compounds in the atmosphere;

Star is a red dwarf, which is long-lived and stable, giving enough time for life to evolve;

Albedo suggests the presence of Ice on the surface of the side facing away from the star;

OR

Answer 2's approach: Rare Earth hypothesis

Rare Earth Hypothesis: Too many factors are against the formation/ sustainability of life in this system;

The planets are too close to the parent stars, which might lead to exposure to solar flares that would ionize the atmosphere, making the planet inhospitable. The system has a gas giant closer to the star and the terrestrial planet further away, and would not act as an effective 'shield' against stray deep space objects such as body C.

All planets are way too massive, and the average density is not a good measure of the actual radius; e.g. a small and dense 'cannonball' iron planet with a large hydrogen atmosphere!

Mean surface temperature is a bad estimate; tidally locked planets could very well be extremely warm on one side and cold on another side; e.g. (-110°C + 124°C)/2 = 7°C;

Atmosphere content is not entirely indicative of how thick it is, nor is there sufficient data to distinguish the relative percentage composition;

No indication of a large natural satellite thus far on any of the planetary bodies.

State the condition present for a planet that is present in the circumstellar habitable zone of a star. Must a celestial body be within this location to be capable of sustaining life? Explain your answer. You are to give a hypothetical or real example to support your answer, if any.

The circumstellar habitable zone is defined as the region around a star whereby water is capable of existing in its liquid state;

No; the habitable zone is an arbitrary zone; many other factors contribute to the existence of liquid water, ranging from the greenhouse effect to plate tectonics to air pressure on the celestial body;

A planet outside the habitable zone might sustain life because of a runaway greenhouse effect; it might have an expected sub-zero surface temperature, but in reality it has a surface temperature within the range for liquid water to exist;

A planet with a higher surface gravity and higher atmospheric pressure will be more likely to retain water as a liquid instead of water vapour, and vice versa;

Europa/ Enceladus are examples of moons of gas giants in our solar system that are highly likely to possess underwater oceans, due to tidal heating, and proposed to be capable of sustaining life despite lying outside of the habitable zone;

Polaris and the cosmic distance ladder (17 marks)

While the Astronomical Unit was first calculated (to reasonable accuracy) with transits of Venus, the advent of radar astronomy has allowed us to give a more accurate estimate of the Astronomical Unit. By bouncing radar pulses off Venus, we can directly compute the distance between Earth and Venus and hence the Astronomical Unit.

a) In April 1961, Venus was in inferior conjunction relative to Earth. During this time, a team of astronomers bounced radar pulses off Venus at a frequency of 440 MHz. Their results on April 8th are shown below.
(Data from http://articles.adsabs.harvard.edu//full/1963AJ.....68...15S/0000016.000.html)

Time of transmission (UT)	Measured round-trip travel time/s	Standard deviation of measurement error/s	Measured Doppler shift/Hz
20 h 27 min 35 s	283.67188	0.00007	2247.76

Assuming that Venus was in inferior conjunction at this point in time, calculate the implied value of the Astronomical Unit. For simplicity, Venus and Earth may be treated as having circular orbits and negligible mass. Use the data given in the Formula Book. **[4 marks]**

Step 1: Find the semi-major axis of Venus in terms of AU.
By Kepler's Third Law:

$$T^2 \propto a^3 \rightarrow (\frac{T_E}{T_V})^2 = (\frac{a_E}{a_V})^3$$

Since the semi-major axis of Earth is 1 AU (by definition, we can solve $a_V = 0.7233\ AU$

We know it takes light 283.67188s to travel from Venus and back. This is equal to $(1 - 0.7233\ AU) \times 2 = 0.553\ AU$

As we know the speed of light, you can find that this is equal to 85.1 million km. Doing the math, we get $1\ AU = 153.8$ million km

b) Comparing this value against your formula booklet, you realise that there is a slight discrepancy between the two values. Using data where available, suggest two possible sources of error and briefly explain how they can account for this discrepancy. **[4 marks]**

Any two of the following:

1: The orbits of Earth and Venus are not perfectly circular. If Earth is further from the Sun than expected and vice-versa for Venus, the distance between the two planets is increased from what is expected.
2: The orbit of Venus and Earth are not in the same plane. This increases the distance between Earth and Venus from what you would expect.
3: Venus is not exactly at inferior conjunction, as can be seen from the Doppler shift exhibited by Venus.

Unknown to many, Polaris is actually the closest Cepheid variable. Hence, understanding stars like Polaris is vital for calibrating the period-luminosity relationship for Cepheids.

c) The Hipparcos satellite found that Polaris displayed an annual parallax of 7.54 milliarcseconds. Find the distance to Polaris and express it in terms of light years. **[1 mark]**

Apply the formula in the formula book: d = 1/p =1/(7.54×10^{-3}) = 132.6 parsecs = 434 light years

Polaris itself is a multiple star system. Polaris Aa (the Cepheid variable) forms a tight binary with Polaris Ab, a main sequence star. Polaris B orbits around the center of mass of this binary star system, and is joined by Polaris C and D. Some details about Polaris Aa and Ab are shown in the table below. You may treat the apparent bolometric magnitude of both stars as equal to their visual magnitude.

	Polaris Aa	Polaris Ab
Apparent magnitude, m_v	1.98, variable	9.2
Spectral type	F7Ib	F6V
Semi-major axis, a/ arcsec	0.133	
Period	29.59 years	

d) Find their semi-major axis in AU. Hence, determine the total mass of the system. **[3 marks]**

Constructing a long triangle, we find that:

$$\tan 0.133 arcsec = \frac{a}{132.6\, parsec}$$

$$a = 8.10 \times 10^{-5}\, parsec = 16.71\, AU$$

Using the exact form of Kepler's third law and expressing everything in SI units

$$(29.59 \times 365.24 \times 24 \times 60 \times 60)^2 = \frac{4\pi^2}{G(M_{Aa} + M_{Ab})}(2.4999 \times 10^{12})^3$$

$$M_{Aa} + M_{Ab} = 1.06 \times 10^{31}\, kg = 5.33\, solar\ masses$$

e) By using the mass-luminosity relation where appropriate, estimate the individual masses of Polaris Aa and Polaris Ab. **[5 marks]**

Compute the distance modulus: $m - M = 5\log\frac{132.6}{10} = 5.61$

So Polaris Ab has an absolute bolometric magnitude of 3.58, against the Sun's absolute bolometric magnitude of 4.75. We can use this to estimate the relative luminosity of Polaris Ab.

$$\frac{L_{Ab}}{L_{Sun}} = 10^{\frac{4.75-3.58}{2.5}} = 2.918$$

Hence, by the mass-luminosity relation, the ratio of the masses of Polaris Ab and the sun is:

$$\frac{M_{Ab}}{M_{Sun}} = (\frac{L_{AB}}{L_{Sun}})^{\frac{1}{3.5}} = 2.918^{\frac{1}{3.5}} = 1.358$$

Polaris Aa can then be estimated by simply taking the difference from the total. This gives 3.97 solar masses

Note: Being a Cepheid variable, Polaris Aa is not a main sequence star! The mass-luminosity relation is only for main sequence stars.

Astronomy on Mars (20 marks)

What would happen if humans colonized Mars? This question attempts to explore how astronauts in the northern hemisphere of Mars would experience the night sky.

a) The human eye has a pupil diameter of approximately 6 mm during twilight. What would be the angular resolution of the naked eye under such conditions, assuming that the peak emitted wavelength for most celestial objects lies around 480 nm? Express your answer in arcminutes. **[2 marks]**

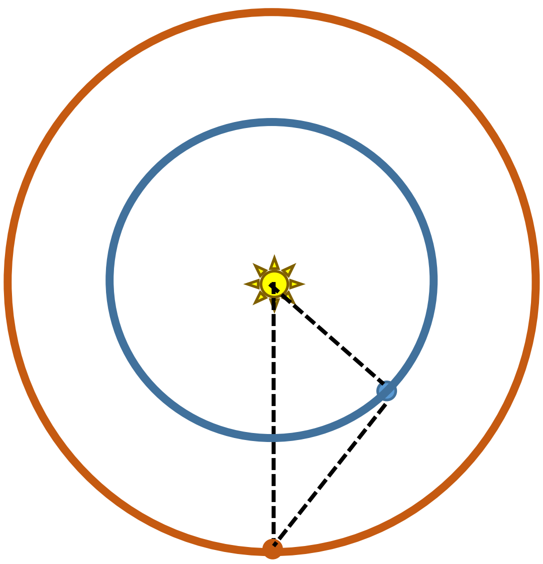
$$\sin\theta = 1.220\frac{\lambda}{D} = 1.220\frac{480nm}{6\,mm} = 9.76\ \times 10^{-5}$$

$$\theta \approx 9.76\ \times 10^{-5}\,rad = 0.3355\,arcmin$$

b) When Earth is at its greatest elongation from the Sun, what is the maximum possible angular separation of the Moon and Earth from Mars? Hence, is it possible to resolve the Earth and Moon into separate objects with the naked eye? Assume all objects are on the same plane, have circular orbits & Earth is being viewed under twilight conditions **[5 marks]**

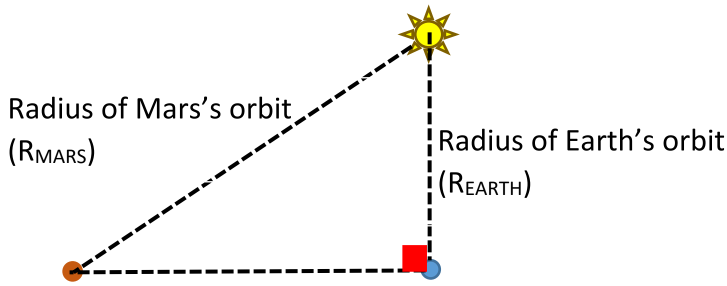
Yes it is possible (0.5 mark)

Most participants were unable to identify what a greatest elongation entailed. For inferior planets, this refers to the point in time where the planet appears to have the greatest angular distance from the Sun. This situation is illustrated in the diagram below.



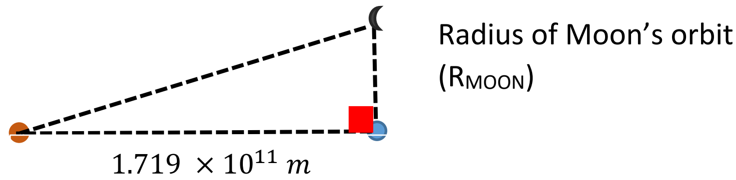
With this in mind ,we can split the problem into separate parts

Part 1: Determine the distance between Mars and Earth when Earth is at its greatest elongation. This is easily done by observing the diagram above. We can construct the following right angled triangle.



From this, it is easy to see that the distance is $\sqrt{{R_{MARS}}^2 - {R_{EARTH}}^2} \approx 1.719 \times 10^{11}\ m$ (2.5 marks)

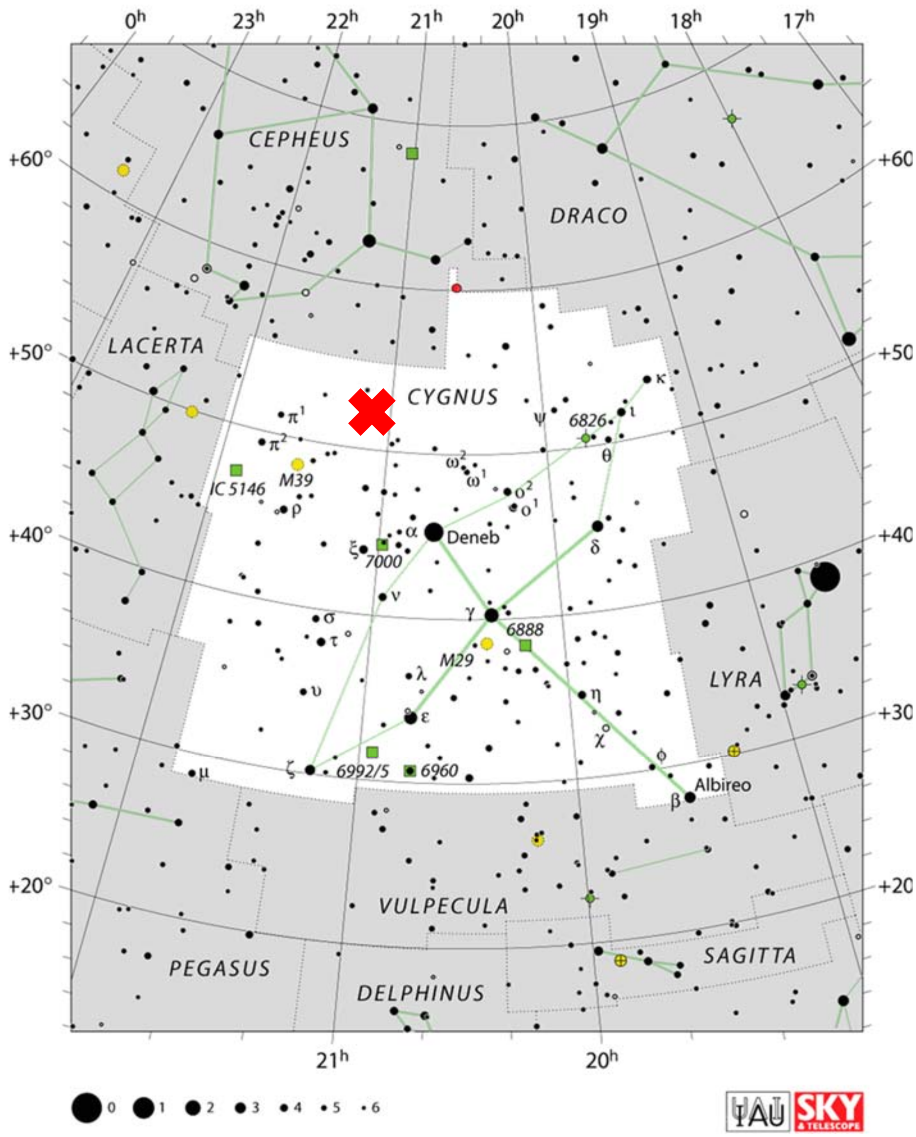
Part 2: The moon and Earth have their greatest angular separation when a line connecting the Earth and Moon is perpendicular to a line connecting Earth and Mars. This gives us another right angled triangle:



The angular separation is then arctan $\frac{R_{MOON}}{1.719 \times 10^{11}} = 0.1281\ rad = 440.38\ arcmin$ (2 marks)

c) The Martian North Celestial Pole is located in Cygnus and has coordinates R.A. $21^h\ 10^m\ 42^s$ / Dec. +52° 53.0′. Mark this on the star map at the right. **[1 mark]**

What is the best way to navigate to the Martian North Celestial Pole? **[1 mark]**

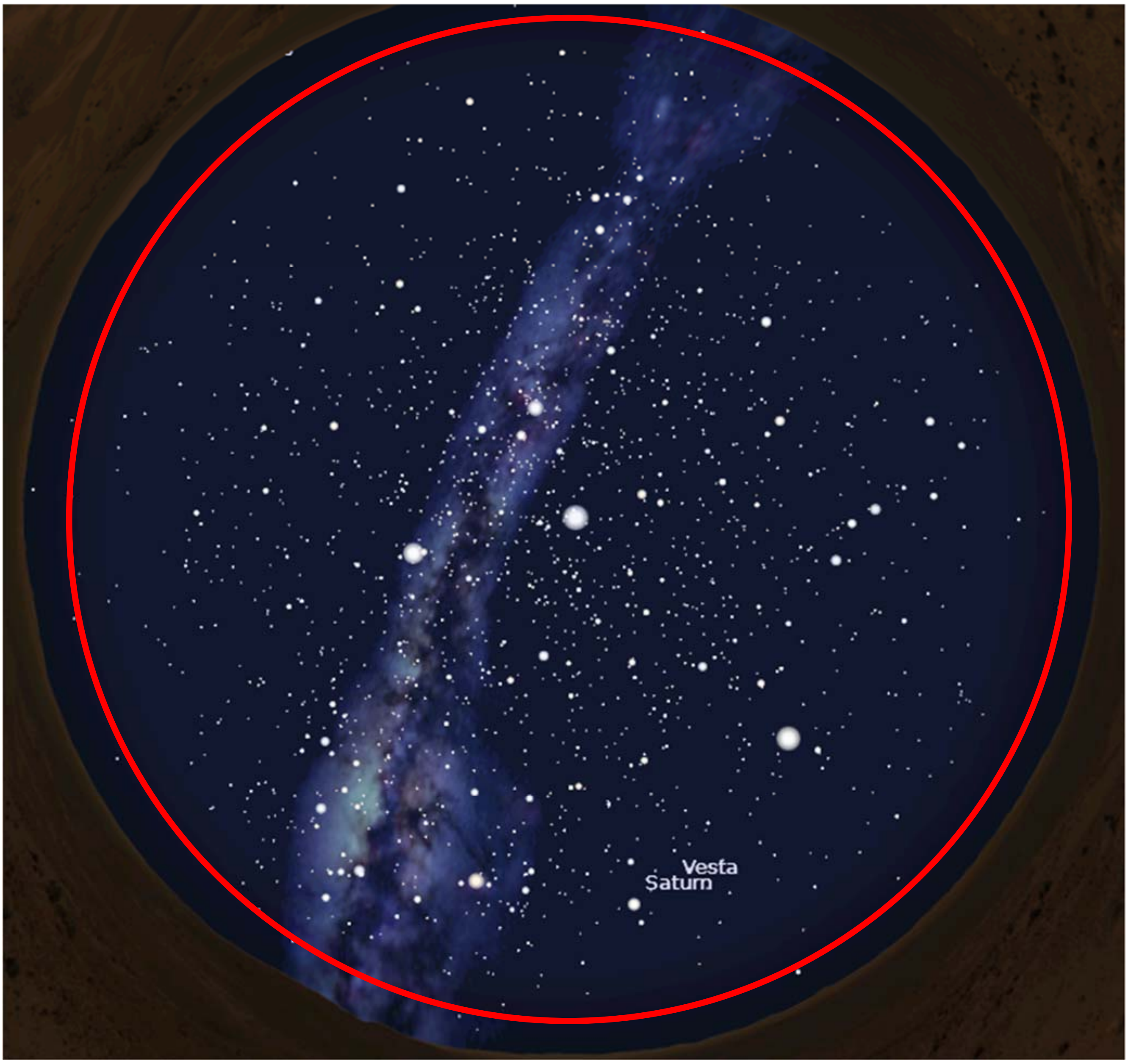


Extend a line from Sadr to Deneb. The Martian North Celestial Pole is approximately the same distance further from Deneb as Sadr is from Deneb.

d) How could one do polar alignment on Mars at night? Assume that you don't have any equipment to tell you where Martian north is. **[4 marks]**

This requires the use of drift alignment [2]. First, align the telescope roughly to the north and then point the scope at a star [1]. Observe North/South drift then adjust altitudinal axis, followed by observing the East/West (RA) drift (and adjusting the azimuthal axis) [1]

e) One night, you decide to hold an observation session outdoors. As you step outside, the following scene greets you. (Planetary bodies have been labelled to avoid confusion)



You have a 6 inch telescope in your hands (focal length 1500 mm), a 32 mm eyepiece and a 10 mm eyepiece. No light pollution is evident. Given the sky right now, name 7 ***visually observable*** deep sky objects that are within reach of your telescope. In your answer, be sure to name the constellation as well as what you should expect to see (galaxy, open cluster, nebula etc.). Also, the region within the red circle contains objects that have an altitude above 5 degrees. Due to atmospheric extinction, you should not venture beyond this zone.

Be specific when naming your targets! Oh, and you are sick of Cygnus, so please don't give any deep sky objects within said constellation **[7 marks]**

PS: If you can't identify which constellation the object belongs to, mark its **exact** location on the star map and indicate the object **clearly**.

In general, Messier/Caldwell objects are accepted, as well as most bright planetary nebulae. Each correctly identified object with location is awarded 1 mark. If teams get the constellation wrong/identify the object wrongly but the answer is otherwise correct, half a mark is awarded.
A sample answer:

M81 (galaxy), Ursa Major

M82 (galaxy), Ursa Major

M97/Owl Nebula (planetary nebula), Ursa Major

M57/Ring Nebula (planetary nebula), Lyra

M8/Lagoon Nebula (nebula), Sagittarius

M13/Great Globular Cluster in Hercules (globular cluster), Hercules.

M7 (open cluster), Scorpius