Astronomy in another land [22 marks]

Elliot, a budding astronomer from country X, has recently purchased a telescope. Its specifications and accessories are stated below. However, he has no idea of what his equipment can do.

Aperture	6 inches (150 mm)
F-ratio	?
Focal length	750mm
Optical Design	Newtonian Reflector
Eyepieces and Accessories	1.25" star diagonal 32 mm eyepiece 25 mm eyepiece 10 mm eypiece 6 mm eyepiece 2x Barlow lens 5x Barlow lens LP Filter

Setting up [9 marks]

a) What is the F-ratio of the telescope? [1 mark]

750/150 = f/5

b) The manufacturer claims that this set can obtain a magnification power exceeding 600x. Given the telescope accessories provided, what is the highest magnification that can be obtained? Hence, is the manufacturer's claim true? [2 marks]

Use the eyepiece with the shortest focal length and the 5x Barlow, you should get 625x

c) Should Elliot try to observe objects through this telescope at 600x? Explain. [2 marks]

No! Fuzzy/blurry images will result.

d) List an advantage and disadvantage of a Newtonian compared to a refracting telescope. [2 marks]

Pros: No chromatic aberration. Generally cheaper than a refractor of the same aperture. Better optical quality in general.
Cons: Requires regular collimation (which is rare for refractors). Suffers from coma

e) Determine the resolution of the telescope if an astronomer is observing at a wavelength of 500 nm. [2 marks]

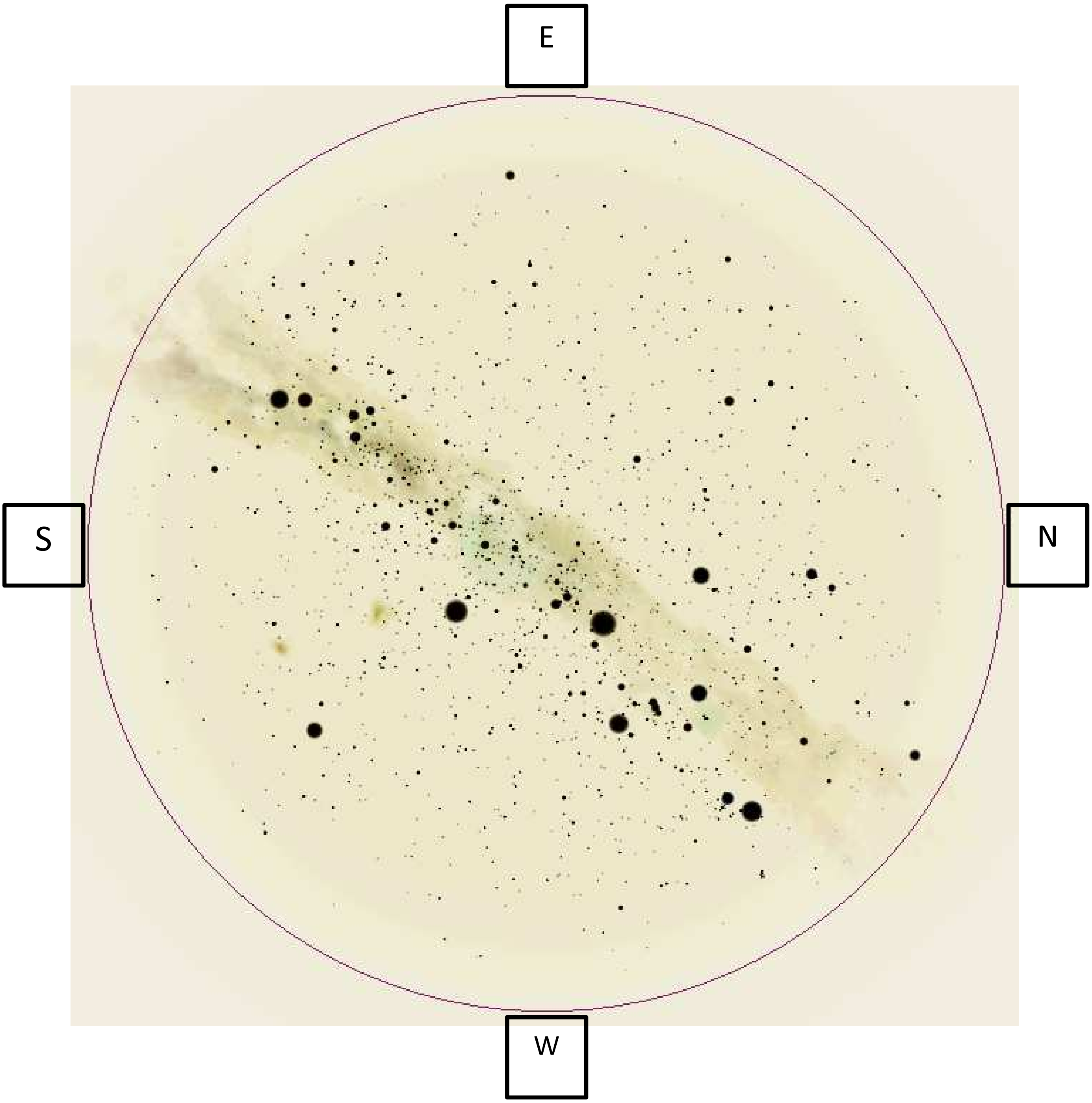
$$\sin\theta_{min} = 1.22\frac{500\times10^{-9}}{0.15}$$

$$\theta_{min} \approx \sin\theta_{min} = 1.22\frac{500\times10^{-9}}{0.15} = 4.07\times10^{-6}\ rad = 0.839\ arcseconds$$

The Observation Session [13 marks]

Thanks to you, Elliot's telescope is finally ready for first light. There's a catch however: Elliot was lost amidst the stars… (Time and date of observation : 20th Dec 2012 1.30 AM 32S 125E)

f) In order to guide him, please fill in the cardinal points on the diagram below [2 marks]



g) Is Elliot in the Southern or Northern Hemisphere? Briefly justify your answer [2 marks]

Southern Hemisphere. If you extend a line from the (visible) Southern Cross, we can see that the South Celestial Pole is above the horizon

h) Can any naked eye planet(s) be observed right now? If yes, please **circle** them. [2 marks]

A planet (Jupiter) is visible next to Aldebaran.

NB: You do not have to name the planet! It's impossible to do this with the black and white photo given above. We were expecting you to label the planet as 'Planet', not make a stab at its actual identity.

i) Name a double/multiple star system that's currently up in the sky AND can be visually separated with the given telescope. [1 mark]

Any reasonable answer: e.g. Rigel.

j) Name a galaxy that's currently up in the sky AND can be seen in the given telescope. [1 mark]
Any reasonable answer: e.g. LMC/SMC

NB: The Milky Way was rejected as an answer: given that all the stars you can see with the naked eye are members of the Milky Way, this statement is true by definition and thus gives no new information to the reader.

k) Name a nebula that's currently up in the sky AND can be seen in the given telescope [1 mark]
Any reasonable answer: e.g. Orion Nebula (M42)

l) Name a star cluster that's currently up in the sky AND can be seen in the given telescope [1 mark]
Any reasonable answer: e.g. the Hyades/Pleiades

m) Identify 3 other <u>deep sky objects</u> that are currently up in the sky AND can be seen in the given telescope. To obtain full credit, **<u>name</u>** the constellations they are in [3 marks]

Any reasonable answer: e.g. the Jewel Box in Crux, Omega Centauri in Centaurus and M41 in Canis Major

Goldilocks in space: The Habitable Zone (20 marks total)

Red – Qi En (*Author notes*/ answers/ optional story)   Black – Finalised question content

Suppose a hypothetical scenario whereby you and your crew aboard the spaceship, Goldilocks-007, visits an extrasolar planet system located in the Ursa Major constellation. Your crew chanced upon a stellar neighbourhood and found a suitable system with Earth-like planets surrounding Star X. The spectrophotometer on board Goldilocks-007 reveals the peak wavelength of Star X to be 411 nm. Deduce the effective surface temperature of X. (0.5m)

Using Wien's Displacement Law: 2897.76829/0.411 = approx. 7051 K (0.5m)

Table 1: Spectral class of stars corresponding to effective surface temperature

Spectral Class	Apparent Color	Effective surface temperature (K)
O	Blue	30,000 – 50,000
B	Deep Blue-white	10,000 – 30,000
A	Light Blue-White	7,500 – 10,000
F	White	6,000 – 7,500
G	Yellow White	5,000 – 6,000
K	Yellow Orange	3,500 – 6,000
M	Light Orange Red	2,500 – 3,500

Using information from the above table, and given that Star X is a main sequence star, deduce its spectral class based on your calculations, and show whether it will be more, equally or less massive compared to our sun. (1m)

F (0.5m); more massive, given $L \propto M^{3.5}$ (0.5m) (*should we subtract marks if they don't mention this proportionality in one form or another?*)

Despite your theoretical calculations, authorities from Wizard-01, a nearby local space station, insists the star to be a yellow white star with a surface temperature of 7000 K. Use this value for all your subsequent calculations.
(This line is added to ensure I don't get ridiculous results from error carried forward from messing up units at the first step)
Thankfully, the radius of Star X was determined by them to be exactly 1.17 times that of our Sun. With this new information, calculate its Luminosity and provide the appropriate units. How many times, then, would Star X be brighter than our Sun? (2m)

Using Stefan-Boltzmann Law: $L = 4\pi R^2 \sigma T^4$
$L = 4\pi (1.17\times6.963\times10^{8})^2 (5.67\times10^{-8}) (7000)^4$
$= 1.1135\times10^{27}$ W (1m methods, 0.5m answers with correct units. Accept S.I. units.)
$1.135\times10^{27} / 3.846\times10^{26} = 2.95$ (0.5m) (*if exact answer*)

➔ approximately 3 times! *(deduct marks if units are left out or if question is incomplete)*

Wizard-01 requests for assistance in verification of the following claim submitted by a lazy scientist on board:

Suppose you have a planet of a distance D from any star of luminosity L.

Assuming the star radiates isotropically, and the planet is of sufficient distance from the star, the power absorbed by the planet can be given by treating the planet as a disc of radius R (in metres), which intercepts some of the power spread over the surface of a sphere of radius D (the distance of the planet from the star, also in metres). At the same time, the planet reflects some of the incoming radiation; this is accounted for with an albedo parameter. An albedo of 1 means all radiation will be reflected, while an albedo of 0 means all of it is absorbed. The expression for absorbed power is thus:

$$E_{absorbed} = \frac{L \times R^2 \times (1-\alpha)}{4D^2}$$

The next assumption we can make is that the entire planet is at the same temperature T, and that the planet radiates as a blackbody. The Stefan-Boltzmann law gives an expression for the power radiated by the planet:

$$E_{emitted} = \sigma T^4 \times 4\pi R^2$$

<u>Prove that the Temperature of an object (in Kelvin) of a distance D away from the star can be determined by the following equation, and state the key assumption made to obtain the following equation.</u> (2m)

$$T = \sqrt[4]{\frac{L \times (1-\alpha)}{16\pi\sigma D^2}}$$

<u>The planet is assumed to be a blackbody. (*-0.5 mark if they did not write this.*) Thus, you can equate the two equations together, by definition:</u>

<u>Energy absorbed = Energy emitted</u>

<u>The effective temperature of a planet can thus be calculated by equating the power received by the planet with the power emitted by a blackbody of temperature T.</u>

(2m goes into the proving process, QED. I made this a proof question, not a derive question to again prevent massive error carried forward later on.)

Next, Wizard-01 mailed you some data regarding the three planets orbiting star X, which your crew designated as Papa, Moma and Cub based on their relative size and distance. Unfortunately, due to an unknown interference, some of the data has been lost.

Table 2: Data for celestial objects orbiting Star X

Observed object	Planet A (Papa)	Planet B (Moma)	Planet C (Cub)
Semi-major axis (AU)	0.402	1.487	1.780
Radius of orbit at periapsis (AU)	0.393	1.467	1.645
Radius of orbit at apoapsis (AU)	0.411	1.507	1.925
Eccentricity	0.0224	0.0135	0.0815
Mass (M⊕)	18.2	5.1	2.8
Density (g/cm^3)	1.09	4.43	3.22
Primary composition of Atmosphere	Hydrogen, Helium, Methane, Ammonia	Methane, Water, Nitrogen	Hydrogen, Methane, Water, Nitrogen
Average Albedo	0.46	0.31	0.22
Mean surface temperature (°C)	913	20	-19

Fill in the missing information in the table. / Using given data, re-calculate the missing information. (1.5m)

Semi-major axis (in AU) of Planet B (Moma): (0.5m)

$\varepsilon = (r_a - r_p) / (r_a + r_p)$ $\varepsilon = (1.507 - 1.467) / (1.507 + 1.467) = 0.0135$

Radius of orbit at periapsis (in AU) for Planet C (Cub): (0.5m)

$\varepsilon = (\alpha - r_p) / \alpha$ $0.0815 = (1.780 - r_p)/ 1.780$ $r_p = 1.645$

Radius of orbit at apoapsis (in AU) for Planet C (Cub): (0.5m)

$\varepsilon = (r_a - \alpha) / \alpha$ $0.0815 = (r_a - 1.780)/ 1.780$ $r_a = 1.925$

When a planet is of a distance away from its parent star equal to its semi-major axis of orbit, it is possible to deduce its average effective temperature. Calculate the average effective surface temperature of Planets A, B and C using the given equation, and compare the values you obtained with those given in Table 2. Which planet has the largest disparity in theoretical and actual temperature? Assume absolute zero to equal -273.15 K. (4m)

$$T = \sqrt[4]{\frac{L \times (1-\alpha)}{16\pi\sigma D^2}}$$

Punch the following into the calculator: (0.5m for each method and answer, + 1m for final statement. Allow room for any error carried forward.)

Planet A (Papa)

$$T = \sqrt[4]{\frac{1.16854 \times 10^{27} \times (1-0.46)}{16\,(5.67 \times 10^{-8})\,\pi\,(0.402 \times 1.495978707 \times 10^{11})^2}} = 494\ K\ OR\ 221\ °C$$

Planet B (Moma)

$$T = \sqrt[4]{\frac{1.16854 \times 10^{27} \times (1-0.31)}{16\,(5.67 \times 10^{-8})\,\pi\,(1.487 \times 1.495978707 \times 10^{11})^2}} = 273\ K\ OR\ 0\ °C77$$

Planet C (Cub)

$$T = \sqrt[4]{\frac{1.16854 \times 10^{27} \times (1-0.22)}{16\,(5.67 \times 10^{-8})\,\pi\,(1.780 \times 1.495978707 \times 10^{11})^2}} = 257\ K\ OR - 16\ °C$$

Scale by 0.751125544… in case the junior fails to figure out the missing pi(e)!

Planet A (Papa) has the largest disparity in actual mean surface temperature and theoretical values!

Give two short explanations to account for the disparities between temperatures from theoretical calculation and the actual measured effective surface temperature on the planets in general. (2m)

Any two of these, 1m each:
--> Atmospheric effects (e.g. Greenhouse gases)
--> Planets are 'greybody systems' (non-idealised blackbody)
--> Tidal heating due to moon(s)
--> Average albedo is an average value, and not a good indicator (e.g. high albedo in poles)
--> Star Luminosity is not necessary constant

+ any other reasonable answers. This is a giveaway!

Suppose for this star system, the lower bound for the habitable zone is defined to be 1.28 AU away. It is then estimated that if Planet C is at a distance with an effective temperature of -20°C or less, ice formation would exceed its equilibrium rate. Using this temperature and planet C as a reference, calculate the upper bound of the habitable zone. Would Planet C's orbit cause it to exit the habitable zone at any point in time? (2m)

Again, 1m for method and 0.5m for answer:

$$T = \sqrt[4]{\frac{L \times (1-\alpha)}{16\pi\sigma D^2}}$$

$$253.15 = \sqrt[4]{\frac{1.16854 \times 10^{27} \times (1-0.22)}{16\,(5.67\times 10^{-8})\,\pi\,(\boldsymbol{D} \times 1.495978707 \times 10^{11})^2}}\,;\ \boldsymbol{D} = 1.84\ AU$$

i.e. Upper bound of habitable zone: 1.84 AU

(This is just manipulating equations to make D the subject again)

At periapsis, C is 1.925 AU away, i.e. it WILL leave the habitable zone! (0.5m)

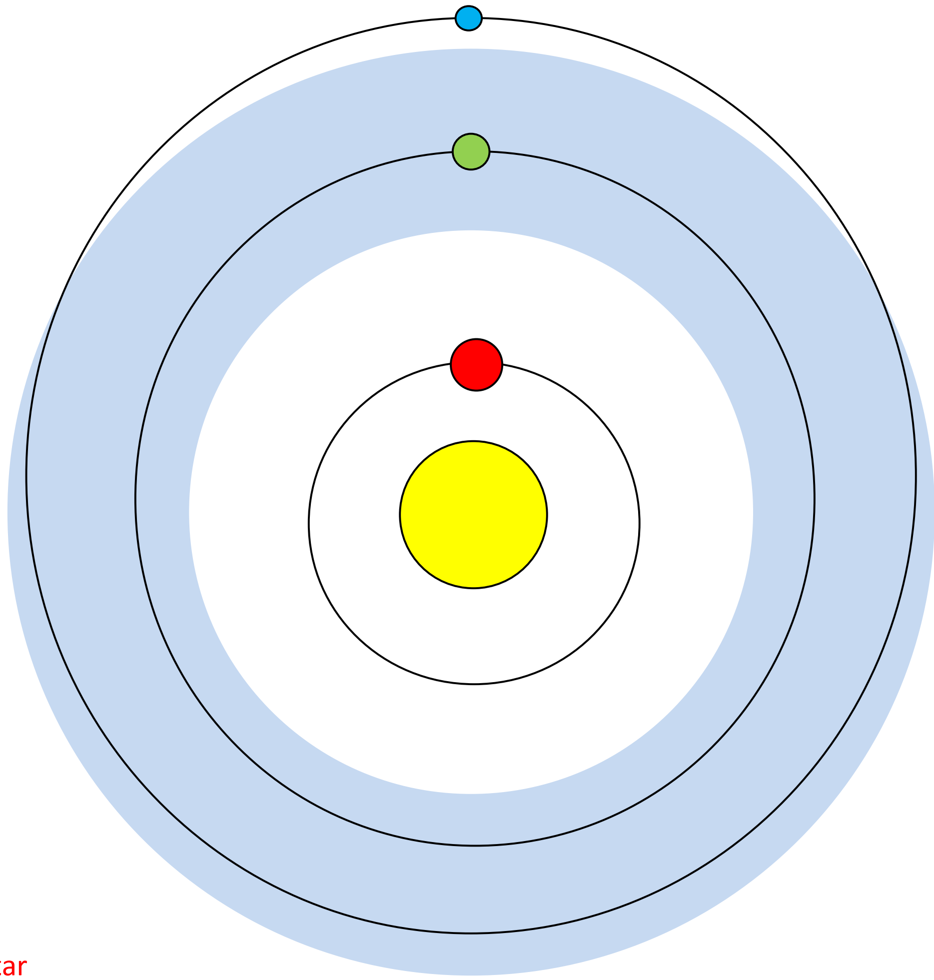
Write a short explanation on the eventual fate of Planet C and its ability to sustain life. You may also instead propose a mechanism, showing clearly parameters involved. Next, propose to explorers from Wizard-01 the best planet to investigate for life in this extrasolar planet system. (2.5m)

--> Snowball planet effect: (2m)

Ice formation increases → albedo increases (show how increasing α will lead to lower effective temperature, given the same distance using given equation), which leads to an even lower effective temperature as a vicious cycle

Investigate Planet B (Moma) instead! (0.5m)

The explorers aboard Wizard-01 are worried they might be lost. Draw a simple top-down map or diagram to show the relative position of Star X, each planet and their orbits, as well as the habitable zone. It need not necessarily be drawn to scale, but do indicate relative sizes of each object and the upper/lower limits of the habitable zone with respect to the planet's orbits clearly. (2.5m)



Legend:

Yellow: Star

Red: Planet A Green: Planet B Blue: Planet C Area: Habitable zone

1m: planets are all in right order and size, as well as properly labelled.

1m: habitable zone upper and lower bound are defined.

0.5m: giveaway if all objectives are met

History in Astronomy: Jupiter's moons (20 m total)

Figure 1: The hall of fame goes to Galileo Galilei (left) and Ole Rømer/ Roemer (right) whose discoveries are still in use for astronomy and this question in particular today.

The four large Galilean moons of Jupiter have captured mankind's attention ever since their initial observation by Galileo Galilei in 1610. Subsequently, he observed the precise orbits and eclipse timing of Jupiter's moons as a method to tell time. This is subsequently investigated by Ole Roemer, who initially tracked eclipses of Jupiter's moons as a mean to determine longitude to help seafarers discern their location (not to mention in his times, it would be a fast track to getting rich and famous). But by accident, he worked out the first estimate of an important constant: The speed of light!

Because any more historical information will be extremely boring, it is now your group's turn to write a short, interesting feature article about any one of the four Galilean moons to ensure your grader won't fall asleep. Include any key features, landmarks, and interesting trivia you deem to be of interest to the reader. (4m++)

(p.s. if your grader falls asleep anyway because your article is too long or boring, a penalty will be imposed. As a rule of thumb, having 4 distinctive traits or 2 really interesting features with elaboration would be enough to score full marks for this column. If your article is on par with a short column in a science newsletter we might consider giving up to 1 bonus mark.)

Suggested points:

Calisto:
- Furthest of the Galilean moons
- 3rd largest moon in solar system and 2nd largest Galilean moon
- Does not experience tidal heating/ not within magnetosphere
- Is tidally locked
- Really thin atmosphere of CO_2

- Might have an underwater ocean but unlikely to support life without tidal heating
- Heavily cratered: due to inactive surface, exposure to / lack of shielding from impacts, combined with lack of volcanism and tectonic activity
- Most suitable place to construct a human base for missions on other moons of Jupiter with elaboration (e.g. HOPE project)

Ganymede:
- Largest moon in the solar system (and anything about size)
- Shares high possibility of internal ocean with Europa
- Thin atmosphere of Oxygen
- Experiences tidal heating and tectonic activity
- Orbital resonance with Europa, Io
- Moon with a magnetosphere – convection within liquid iron core
- Any planned missions to it

Europa:
- Smallest of Galilean moons
- Ice covered surface with occasional geysers; underwater ocean
- Lineae features as a result of tidal flexing (comparison with ocean ridges)
- Orbital resonance with Io, Ganymede
- *Most likely candidate for Extraterrestrial life, and this can go on and on. Max 3 m including planned missions in the future, talks about how life is sustainable in deep sea hydrothermal vents, etc. Most Biology students should be able to spam info here.*

Io:
- 3rd largest Galilean moon
- Highest density of all moons
- Most dry object in solar system (least water)
- Most geologically active object in solar system. Anything about volcanoes, really, from how extreme tidal heating causes activity to height and frequency of eruptions
- Sulfur rich
- Plasma torus

Orbital resonance with Europa, Ganymede
- Used by Ole roemer (and the candidates later) to determine the speed of light

++ any other reasonable points

Before we can attempt Ole Roemer's experiment using various objects in space, we need to first consider the distance traveled in space by Earth and Jupiter.

Using information in the data booklet, calculate:

i) The distance travelled by Earth within one complete orbit, in AU. (1m)

S = R θ,   S = 1 * 2π = approx. 6.28 AU.

ii) Jupiter is known to be 5.2 AU away from the Sun. Thus, calculate the distance travelled by Jupiter in the same time taken for Earth to complete one orbit, in AU. (2m)

Jupiter's period (from data booklet) is 11.86 years, compared to Earth's, which is 1 year.
S = R θ,   S = 5.2 * 2π = approx. 32.67 AU travelled in 11.86 years.
Thus,   32.67/11.86 = approx. 2.75 AU.

It is also possible to calculate based on how many degree radians Jupiter travels in the period of 1 year to arrive at the same answer.

Ready? Now, yet another history lesson begins.

For the sake of simplicity and more possible observations, Io was chosen by Ole Roemer for observation due to its short period. He initially started by predicting the time taken for Io to eclipse (move behind Jupiter's shadow), based on known data of Io's synodic period (the time taken for it to complete one orbit).

He observed the eclipse of Io at two different times, once when Earth and Jupiter are really close and once when they are further away from each other. And then something unexpected happened: **There existed a difference in the time taken, about several minutes, between his estimated transit timing and its actual occurrence!**

Baffled, he hypothesized that if the observation of said transit does not occur on schedule when given a different Earth-Jupiter distance… is because the speed of light is not instantaneous! (i.e. the information contained by light that Io has eclipsed behind Jupiter's shadow will take a longer time to travel to an observer who is further apart on Earth). Thus, said speed of light can be calculated using the difference in Earth-Jupiter distance divided by the difference in transit time, as given by the following equation:

$$\text{Speed of light} = c = \frac{\Delta D}{\Delta T}$$

… In case you're feeling sleepy, here's a pop quiz. Draw and label a diagram showing the relative position of the Sun, Earth and Jupiter at 4 different timings:

(A) During a superior conjunction

(B) 3 months after a conjunction

(C) 1 month before an opposition

(D) During opposition

You may draw 4 separate sub-diagrams or one large diagram. Be sure to label the position of all celestial objects, especially Earth clearly. Use the information calculated from the above section (i and ii) to make a more informed decision on relative position of the planets. The diagram need not be drawn to scale, but make it as neat as possible for reference purposes in the subsequent section. (4m)

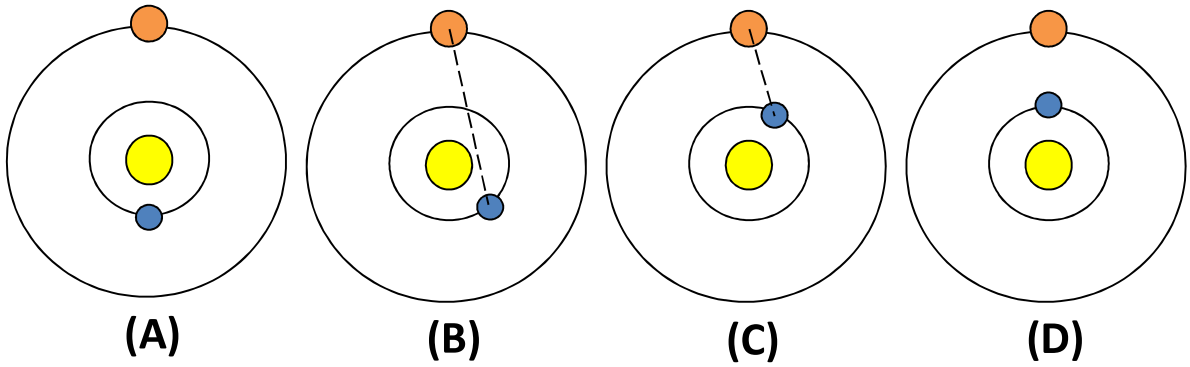


Diagram showing Earth's (Blue) position with respect to Jupiter (Orange) and the Sun (Yellow) at different times due to the difference in the orbit of both planets. Dotted lines represent the two distances (Far and Near) used in this observational experiment. (A) Superior conjunction of Jupiter and Earth. (B) 3 months after conjunction, the first date used for observation for a larger Earth-Jupiter distance. (C) 1 month before opposition, or the second date. (D) Opposition of Jupiter and Earth.

Ole Roemer observed transits only 3 months following a conjunction (B) and 1 month before an opposition (C), but not during the conjunction itself (A). In theory, the Earth-Jupiter distance should be the greatest during superior conjunction. So why didn't – or couldn't – he observe an eclipse at this time? Explain this in one sentence. (Hint: take a close look at your diagram) (1m)

It is impossible to observe Io's eclipse from Earth as Jupiter is located behind the Sun during a superior conjunction. (QED)

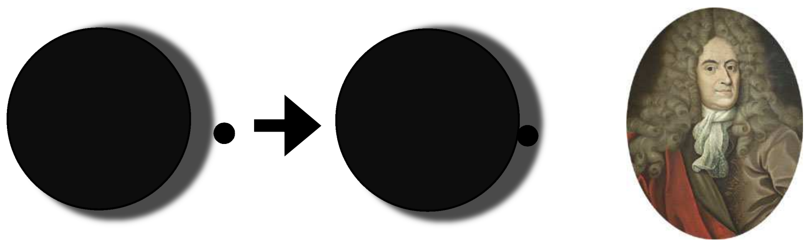
The following is a dataset corresponding to real-life dates with the theoretical positions of Earth and Jupiter, from A-D.

Phenomena	Date	Time
(Superior) Conjunction (A)	24th July, 2014	20:43:58 UT (JD: 2456863.363862)
Opposition (D)	6th Feb, 2015	18:19:51 UT (JD: 2457060.263782)

Phenomena	Date	Start of Io Transit	Exact Earth-Jupiter Distance
3 months after a conjunction (B)	25th Oct, 2014	01:11:23 UT (JD: 2456955.549572)	5.52113 AU (8.25949 × 10^{8} km)
1 month before an opposition (C)	5th Jan, 2015	14:36:26 UT (JD: 2457028.108634)	4.49916 AU (6.73065 × 10^{8} km)

Table 2: Real-life observation data obtained. Two transits were observed during Earth-Jupiter positions at (B) and (C). JD refers to Julian Days, which is the number of days elapsed since a reference point, the Julian period.

Now, all the ingredients are in place and you can work out Ole Roemer's famous cryptic working (don't worry, this will be guided) to calculate the speed of light! For the purpose of accuracy, treat the synodic period of Io to be 1.769861 days for calculation purposes.



"Hey, why did Io disappear before it is supposed to today?!"

Figure 2: Ole Roemer's experiment in a nutshell

First, calculate how many orbits Io *seemed* to have made on the 5th of Jan, 2015 since 25th Oct, 2014. (1m) (Hint: use the difference in Julian days to make life a lot easier. Do NOT round up as precision is key in this step!)

Difference in days = 2457028.108634 – 2456955.549572 = 72.559062.

Number of Orbits = 72.559062 days / 1.769861 days per orbit = 40.997.

But wait… Io's synodic period should be a **precise constant** – so the only reason why you would not observe an exact whole number is because of the **time difference for information to travel in the form of light to us**! That is why the number of orbits fell short of expectation by a tiny margin. <u>*Now*, calculate the predicted time whereby the eclipse will occur, using the nearest whole number of orbits for predicted time. Then find the Time Difference (ΔT) between the times of the observed eclipse and the predicted eclipse. (1m)</u>

Expected eclipse time = 41 × 1.769861= 72.5643 days.
ΔT is thus 72.559062 – 72.564301 = -0.005209 days = -7.54416 minutes.

<u>Next, find the difference in Distance (ΔD) between Earth and Jupiter on both relevant dates in Astronomical Units (AU). (1m)</u>

5.52113 – 4.49916 = 1.02214 AU.

Using the formula, Speed of light = c = ΔD / ΔT, and letting 1 AU be 149,597,870,700 meters, <u>calculate the speed of light and determine by how many percent is it greater or less than the theoretical value. (1m)</u>

1.02214 / 7.54416 = 0.135488 AU/min = 3.37811×10^8 m/s
Let c to be 2.99792×10^8 m/s.
This gives $[(3.37811\times10^8 - 2.99792\times10^8)/ 2.99792\times10^8] \times 100\%$ = about <u>12.68%</u>.
(Accept and close one eye if c = 3×10^8 m/s is used)

<u>Give an explanation why the speed of light is different from the presently accepted value. (2m)</u>

- uncertainty in distance measurements between the Earth and Jupiter
- uncertainty such as the exact occurrence of the eclipse
- theoretical period of Io itself
- only one pair of observation was made; many paired observations coupled with a statistical average should be compiled
- shift in position of Jupiter's shadow not taken into account

If you have enough of Io for today, how about wishing that it is going to disappear? At present, Io is 421,700 km away from Jupiter's center and 350,000 km from Jupiter's cloudtop. It has a density of 3.53 g/cm^3. Calculate the distance due to the Roche limit for it to be reduced to fragments. Would you expect it to end up in pieces around Jupiter anytime soon? (Assume Jupiter to be a perfect sphere of volume 4/3 π R^3, where R refers to its radius.) (3m)

Io's density: 3.53 g/cm^3 = 3530kg/m^3.

Density of Jupiter:
1.899*10^27/ ((4/3) * π *(7.149*10^7)^3) = 1240.8 kg/m^3

Calculating the Roche limit, 1.26*(7.149*10^7)*((3530)/(1240.8))^(1/3)
= 1.27636*10^8 m = 127636 km

Io is not within the Roche limit and will not fragment. It will stay to torment the next batch of Astrochallengers!

An introduction to Observation Plans

Apparent magnitude and Surface Brightness

We know that the apparent magnitude of a star is simply how bright it looks. But how exactly do we quantify it? It turns out that the apparent magnitude of an object is given by the following formula

$$m = -26.74 - 2.5\log\frac{B}{B_{Sun}}$$

Where $\frac{B}{B_{Sun}}$ indicates the apparent brightness of the object, relative to that of the Sun. For instance, if $\frac{B}{B_{Sun}} = 2$, the object appears to be as twice as bright as the Sun

However, most deep-sky objects are not point sources. In particular, galaxies and nebulae are diffuse objects: their light is spread out over a certain area. Because their light is spread out, large objects can be extremely dim and hard to see, even if their apparent magnitude appears to be relatively bright.

This is where surface brightness comes in. Surface brightness is the average apparent magnitude of a one square arcminute patch of the object. Intuitively, we are taking the apparent brightness of the object and spreading it out evenly across its area. By measuring the apparent magnitude of a unit area of this object, we have a measure of how easily visible these objects are.

This implies that surface brightness (S) is defined by the following formula:

$$S = -26.74 - 2.5\log\frac{B_1}{B_{Sun}}$$

Where $\frac{B_1}{B_{Sun}}$ is the apparent brightness of an average square arcminute of the object, relative to the Sun. The next few questions will show that not only is surface brightness easily determined, it has a particularly nice property.

a) Suppose a nebula has an apparent area of A square arcminutes and magnitude m_{nebula}. Prove that the surface brightness (S) is given by the following expression **[3 marks]**

$$S = m_{nebula} + 2.5\log \mathrm{A}$$

If A square arcminutes have the total brightness of of m_{nebula}, then

$$B = AB_1 \rightarrow B_1 = \frac{B}{A}$$

Substitute this in, collect the terms and simplify

$$S = -26.74 - 2.5\log\frac{B}{AB_{Sun}}$$
$$S = -26.74 - 2.5\left(\log\frac{B}{B_{Sun}} - \log A\right)$$

$$S = -26.74 - 2.5\log\frac{B}{B_{Sun}} + 2.5\log A$$
$$S = m_{nebula} + 2.5\log \mathrm{A} \ \ (Q.E.D)$$

Now, suppose that the distance to the nebula is increased by k times. By definition, the absolute magnitude of the nebula (M_{nebula}) remains unchanged, but the apparent area of the nebula now shrinks to $\frac{A}{k^2}$ (You do not need to prove this)

b) What is the new apparent magnitude (m'_{nebula})? Express your answer in terms of k and m_{nebula} only **[3 marks]**

Apply the distance modulus formula

$$m_{nebula} - M_{nebula} = 5\log\frac{d}{10}$$
$$M_{nebula} = m_{nebula} - 5\log\frac{d}{10}$$

$$m'_{nebula} - M_{nebula} = 5\log\frac{kd}{10}$$
$$m'_{nebula} - \left(m_{nebula} - 5\log\frac{d}{10}\right) = 5\log\frac{kd}{10}$$
$$m'_{nebula} = m_{nebula} - 5\log\frac{d}{10} + 5\log\frac{kd}{10}$$
$$m'_{nebula} = m_{nebula} - 5\log\frac{d}{10} + 5\log\frac{d}{10} + 5\log k$$
$$m'_{nebula} = m_{nebula} + 5\log k$$

c) Hence, determine the new surface brightness of the nebula, S'. Express S' in terms of S. What do you notice? **[2 marks]**

$$S' = m_{nebula} + 5\log k + 2.5\log\frac{\mathrm{A}}{k^2}$$

$$S' = m_{nebula} + 5\log k + 2.5\log A + 2.5[-2\log k]$$

$$S' = m_{nebula} + 2.5\log A = S$$

The surface brightness remains constant!

I hope you can see that surface brightness is far more useful than apparent magnitude!

Observation Plans

What is an observation plan? Essentially, it is a list of objects that you wish to observe, along with other important information that would help you during your observation session.

You are given a list of the following planetary nebulae for an upcoming observation session near the Equator

Object	RA	Dec	Apparent Magnitude, m	Area (square arcminutes)
M27 (Dumbbell Nebula)	19h 59m	+22° 43′	7.5	44.8
M57 (Ring Nebula)	18h 53m	+33° 01′	8.8	14.7
M76 (Little Dumbbell Nebula)	01h 42m	+51° 34′	10.1	4.9
M97 (Owl Nebula)	11h 14m	+55° 01′	9.9	11.2
C39 (Eskimo Nebula)	07h 29m	+20° 54′	10.1	0.6
C63 (Helix Nebula)	22h 29m	−20° 50′	7.6	180

d) What are planetary nebulae? Briefly describe how they are formed. **[3 marks]**

They are emission nebulae surrounding the cores of old red giants [1 mark]. As these stars exhaust their fuel, the core contracts under its own gravity, increasing its temperature and increasing the luminosity. These conditions cause the red giant star to lose most of its outer layers to space, which are subsequently ionised by the hot remnant core/white dwarf to form these planetary nebulae. [2 marks]

e) <u>A proper observation plan should be sorted by the time objects rise.</u> Sort the objects in this list by the order in which they rise, with the earliest object first. **[3 marks]**

M76 -> C39 -> M97 -> M57 ->M27 ->C63

f) Rank these objects by surface brightness, with the highest surface brightness object first. **[4 marks]**

1 mark for correct calcs, 3 marks for correct order (-0.5 marks for 1 misplaced object)

Nebula (sorted)	Surface Brightness (2 d.p.)
C39 (Eskimo Nebula)	9.55
M27 (Dumbbell Nebula)	11.63
M57 (Ring Nebula)	11.72
M76	11.83
M97 (Owl Nebula)	12.52
C63 (Helix Nebula)	13.24

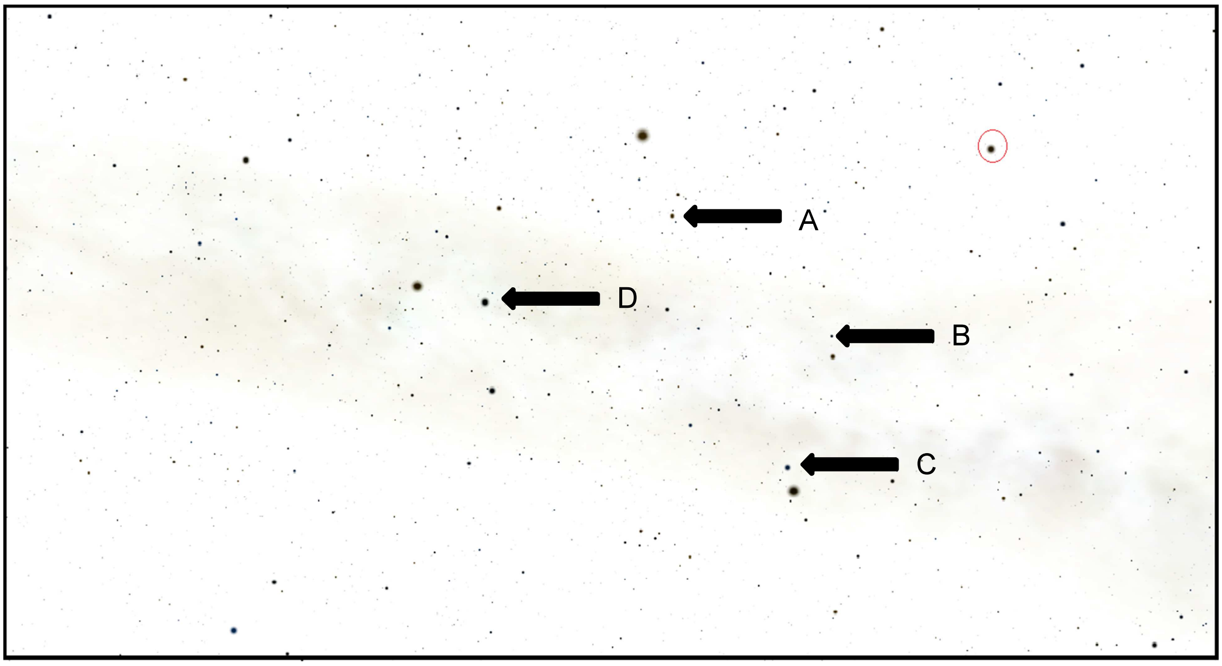
g) Suppose that during one night, you notice that the Owl Nebula is setting. Other than the Owl Nebula, what objects are above the horizon right now? **[1 mark]**

M57, M27, C63

h) On another night, you notice that Object X in your list is currently near the zenith (i.e. it is crossing the meridian). As it does so, you also notice that the Little Dumbbell Nebula is setting in the West. What is Object X? **[1 mark]**

C39

<u>Occultation of Jupiter (17 marks)</u>



On a certain night in June, Clarence and Qi En held observation sessions in 2 different places along the Equator. Clarence in Pontianak (0° S, 109.3° E) is looking at a patch of night sky as shown above.

Clarence managed to find the following 4 stars and matched them to Qi En's catalog (arrows). He also knows that the star Rasalhague (the brightest star in Ophiuchus) is the circled star above. The coordinates of the following stars are shown below.

Star	RA	Dec
Rasalhague	17h 34m	+15° 04'
A	18h 58m	+32° 41'
B	18h 59m	+15° 04'
C	19h 46m	+10° 36'
D	20h 22m	+40° 15'

a) Can we determine which direction is approximately celestial north? Explain why/why not. If yes, mark this direction clearly with an arrow on the diagram. **[2 marks]**

<u>Yes we can!</u> Star A and B have approximately the same RA, and hence a line from B to A will point approximately to celestial north.

b) Clarence notices that right now (0052 hours), Rasalhague is on its upper culmination. When will star C set? **[1 mark]**

By comparing their RA, we know that star C will cross the meridian (and set) 2 hours and 12 minutes after Rasalhague. Since we are on the equator, we also know that Rasalhague will take 6 hours to set from its upper culmination. Add this all up, and we get 0904 hours

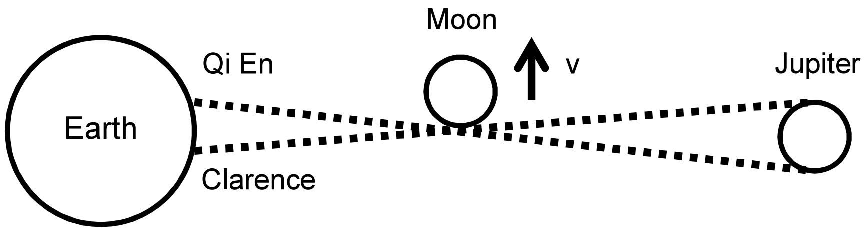
After gazing at this patch of sky for a while, Clarence gets bored and now intends to find some deep sky objects nearby.

c) Mark at least 1 deep sky object on the image and give its name. Clearly state what should Clarence expect to see when he has found the object. In order to help Clarence find your object, please attach a set of finding instructions/draw a finding chart. **[3 marks]**

There are plenty of objects to choose from. One of the easiest is probably M57.

While Clarence was waiting for you to finish, he read some astronomy news and found out that there would be an Occultation of Jupiter on that night. He informs Qi En, and they decide to observe the Jupiter Occultation simultaneously from their locations. When Clarence sees the end of the occultation, Jupiter has completely exited the edge of the Moon (surfaces tangent/barely touching each other).

However, from Qi En's location, the occultation is still ongoing. Just when Clarence spots that Jupiter completely exited the edge of the Moon, Qi En simultaneously notes that Jupiter starts to appear from behind the Moon. A simple sketch is given below (not to scale).



For the rest of this question, assume that:

- all observers and objects are on the same plane (i.e. the celestial equator).
- all objects have circular orbits
- both Clarence and Qi En are equidistant from the line connecting the center of Earth, the edge of the Moon and the center of Jupiter

d) You know that when the occultation ended, the Moon is exactly at Last Quarter. What is the distance between Jupiter and Earth, as well as Jupiter and the Moon? **[4 marks]**

If the occultation occurred during Last Quarter, then Jupiter must be at quadrature (specifically, western quadrature). Hence, the Earth, Jupiter and the Sun form a right angled triangle (with Earth at the right angle). Therefore (2 marks each):

$$d_{Earth-Jupiter} = \sqrt{(7.785 \times 10^{11})^2 - (1.496 \times 10^{11})^2} \approx 7.6399 \times 10^{11}\ m$$
$$d_{Moon-Jupiter} = d_{Earth-Jupiter} - d_{Earth-Moon} \approx 7.6361 \times 10^{11}\ m$$

e) Hence or otherwise, what the distance between Clarence and Qi En? Remember to account for the curvature of the Earth. **[6 marks]**

If you inspect the diagram, you'll notice that the angular diameter of Jupiter from the Moon is approximately equal to the angular separation of Clarence and Qi En from the Moon. Let this value be θ. Since θ is small, we can do the following

$$\tan\frac{\theta}{2} = \frac{R_J}{d_{Moon-Jupiter}} \rightarrow \tan\theta = \frac{2R_J}{d_{Moon-Jupiter}} \rightarrow \theta = \frac{2R_J}{d_{Moon-Jupiter}}$$

$$\theta = 1.87243 \times 10^{-4}\ rad = 0.010728°$$

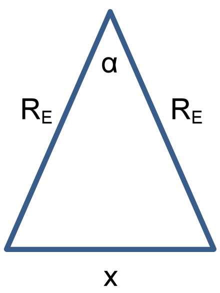
Let x denote the straight-line distance between Clarence and Qi En. Since θ is small:

$$\tan\frac{\theta}{2} = \frac{0.5x}{d_{Earth-Moon} - R_E} \rightarrow \tan\theta = \frac{x}{d_{Earth-Moon} - R_E} \rightarrow \theta = \frac{x}{d_{Earth-Moon} - R_E}$$

$$x = \theta(d_{Earth-Moon} - R_E) \approx 70765m$$

(3 marks total for these calculations, 1.5 marks each)

How do we find the actual distance between Clarence and Qi En? Consider the following triangle from the center of Earth.



Angle α can be found easily through the cosine rule or other methods (2 marks)

$$x^2 = {R_E}^2 + {R_E}^2 - 2{R_E}^2 \cos\alpha$$
$$\alpha \approx 0.637°$$

If we know the angle subtended by Clarence and Qi En, we can simply compute the actual distance D as a fraction of the Earth's circumference (1 mark)

$$D = \frac{\alpha}{360°} \times 2\pi R_E = 70.8\ km$$

f) Clarence observed that sunrise occurred at 0640 hours. Hence, approximately when did the occultation end for Clarence? Justify your answer **[1 mark]**

It must have ended slightly before sunrise. If the Moon is both at last quarter and near the zenith (see diagram), then the Sun must be near the horizon.