AstroChallenge 2022
Senior MCQ Round

SOLUTIONS

Saturday 4th June 2022

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. This paper consists of **39** printed pages, including this cover page.
2. You are required to keep your microphone and camera on at all times throughout the round.
3. You are not allowed to use your keyboard at all times, but you may use your mouse to scroll through the question paper as well as switch to the formula booklet.
4. Any materials other than the Question Paper and Formula Booklet are strictly prohibited.
5. You have **2 hours** to attempt **ALL** questions in this paper. If you think there is more than one correct answer, choose the *most* correct answer.
6. Write your answers on a piece of A4 paper. Write your **Name**, **School**, and **Team Number** at the **bottom right corner** before taking a photo to submit. Failure to conform to this may result in us being unable to find the owner of the script.
7. It is *your* responsibility to ensure that your answer script has been submitted.

1. "It is a beautiful winter night. Perseus is visible in the North. With a pair of binoculars aimed at the brightest star in Perseus, you would be able to see the Alpha Persei Cluster. Looking further East, the red dwarf Capella is bright and visible. Tracing the stars nearby in an irregular hexagonal shape, the constellation Auriga takes shape. Looking upwards from Auriga and Perseus towards the South, the brighter stars of Sagittarius form the distinct teapot asterism."

   Which sentence in this paragraph is wrong if the rest of the paragraph is correct?

   (A) "It is a beautiful winter night."

   (B) "Perseus is visible in the North."

   (C) "Tracing the stars nearby in an irregular hexagonal shape, the constellation Auriga takes shape."

   (D) "Looking upwards from Auriga and Perseus towards the South, the brighter stars of Sagittarius form the distinct teapot asterism."

   (E) Trick question: All sentences in the paragraph are correct.

   Solution:

   Sagittarius is a summer constellation and cannot be seen in the same sky as Perseus and Auriga. Perseus and Auriga can be seen in a Winter sky.

2. Which of the following deaths of the universe can be best explained by the second law of thermodynamics?

   (A) Big Crunch

   (B) Big Rip

   (C) Big Slurp

   (D) Big Chill

   (E) Big Bounce

   Solution:

   The maximum entropy principle states that once thermodynamic equilibrium is reached, entropy is at its absolute maximum. Then, due to the second law of thermodynamics, no more work can be done in the universe. This is what leads to the Big Chill, formally known as the Heat Death of the Universe

3. During a conversation at 12 a.m. local solar time in Singapore, your crush said that it was her birthday month and she wants to see her zodiac sign, Taurus in the night sky. Trying to impress her, you took up the challenge and tried to look for her zodiac constellation in the night sky. Will you be able to succeed?

(A) Yes. The zodiac constellation is visible by naked eye.

(B) Yes. The zodiac constellation is visible but only with a pair of binoculars or telescope.

(C) No. The zodiac constellation is below the horizon.

(D) No. Zodiac constellations are fake and you should leave her immediately.

(E) No. The zodiac is near the horizon and blocked by buildings and atmosphere.

Solution:

The Sun lies near the zodiac constellation in the associated month. So, the constellation will not be in the night sky.

4. You asked your crush out for another stargazing session but she said she will only come out when the two brightest superior wandering stars align. Assuming the previous time these two wandering stars aligned was yesterday, how long will you need to wait to finally meet her again?

(Hint: Synodic period formula is $\frac{1}{P} = \frac{1}{E} - \frac{1}{S}$, where P is the period between conjunction, E is the orbital period of the inner planet and S is the period of the outer planet.)

(A) About 2 days :)

(B) About 2 weeks :|

(C) About 2 months :(

(D) About 2 years :'(

(E) About 20 years T.T

Solution:

The 2 planets are Mars and Jupiter. Relevant data of Mars and Jupiter are available in the formula booklet. The synodic period is 26.8 months, or about 2 years 3 months. (At least she didn't say Uranus and Neptune conjunction, or Venus Sun transit.)

5. Chuan Ming is observing an unknown bright star through a refractor on an equatorial mount with a 2-degree true field-of-view. He observes that the star takes roughly 9.23 minutes to fully cross the diameter of the eyepiece field of view. Estimate the declination of the star. Assume the star is in the northern hemisphere.

(A) 0°

(B) 12°

(C) 21°

(D) **30°**

(E) 45°

Solution:

The "arc length" swept out by declination lines is proportional to the distance to the central axis.

Thus, tangential speed at a certain declination = angular speed of earth's rotation $\times \cos(\text{dec})$ (for a unit sphere with radius 1).

$$\text{dec} = \arccos \frac{\text{star's tangential speed}}{\text{earth rotation angular speed}} = \arccos \frac{\frac{2}{260} \times \frac{2\pi}{9.23\text{mins}}}{\frac{2\pi}{24 \times 60\text{mins}}} = 29.96°$$

Note: The result is only accurate near the celestial equator as near the celestial poles the eyepiece FOV would be too large relative to the declination circle, resulting in a poor approximation of the star's tangential speed

6. The Saros Cycle is an 18-year eclipse cycle within which the relative positions of the sun, moon and earth return to a roughly identical relative geometric position. This cycle has been historically used by the Ancient Greeks and Chaldeans to predict solar and lunar eclipses. Which of the following phenomena are significant factors in the calculation of the Saros Cycle?

I Precession of the Equinoxes

II Precession of Lunar nodes

III Apsidal precession of the Lunar orbit

IV Apsidal precession of Earth's orbit

V Axial precession of the moon

(A) I, II and III only

(B) II, III and V only

(C) II and III only

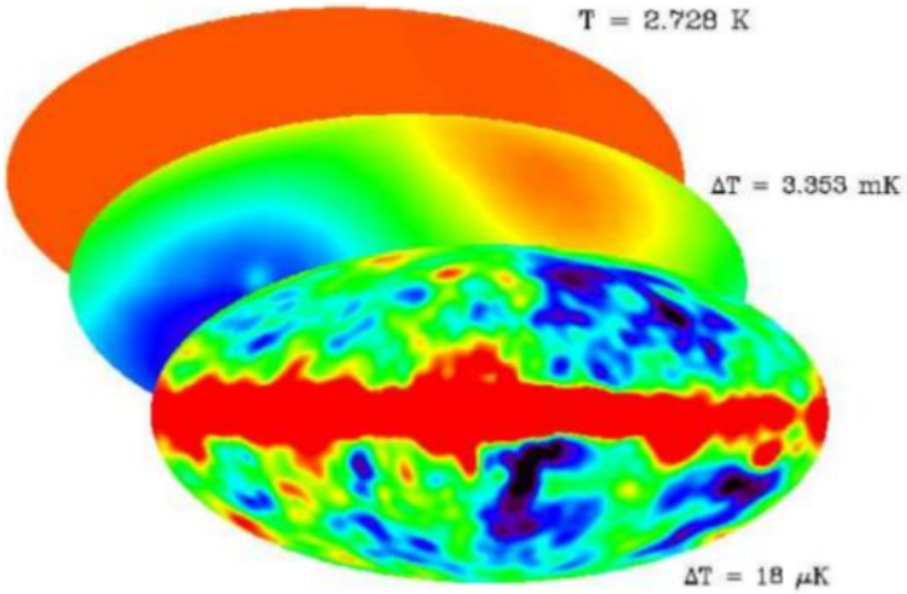
(D) I, II, III and IV only

(E) All of the factors.

Solution:

The Saros Cycle is concerned with regularities in the relative geometric positions of the Sun-Earth-Moon system, and thus the axial orientation of both the Earth and Moon are irrelevant(I and V). While the apsidal precession of the Earth (IV) does indeed impact the relative distances of the sun, earth and moon, it occurs on a timescale far longer than II and III, and inaccuracies in the Saros cycle render this effect irrelevant anyways.

7. In the study of Cosmic Microwave Background (CMB), there are a few large scale variations usually known as foreground or secondary anisotropy that need to be first eliminated as they are unrelated to the CMB. The figure below shows the predicted map of such variations generated from computer simulations.



Which of the following is **NOT** one of the reasons for this large-scale variation?

(A) The motion of our galaxy in the Local Group with respect to the CMB.

(B) The variation in density due to "sound waves" passing through the CMB.

(C) The scattering of CMB photons as they pass through the centre of our Milky Way Galaxy.

(D) **The lensing caused by the large scale curvature of the universe.**

(E) The gravitational lensing by massive structures such as galaxy clusters and super-clusters.

Solution:

Option A is the variation of the CMB due to the proper motion of the galaxy with respect to the CMB. This will lead to a dipole like variation typical of any Doppler effect variation.

Option B is called Baryonic Acoustic Oscillation and it is a result of the fact that information can not reach far enough in the early stage of the universe and as a result only regions where light is able to reach within the early stages of the universe will have similar property (this is often measured using something called correlation length). Typical size of these regions are about 150–200 Mpc in size.

Option C is called the zone of avoidance which is due to the emission, extinction and reflection by interstellar dust in the core of the Milky Way and is observed in most spectral analysis across many spectrum including the spectrum used for CMB radiation observation.

Option D is a wrong reason because there should be no large scale curvature of the universe thanks for the incredibly fast expansion of the universe in its big bang phase. Although lensing by smaller object such as galaxy clusters and superclusters are still seen as an result of their incredible mass, resulting in Einstein's Ring like structure which need to eliminated when looking at CMB spectrum making Option E also a correct reason.

8. Recently, European Southern Observatory (ESO) scientists announced that they spotted a stellar-mass black hole outside the Milky Way which forms a binary system with a star. However, experts have suggested that the system might not actually be a black hole. What is one possible reason for this claim?

(A) It is impossible to detect an extra-galactic black hole by measuring its influence on the motion of a nearby star.

(B) The companion star's mass might be much lower than expected.

(C) It is impossible to find a black hole through radial velocity measurements outside the Milky Way.

(D) There is a faint X-ray signal detected around the star by the Chandra X-ray Observatory.

(E) None of the above.

Solution:

A False. Detecting an extra-galactic black hole by measuring its influence on the motion of a nearby star is difficult, but not impossible.

B True. The researchers speculate that the luminous star is actually similar in mass to our Sun, rather than five times more massive. The star's companion has stripped most of the mass from the star, leaving only a low-mass, bloated core. Such stripped stars appear more luminous than they should, appearing more massive than they are. If this is indeed the case, and the star's mass is much lower than thought, then the mass of the companion will also be low enough such that a black hole is not possible.

C False. Finding a black hole through radial velocity measurements outside the Milky Way is difficult, but not impossible.

D False. This evidence actually supports the speculation of a black hole.

9. Consider a binary system of celestial objects A and B with the same size. Take the centre of mass (CM) of A as the origin of our coordinate system.

   Let

   - x_b be the distance of the barycentre of A and B from the origin;
   - x_CG be the distance of the centre of gravity (CG) of B from the origin

     (For simplicity, we define CG this way: If the total gravitational force acting on B acts through the **CG** of B, then the torque of this total force will be identical to the sum of the torques each originating from the gravitational force acting on each point of B. **All torques are only evaluated with respect to the origin.**);
   - x_CM be the distance of the CM of B from the origin; and
   - x_eq be the distance of the equilibrium point between A and B from the origin.

   If $m_\mathrm{A} \geq m_\mathrm{B}$, which of the following relationships always holds, assuming the celestial bodies to be spherically symmetric?

   (A) $x_\mathrm{b} < x_\mathrm{eq} < x_\mathrm{CG} < x_\mathrm{CM}$

   (B) $x_\mathrm{b} < x_\mathrm{eq} < x_\mathrm{CG} = x_\mathrm{CM}$

   (C) $x_\mathrm{b} \leq x_\mathrm{eq} < x_\mathrm{CG} < x_\mathrm{CM}$

   (D) $x_\mathrm{b} < x_\mathrm{eq} < x_\mathrm{CM}$ and x_CG is not uniquely defined

   (E) $x_\mathrm{b} \leq x_\mathrm{eq} < x_\mathrm{CM}$ and x_CG is not uniquely defined

   Solution:

   x_CG is not uniquely defined because any point on the line joining the CMs of A and B fulfills the definition given. This is because the sum of torques due to gravitational forces acting on individual points is zero.

   Note that $x_\mathrm{b} = x_\mathrm{eq}$ can be fulfilled when $m_\mathrm{A} = m_\mathrm{B}$.

10. Jack and Jill went stargazing in Selangor. They complain of the high obstructed buildings of about 110m, but are nonetheless excited to look for Object X. Object X is an object of interest which Jack and Jill wish to see again after seeing it the night before. They were also interested in Object Y with R.A. 09h 45m 51s | Dec +23° 46' 32" after seeing it culminate near 0600h the day before.

    Which is likely to be Object X?

    Selangor Co-ordinates: N 3° 21' 0.00"

    (A) **R.A. 3h 47m 24s | Dec +24° 7' 0"**

    (B) R.A. 12h 26m 36s | Dec -63° 5' 57"

    (C) R.A. 20h 41m 26s | Dec +45° 16' 19"

    (D) R.A. 2h 41m 39s | Dec +89° 15' 51"

    (E) None of the above could be Object X.

    Solution:

    As Object X was visible the night before, it should culminate a few hours before Object Y and have a similar declination as Object Y. So, Options B and C are ruled out. Option D cannot be the correct choice as it is too close to the horizon and would be blocked by the tall buildings.

11. Refer to the chart below.

	1)	SATURN	URANUS	2)	3)	VENUS	EARTH	NEPTUNE
4)	5427	687	1271	3933	1326	5243	5514	1638
Gravity (m/s2)	3.7	9	8.7	3.7	23.1	8.9	9.8	11
5)	4.3	35.5	21.3	5	59.5	10.4	11.2	23.5
Rotation Period (hours)	1407.6	10.7	-17.2	24.6	9.9	-5832.5	23.9	16.1
Length of Day (hours)	4222.6	10.7	17.2	24.7	9.9	2802	24	16.1
Orbital Inclination (degrees)	7	2.5	0.8	1.9	1.3	3.4	0	1.8
Obliquity to Orbit (degrees)	0.034	26.7	97.8	25.2	3.1	177.4	23.4	28.3
Mean Temperature (C)	167	-140	-195	-65	-110	464	15	-200

Choose the correct answer:

(A) 1) Mercury 2) Mars 3) Jupiter 4) Mass (10^{24} kg) 5) Radius (10^{8} m)

(B) 1) Mercury 2) Mars 3) Jupiter 4) Density (kg · m^{-3}) 5) Escape velocity (km · s^{-1})

(C) 1) Mars 2) Jupiter 3) Mercury 4) Radius (10^{8} m) 5) Mass (10^{24} kg)

(D) 1) Mars 2) Jupiter 3) Mercury 4) Radius (10^{8} m) 5) Escape velocity (km · s^{-1})

(E) 1) Jupiter 2) Mars 3) Mercury 4) Density (kg · m^{-3}) 5) Escape velocity (km · s^{-1})

Solution:

Adapted from https://nssdc.gsfc.nasa.gov/planetary/factsheet/

Most values can be derived from the formula booklet.

12. Which is **NOT** an argument put forth against the recent purported detection of phosphine, PH3, in Venus' atmosphere which possibly points to biological origins?

(A) Phosphine was not actually present as there were ambiguities in the spectral lines.

(B) The telescope equipment and processing techniques were flawed.

(C) The short-lived phosphine was a result of volcanic activities on Venus.

(D) Microbial life simply cannot exist in the harsh conditions of Venus' atmosphere.

(E) Phosphine has a very short half-life in the atmosphere of Venus.

Solution:

Both PH_3 and SO_2 absorb at about 266.94 GHz, which is a factor for misidentification. So, option A is a possible argument.

Greaves' team admitted that ALMA data had been processed incorrectly, and after corrections, they found phosphine at much lower concentrations. However, other independent researchers failed to detect phosphine at all, suggesting the ambiguity of interpreting scientific data. So, option B is a possible argument.

Lunine and Ngoc Truong argued (2021) that phosphides in volcanic materials were carried to the aerosol layer in the atmosphere, which then reacted with sulfuric acid to form phosphine. So, option C is a possible argument.

Option D is circular reasoning which begs the question. In fact, microbial life has been postulated to be possible in the Venusian clouds, although hypothetical. Therefore, option D is the answer.

Phosphine is a reactive compound. It is unlikely that any significant signal could be due to phosphine present. So, option E is a possible argument.

References:

Bains, W., Petkowski, J. J., Seager, S., Ranjan, S., Sousa-Silva, C, Rimmer, P. B., Zhan, Z., Greaves, J. S., & Richards, A. M. S. (2021, October 07). *Phosphine On Venus Cannot Be Explained by Conventional Processes*. Astrobiology, *21 (10)*, p. 1277-1304. Retrieved from https://www.liebertpub.com/doi/10.1089/ast.2020.2352

Dorminey, B. (2021, July 13). *Phosphine In Venus' Atmosphere Points to Volcanics, Not Life, Says Paper*. Forbes. Retrieved from https://www.forbes.com/sites/brucedorminey/2021/07/13/phosphine-in-venus-atmosphere-points-to-volcanics-not-life-says-paper/?sh=7f297acc63f6

Metcalfe, T. (2021, February 5). *'Sign Of Life' On Venus Might Just Be Ordinary Sulfur Gas*. NBC News. Retrieved from https://www.nbcnews.com/science/space/signs-life-venus-might-just-be-ordinary-sulfur-gas-n1256739

Witze, A. (2021, January 28). *Life On Venus Claim Faces Strongest Challenge Yet*. Nature, *590*, p. 19-20. DOI: https://doi.org/10.1038/d41586-021-00249-y

13. In 1924, Edwin Hubble measured the distance to the Andromeda nebula, which firmly established it as a separate galaxy outside our own Milky Way. What did he measure to allow him to calculate the distance of Andromeda?

(A) **Cepheid variables**

(B) Galactic redshift

(C) Gamma-ray bursts

(D) Parallax

(E) Rotational Doppler shift (Tully-Fisher relation)

Solution:

Factual recall. Nevertheless, other options can be eliminated as they are higher rungs on the cosmic-distance ladder used to measure extra-galactic distances. Thus, it does not make sense for them to be used as standard candles before the existence of other galaxies was even established.

14. Currently, GN-z11 is the only galaxy detected so far to have a redshift more than 10 ($z \approx 11.09$), confirmed spectroscopically. Which of the following statements explain(s) why galaxies with redshift more than 10 are incredibly rare?

I Most galaxies have not started forming at that point in time.

II Some of these galaxies have shifted beyond our observable universe, as the Universe is expanding faster than the speed of light.

III These galaxies are faint, not easily distinguishable from other faint galaxies with low redshift and are hard to detect with current technological capabilities.

(A) I only

(B) II only

(C) I and II only

(D) I and III only

(E) I, II and III

Solution:

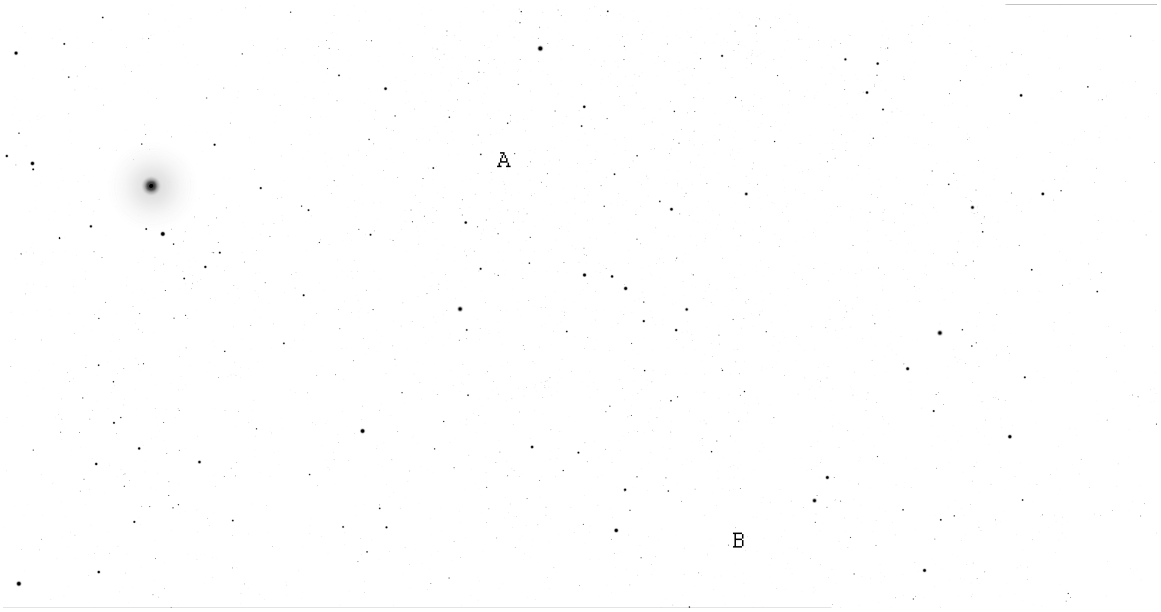
Recall that by Hubble's Law, the higher the redshift a galaxy has, the further it is from us. Also, redshift is a measure of the time elapsed between when the galaxy emits light, and when we observe it.

Statement I is true because the higher the redshift, the greater the amount of time that has elapsed between emission and observation. That means a galaxy with redshift more than 10 must have existed a long time ago, only about 400-600million years after the Big Bang.

Statement II is false. Since we are still able to detect the Cosmic Microwave Background (CMB) with redshift $z \approx 1089$, these galaxies must still be detectable if they existed.

Statement III is true because the sky is crowded with relatively close but intrinsically faint galaxies. They are hard to distinguish from intrinsically luminous but high-redshift galaxies at first glance. The existing Lyman-break technique mostly works for galaxies with redshift $z \approx 3$ 5. Current research is also limited by spectroscopic and wide area, near infrared survey capabilities, that will improve with the launch of WFIRST, James Webb and Euclid.

15. What Deep Sky Objects A and B are respectively? Refer to the figure below.



(A) Beehive Cluster and Leo Triplet

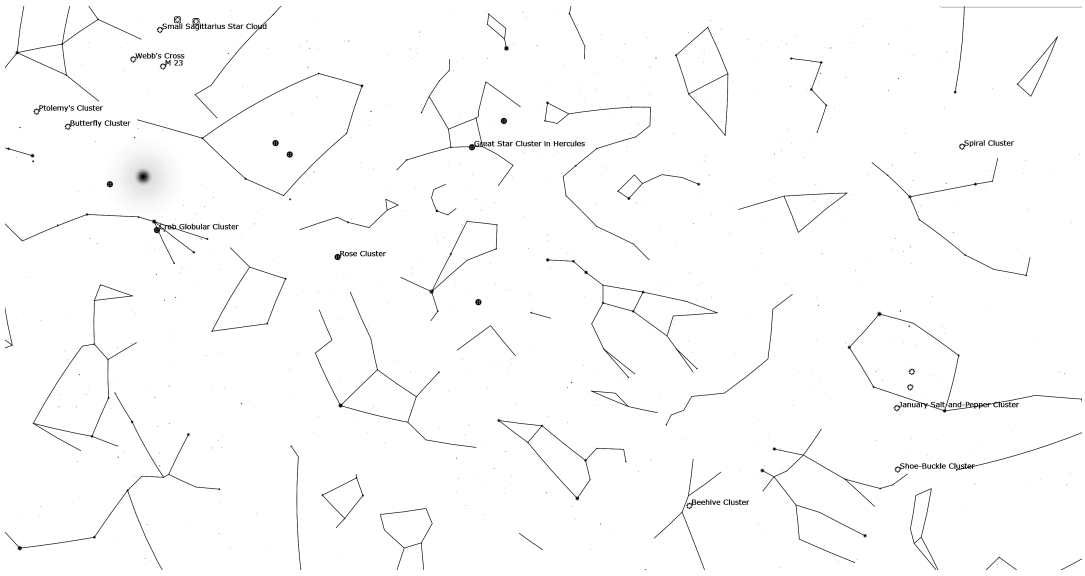
(B) Hercules Globular Cluster and (No Object Present)

(C) **Hercules Globular Cluster and Beehive Cluster**

(D) Sunflower Galaxy and (No Object Present)

(E) (No Object Present) and Beehive Cluster

Solution:



16. It is the June solstice and there is a binocular comet. As the comet is not particularly bright, it can only be visibly observed when the Sun has an altitude below $-12°$ (i.e. after astronomical twilight begins). Coincidentally, the comet has the same right ascension as the Sun on this date, but has a declination of $+55°$. Define the best location as the place where the comet can be visibly observed above the horizon for the largest amount of time in a 24 hour period. Given the definition above, which of the following locations is the best place to observe this comet? Assume perfectly clear weather.

(A) Saint Petersburg (latitude 60.0°N)

(B) Copenhagen (latitude 55.7°N)

(C) Lisbon (latitude 38.7°N)

(D) The Equator

(E) Perth (latitude 32.0°S)

Solution:

Given that the comet and Sun share the same R.A., there are three constraints to consider:

- The comet must rise above the horizon
- AND the Sun must set before the comet does (or equivalently, rise after the comet does)
- AND the Sun must be sufficiently below the horizon at some point (-12 degrees) while the comet is up

The first constraint is not too helpful here as the comet rises in all locations listed. Thus, we turn to the other constraints. The second constraint is easy to check. The fact that the comet shares the same RA as the sun greatly simplifies calculation -spherical trigonometry is not needed at all. Consider the case of the Equator (option D): because the Sun has the same RA as the comet, they both rise and set at the same time, ruling D out as an option. For any location south of the equator (e.g., Perth), the Sun rises before (and sets after) the comet, violating the constraint. The third constraint rules out Options A/B. Note that for Copenhagen (option B), the Sun never attains an altitude below -12 degrees during the summer solstice, hence Copenhagen does not experience astronomical twilight on that day. The same reasoning applies with even more force to Saint Petersburg. You can also verify that option C satisfies this constraint at local midnight, leaving C as the answer.

17. Below are listed several telescopes.

I Hubble Space Telescope (191 nm, $D = 2.4$ m, f/24)

II James Webb Telescope (600 nm, $D = 6.5$ m, f/20)

III Parkes Radio Observatory (1 mm, $D = 18$ m, f/16)

IV X-Ray household telescopes (10 nm, $D = 10$ cm, f/12)

Which one of these telescopes can theoretically resolve Olympus Mons (21.9 km) anytime in their respective wavelengths? Assume circular orbits for the solar system planets.

(A) I only

(B) I and II only

(C) I, II and III only

(D) I, II, III and IV

(E) None of the telescopes

Solution:

It's most difficult to resolve objects when they're farthest. Hence, we take the distance between Earth and Mars during conjunction. Hence the minimum resolution required will be:

$\theta = \frac{21.9\times10^3}{(2.279+1.496)\times10^{11}} = 0.012"$.

Using Rayleigh's criterion, the resolution of the telescopes are:

I HST: 0.020"

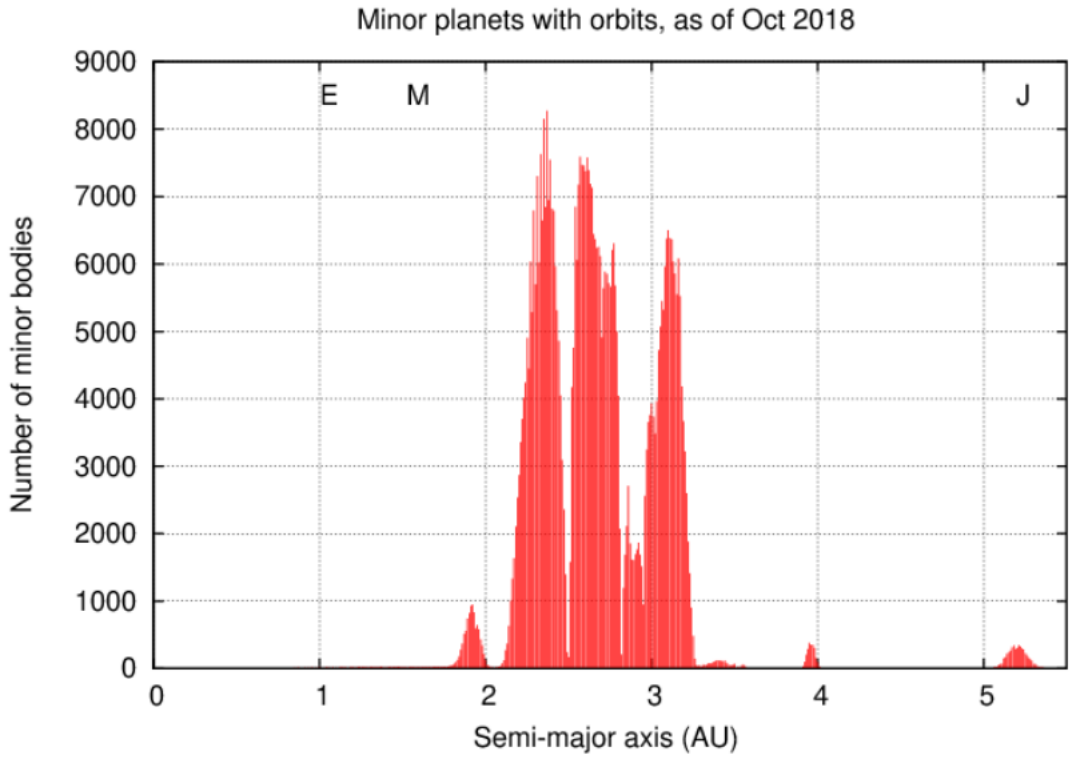
II James Webb Telescope: 0.023"

III Parkes Radio Observatory: 13"

IV X-Ray household telescopes: 0.025"

Therefore, none of the telescopes can resolve Olympus Mons at any time.

18. The figure below shows the distribution of minor bodies, mostly asteroids, with orbiting distance from the Sun. There are thin gaps in the distribution, particularly between a semi-major axis of 2 and 3 AU. Which of the following best explains the lack of asteroids at these distances?



(A) **Orbital resonance with Jupiter**

(B) These are the orbits of several dwarf planets.

(C) Orbits at these distances in a planetary system are inherently unstable.

(D) Gravitational influence from other asteroids

(E) They were remnant artefacts of the early formation of the Solar System, as described in the Condensation Model.

Solution:

The answer is the explanation.

19. Suppose the full moon is observed to lie near the Pleiades star cluster. One sidereal month later, which of the following statements is true for an observer at the same location? You may assume the moon is above the horizon at this time.

(A) **The moon is near the Pleiades, but is currently a waxing gibbous moon.**

(B) The moon is near the Pleiades, and is a full moon.

(C) The moon is near the Pleiades, but is currently a waning gibbous moon.

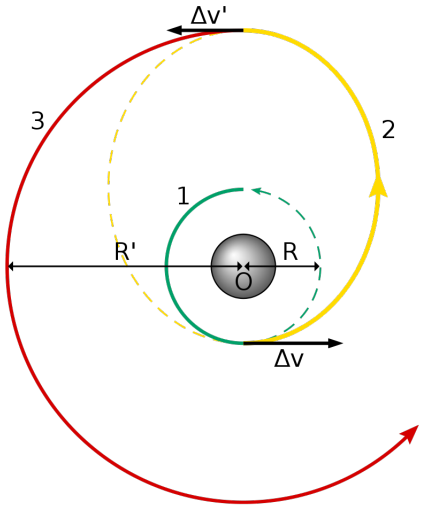
(D) The full moon has moved past the Pleiades. It is now closer to the feet of Gemini.

(E) The full moon has not reached the Pleiades. It is instead closer to Aries.

Solution:

This question tests one's understanding of the practical meaning of a sidereal and synodic month. After one sidereal month, the moon returns to the same place with respect to the stars, eliminating Options D/E. A sidereal month is however shorter than a synodic month (which is the time taken for the moon to display the same phase). Hence, the moon will not yet be full one sidereal month later, leaving the answer as A.

20. You learnt that the Hohmann transfer (yellow orbit shown below) is the most fuel-efficient transfer orbit sequence. With that in mind, you are interested to know how long it will take for you to reach Neptune using the Hohmann transfer. What is the shortest possible time it will take for you to reach Neptune? Choose the closest value.



(A) **11010 days**

(B) 11200 days

(C) 22020 days

(D) 22400 days

(E) 42020 days

Solution:

Use the formula for orbit eccentricity in terms of other parameters and the data of solar system objects in the formula book.

We first find $r_{periapsis}$ of both Neptune and Earth. For Neptune, it is 4.45256×10^{12} m. For Earth, it is 1.4710168×10^{11} m.

Then, we find the semi-major axis of the Hohmann Orbit:

$a_{Hohmann} = \frac{4.45256\times10^{12}+1.4710168\times10^{11}}{2} = 2.29983 \times 10^{12}$ m.

Next, note that the Sun is still at the focus of the Hohmann orbit. Using Kepler's Third Law, the period of the Hohmann orbit is 22020.625 days.

Finally, divide by 2 and we have the answer 11010 days.

21. To get to the outer solar system and beyond, spacecrafts use orbital slingshots to accelerate. These orbital slingshots work off the principle of conservation of momentum and are analogous to elastic collisions. A spacecraft comes in at an angle of 60 degrees to the planet's trajectory. The speed of the spacecraft is twice that of the planet's speed. If the spacecraft gained kinetic energy through orbital slingshot, what is the percentage increase in kinetic energy of the spacecraft?

(A) 100%

(B) 150%

(C) 175%

(D) **200%**

(E) 300%

Solution:

There are two possible scenarios for this question: the spacecraft approaching the planet in generally the same direction and in generally the opposite direction.

The mass of the planet is much larger compared to the mass of the satellite, so the equations for linear elastic collision reduces to:

$v_{\text{satellite}} \approx -u_{\text{satellite}} + 2u_{\text{planet}}$,

$v_{\text{planet}} \approx u_{\text{planet}}$.

According to above equations, for both scenarios, the magnitude of the component of the satellite's velocity perpendicular to the trajectory of the planet will not change. So, the only change in kinetic energy is due to the component of the satellite's velocity tangential to the trajectory of the planet.

For the first scenario, $v_{\text{satellite}} \cos 60° \approx -2u_{\text{planet}} \cos 60° + 2u_{\text{planet}} = u_{\text{planet}} = u_{\text{satellite}} \cos 60°$. So for the first scenario, the satellite will not gain kinetic energy.

For the second scenario, $v_{\text{satellite}} \cos 60° \approx 2u_{\text{planet}} \cos 60° + 2u_{\text{planet}} = 3u_{\text{planet}}$. Then, $(v_{\text{satellite}})^2 = 9(u_{\text{planet}})^2 + (2u_{\text{planet}} \sin 60°)^2 = 12(u_{\text{planet}})^2$. Therefore, the percentage increase in KE is $\frac{12-2^2}{2^2} = 200\%$.

22. Which of the following statements is true and can be accounted for by the Cosmological Principle?

(A) The observable universe is finite in size.

(B) Every observable galaxy is moving away from us.

(C) **The cosmic microwave background radiation is highly isotropic.**

(D) The distribution of the stars in a suitably dark night sky is roughly uniform.

(E) Galaxies exist in filaments separated by voids, in a structure also known as the Cosmic Web.

Solution:

Reference: https://astronomy.swin.edu.au/cosmos/l/Large-scale+Structure

The Cosmological Principle states that on a large enough scale, there should be no irregularities

to the distribution of matter that evolved from the homogeneous and isotropic distribution of matter in the primordial universe. The Cosmic Web is better explained by Inflation and the exponential expansion of the Universe. The distribution of stars in a sufficiently dark night sky appears to be uniform but are in fact not. There are always more stars in the direction of the Milky Way plane. Moreover, all observable stars are from the Milky Way galaxy, which is too small of a scale to be explained by the Cosmological Principle.

23. Jimmy is a budding astrophotographer. Last night, he tried to take photos of the Andromeda Galaxy. He set up his apochromatic refractor with a DSLR camera attached on his motorised alt-azimuth, before polar aligning his set-up. He then chose the settings for his set-up, such that his mount would track the galaxy over the night, and his camera would snap hour-long exposure shots of the galaxy. Unfortunately, the photos came out blurry. What is the most likely cause for this?

(A) Astigmatism

(B) Chromatic Aberration

(C) Coma

(D) Field Rotation

(E) Light Pollution

Solution:

Options A and C are wrong because a refractor is less likely to face these problems, compared to say, a reflector.

Option B is wrong because it's stated that Jimmy's refractor is apochromatic.

Option D is the answer as the alt-azimuth mount is not able to eliminate field rotation. The galaxy would rotate about 15° in an hour, which would be enough to smudge its photos.

Option E will whitewash the photos but won't exactly make them blurry.

24. What is the minimum energy required to launch a geostationary satellite with a mass of 1000 kg from Singapore?

(A) 4.59×10^9 J

(B) 4.70×10^9 J

(C) 4.85×10^{10} J

(D) 5.77×10^{10} J

(E) None of the above.

Solution:

Let total energy of satellite stationary with respect to the ground at the surface of the Earth be E_s and the total energy of the satellite at the geostationary orbit be E_g.

$E_s = \frac{1}{2}mv_s^2 - \frac{GMm}{r_s}$,

$E_g = -\frac{GMm}{2r_g}$,

where
$m = 1000$,
$M = 5.972 \times 10^{24}$ is the mass of the Earth,
$v_s = \omega r_s$,
$r_s = 6.37 \times 10^6$ is the radius of the Earth,
$\omega = \frac{2\pi}{24\times60\times60}$ is the angular velocity of the Earth's rotation, and
$r_g = (\frac{GM}{\omega^2})^{\frac{1}{3}}$ is the radius of the geostationary orbit, derived from equating the centripetal force to the gravitational attraction.

With these, we can solve for the total energy at the two locations. The energy required to launch the satellite is their difference: $E_g - E_s = 5.57 \times 10^{10}$ J.

25. Which of the following statements about the operation of astronomical telescopes is correct?

(A) **Most infrared telescopes are space-based due to the strong absorption by molecules in the air blocking the desired signals.**

(B) The reason the Lagrangian point L2 is used by many space-based telescopes is to use the Earth as shade from the Sun's radiation.

(C) The reason radio telescopes are generally deployed in a large array is to increase the effective aperture size of the telescope. Each small telescope in the array will take a small portion of the sky. The astronomers will then stitch the results together to form a full photo.

(D) Ground based telescopes that use adaptive optics can obtain images with quality comparable to space-based telescope of the same wavelength range.

(E) X-ray telescopes have a different design than typical optical telescopes because the optical design will result in a telescope aperture that is too small.

Solution:

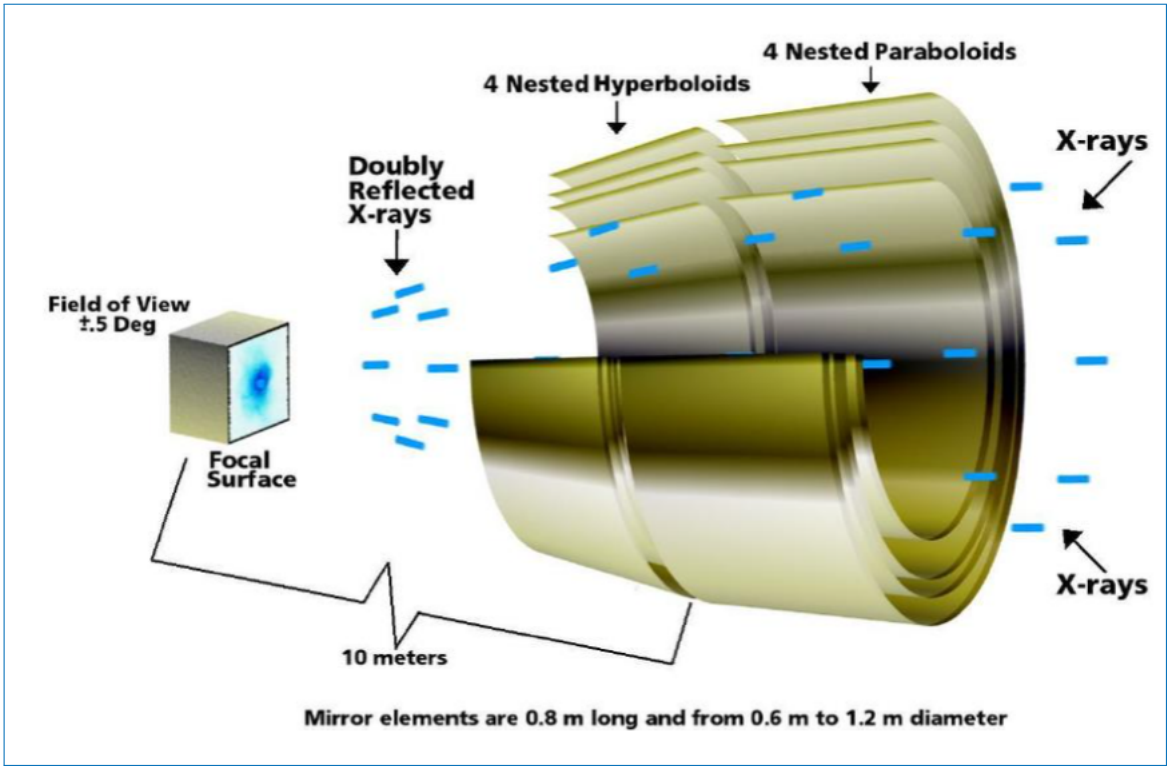
Option A is correct because most organic molecules absorbs in the infrared, thus making observing in the infrared difficult due to confounding absorption by organic molecules in the atmosphere like water and carbon dioxide. Although there are certain frequency in the near infrared that can be used for observation on the ground in the waterhole region.

Option B is not the case. Telescopes still need to have their own sun shield at L2 to avoid being overheated. The Earth will be too small in angular size to completely eclipse the Sun. Furthermore, it will not be in the correct location to block out the sun for most of the orbit there (due to the need to follow a Lissajous orbit).

Option C is wrong because a radio telescope array does not piece together individual images to form an complete image. Instead, they use interferometry to generate the final image.

Option D is wrong because adaptive optics can only correct for a very small portion of the field of view. Space telescopes do not suffer from this limitation and thus generally have better resolution across the whole image than ground based telescopes of similar aperture.

Option E is wrong. X-ray telescopes have a different design because X-rays can penetrate most materials used to make reflecting lenses. X-rays only reflect off surfaces at a large angle of incidence (glancing incidence). Thus, the telescope is designed to make use of this fact. Below is a schematic for a X-ray telescope.



26. The following statements describe the life stages of certain stars. Which of the following statements (arranged in chronological order) describe the major life stages of a solar mass star?

I Helium starts to run out in the core of the star.

II The star expels its outer layers, forming a planetary nebula with a white dwarf in its center.

III Hydrogen fusion begins and the star lies on the main sequence.

IV Carbon fusion begins, creating a carbon star.

V The core of the star undergoes a helium flash, commencing helium fusion.

VI A nebula begins to contract under its own gravity, forming a protostar at its center.

VII Hydrogen in the core begins to run out, and the star evolves into a red giant.

(A) III, VI, V, I, II

(B) III, VI, VII, IV, II

(C) VI, III, VII, V, I, IV, II

(D) VI, III, VII, I, II,IV

(E) VI, III, VII, V, I, II

Solution:

The sun is not massive enough to undergo carbon fusion, so the correct answer should not have step IV. This leaves options A/E. A is wrong because option VI should occur first, leaving E as the right answer.

27. The rules of the AC calendar is the following:

Rule 1: The first day of each month falls on the day where there will be a new moon.

Rule 2: The second month must contain the solar term the vernal equinox.

Rule 3: Each ordinary year in this calendar has 12 lunar months. A leap year would involve adding a thirteenth month to the year to ensure that rules 1 and 2 are always true.

Given that the vernal equinox fell on 20 March 2022 at 1500 hours, the new moon of the second month of the year AC2022 was 3 March 2022 at 0130 hours, which is the first upcoming AC year which would be a leap year?

(A) **AC2023**

(B) AC2024

(C) AC2025

(D) AC2026

(E) AC2027

Solution:

For each Georgian normal year the equinox moves earlier by roughly 6 hours. Using that, you can assume that all vernal equinoxes are on 20 March.

Each normal AC calendar year lasts $29.53 \times 12 = 354.36$ days. This corresponds there to the new moon of the second month moving forward by 10 days 21 hours each year.

In AC2023, the new moon of the second month will lie 11 days earlier. On 20 Feb 2023, vernal equinox will be less than 29.53 days away. In AC2024, the new moon of the second month will occur on 9 Feb 2024, the vernal equinox will be more than 29.53 days away. So, AC2023 has to have a leap month.

28. Blackholes are given their name as their gravitational pull is so strong that even light cannot escape from within. Which of the following is not a way whereby blackholes can be identified?

(A) Accretion of neighbouring stellar materials

(B) Gravitational waves

(C) Type II supernova

(D) Gravitational influence on nearby stars

(E) **Hawking radiation**

Solution:

Gravitational waves are emitted when black holes merge.

Black holes are often identifiable by their accretion disks, which emits in the X-ray.

The luminosity of a Type II Supernova can indicate the Star's mass, allowing us to deduce that a black hole will be formed when a Star of suitable mass collapses (>25 solar masses).

A black hole can often be identified by the orbit of stars around it and its compactness.

Hawking Radiation from blackholes are too faint to be detected and is inversely proportional to the square root of the black hole's mass, making them almost impossible to observe.

29. Which of the following is true about dark and bias frames in astrophotography?

(A) A bias frame is often longer exposure than a dark frame.

(B) **A bias frame is no longer necessary once a dark frame is used but not the reverse.**

(C) The former is taken with the camera shutter closed while the latter with it opened.

(D) One is temperature dependent while the other is not.

(E) Both are meant to remove shot noise from the image.

Solution:

A bias frame is taken with the shortest exposure possible to detect dark fixed-pattern noise, arising from variations in manufacturing of the image sensor. The only way to remove Shot-noise (arising from the randomness of electrons produced by the sensor per unit time) is to stack images, subtracting dark and bias frames only remove offset noise, even the dark and bias frames themselves have to be stacked to remove shot noise. Both are temperature dependent due to thermally generated electrons in the image sensor and with the camera shutter closed.

References:

http://slittlefair.staff.shef.ac.uk/teaching/phy217/lectures/instruments/L14/index.html

https://skyandtelescope.org/astronomy-blogs/imaging-foundations-richard-wright/dark-frames-and-bias-frames-demystified/

30. Which of the following would most likely bring about the greatest increase in the average surface temperature of a rocky planet orbiting a single star? You may assume that the planet is sufficiently far from its parent star.

(A) Doubling its diameter

(B) Having a parent star of spectral class A instead of B

(C) **Halving its orbit radius around its star**

(D) Decreasing its albedo from 0.85 to 0.5

(E) Increasing the surface temperature of the star from 5000 K to 7000 K

Solution:

A Doubling the diameter will quadruple the surface area receiving starlight, but will also quadruple the rate of irradiated heat, negating its effect.

B Spectral class A corresponds to a lower surface temperature than spectral class B. The temperature of the planet will decrease.

C By the inverse square law, halving the orbit radius around the star would increase the value of T^4 by a factor of 4. Hence, temperature would increase by a factor of $\sqrt{2}$.

D With a reduced albedo, the proportion of heat being absorbed increases from 0.15 to 0.5, so the value of T^4 increases by a factor of about 3.33. Hence, temperature would increase by a factor of about 1.35.

E Increasing the surface temperature of the star by a factor of 1.4 will bring about a proportional change in surface temperature of the planet.

Therefore, the answer is Option C.

31. On Oct. 19, 2017, Robert Weryck, a researcher at the University of Hawaii, discovered a strange object with unknown origins, which Astronomers later named Oumuamua. Oumuamua had an orbit around the Sun with eccentricity 1.20 and had a non-gravitational acceleration. There was also no observable out-gassing. Below are some statements about Oumuamua:

I An orbital eccentricity of 1.20 suggests that Oumuamua is a comet from the Oort Cloud.

II An orbital eccentricity of 1.20 suggests that Oumuamua is an interstellar object.

III Non-gravitational acceleration with no observable out-gassing suggest that Oumuamua contains water ice.

IV Non-gravitational acceleration with no observable out-gassing suggest that Oumuamua contain hydrogen ice or nitrogen ice, but not water ice.

V It is more likely for Oumuamua to contain nitrogen ice than hydrogen ice because if it had hydrogen ice, they would have evaporated on its journey to the solar system.

VI It is more likely for Oumuamua to contain nitrogen ice than hydrogen ice because nitrogen is more abundant in its parent stellar system.

Which statement(s) is/are well-reasoned and supported by observation, forming a plausible theory about Oumuamua?

(A) I and III only

(B) I, IV and V only

(C) II and III only

(D) **II, IV and V only**

(E) II, IV and VI only

Solution:

A comet is defined to undergo observable out-gassing. Therefore, all options containing statement I are wrong.

An orbital eccentricity of 1.20 strongly suggests a hyperbolic trajectory originating from interstellar space. So, statement II is correct.

Out-gassing of nitrogen and hydrogen will not be detectable with current technology, while water is detectable. Therefore, statement IV is correct but not III.

The origins of Oumuamua is unknown, so statement VI is poorly reasoned. In contrast, statement V is a plausible argument.

Therefore, Option D is the answer.

32. Which of the following statements is generally **false** about globular and open clusters?

(A) Open clusters have irregular shapes.

(B) Globular clusters are older than open clusters.

(C) Open clusters normally appear in the disk of the galaxy.

(D) Globular clusters contain more stars than open clusters.

(E) None of the above

Solution:

A, B, C and D are all generally true facts about open and globular clusters.

33. Many ancient monuments were built to align to equinoxes, solstices, or stars. Which of the following alignments would still be accurate today?

I Stonehenge was built in 3000 BC such that during the summer solstice, the sun rises directly above the Heel Stone.

II The Mnajdra temples on Malta consist of three structures built over a period of a thousand years beginning around 3600 BC. It was built such that the spring equinox sunrise bisects the entrance to the Lower Temple, shining light through the main passageway and into a small shrine.

III Great Pyramids of Sneferu, Khufu and Khafre built about 4700 years ago was hypothesised to be built such that they directly pointed to the star Thuban.

(A) I only

(B) III only

(C) **I and II only**

(D) I, II and III

(E) None of the alignments

Solution:

Monuments based on equinoxes or solstices would still be accurate as the relative position of the Sun and the earth is preserved by definition. Monuments based on the stars would no longer be accurate due to precession.

34. Two deep sky objects are described below. Hence or otherwise, identify both deep sky objects.

Object X	With an integrated apparent magnitude of 2.5, this cluster is bright and easily seen in Singapore. The cluster lies deep within the southern sky, and is relatively easy to find due to its proximity to the False Cross. However, when viewed in a telescope, the cluster is seen to have little central condensation and has relatively few stars. For this reason, Object X is best viewed through binoculars.
Object Y	Object Y has an integrated apparent magnitude of 3.8. Theoretically, the cluster should be visible through binoculars due to its brightness. However, even if you centred your binoculars at its dominant central star, you would never notice the cluster. This is because the cluster is extremely centrally condensed, packed into a region that is merely 6 arc-minutes across! The brilliance of the dominant central star plus the cluster's small size means that the members of the cluster can be hard to observe without the use of averted vision, even with a telescope. This interesting object is also relatively easy to find, being only 3 degrees northeast of the bright star Wezen.

(A) Object X: Jewel Box (C94)
Object Y: Hyades (C41)

(B) Object X: Omicron Velorum Cluster (C85)
Object Y: Tau Canis Majoris Cluster (C64)

(C) Object X: Southern Beehive Cluster (C96)
Object Y: Beehive Cluster (M44)

(D) Object X: Southern Pleiades (C102)
Object Y: Alpha Persei Cluster

(E) Object X: Jewel Box (C94)
Object Y: Alpha Persei Cluster

Solution:

Of all the possible choices for Object X, only two are close to the False Cross (Omicron Velorum and the Southern Beehive). This leaves B/C.

Similarly, of all the possible choices for Object Y, only the Tau Canis Majoris cluster displays central condensation – the other clusters are in fact rather large and lacking central condensation. Further, the other clusters are all easily seen in binoculars, leaving Bas the right answer.

35. An impostor is ejected at a speed of $0.1c$ towards the Earth relative to an interstellar spacecraft. The spacecraft travels away from Earth relative to an observer on Earth at $0.5c$. If the impostor's rest mass is 20 kg, determine his kinetic energy in the frame of the observer. (Hint: γmc^2 gives the total energy of an object in motion).

(A) 1.84×10^{16} J

(B) 1.48×10^{17} J

(C) 3.76×10^{18} J

(D) 2.67×10^{19} J

(E) None of the above

Solution:

Using the velocity addition formula, we find that the speed of the imposter relative to the observer is $\frac{0.5-0.1}{1-0.5\times0.1} = 0.42105c$.

The imposter's kinetic energy is $(\gamma - 1)mc^2$, which solves to be 1.84×10^{17} J.

36. The Star of Bethlehem said to have guided the Magi westward to Bethlehem (31.71° N, 35.20° E) during Christmas midnight (25 December). Which one of these stars is the most possible contender for the star? Note that the star must at least **NOT** appear in the eastern half of Bethlehem's sky to be pointing westward. You shall ignore the effect of precession.

(A) Acrux

(B) Aldebaran

(C) Alioth

(D) Altair

(E) Arcturus

Solution:

Firstly, due to being situated near the south celestial pole, Acrux (roughly 63° S) never rises in Bethlehem and cannot be a contender.

Next, we calculate that the local sidereal time on Christmas midnight to be 6 h 10.5 min. Remember that local sidereal time is 0 at the midnight of the autumn equinox.

This value tells us that the point of Aries (which is in Pisces) was almost precisely pointing to the west horizon. Hence, stars with right ascension between 0 h and 6 h will likely appear at the "west".

This eliminates Alioth (roughly 13 h), Arcturus (roughly 14 h), and Altair (roughly 20 h); thus, leaving Aldebaran (4 h 35 min, 16.5° N) as the sole contender.

37. What is one possible reason why Johannes Kepler realised that the orbits of planets are not circular, but elliptical?

(A) He was not convinced by the then popular mystical belief that the circle is the Universe's perfect shape.

(B) He observed that Uranus appeared to suddenly reverse course.

(C) He realised that the radial line drawn from the Sun to a planet sweeps out equal areas in equal intervals of time and the planet travelled faster near the Sun.

(D) He realised that the arc lengths travelled by a planet in equal intervals of time are equal.

(E) Trick question. The orbit of any planet is circular because the planet must be at a constant radius away from the Sun in order to maintain a stable orbit.

Solution:

A False. Kepler himself was religious, and had held the belief that the circle was the Universe's perfect shape, and that as a manifestation of Divine order, the planets' orbits must be circular.

B False. B In Kepler's time, people observed that Mars appeared to suddenly reverse course.

C True. This is Kepler's second law of planetary motion. Combining this with a non-uniform motion suggests an elliptical orbit.

D False. Should be area, not arc length.

E Not a trick question.

38. Given that on a certain day, the Sun rose over Edinburgh at 0734 Local Mean Solar Time. On the same day, which of the below locations will observe a sunrise at an earlier local solar time than Edinburgh. You are given that Edinburgh is located at 55 ° 57' N / 3 ° 12' W.

(A) Porto (41°9' N 8°37' W)

(B) Reykjavik (64°08' N 21°56' W)

(C) St Petersburg (59°56' N 30°18' E)

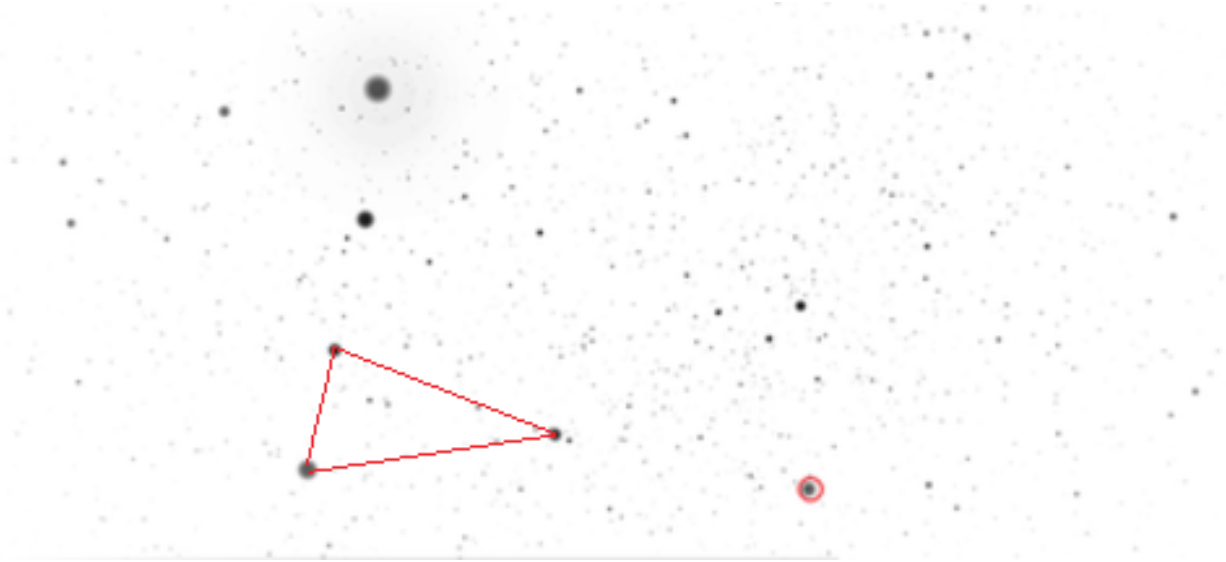
(D) None of the above

(E) Not enough information to determine an answer

Solution:

The current day is past the autumnal equinox because the sun rises in Edinburgh past 0700 local mean solar time. As such, the sun will rise at a later local solar time further in the north compared with the south. If the time past the vernal equinox, then the sun will rise at an earlier local solar time further in the north.

For Questions 39 and 40, please refer to the following figure.



39. Identify the object circled in red.

(A) Betelgeuse

(B) Fomalhaut

(C) Regulus

(D) **Vega**

(E) None of the above

Solution:

40. Identify the asterism formed by the red triangle.

(A) Spring Triangle

(B) Summer Triangle

(C) Autumn Triangle

(D) Winter Triangle

(E) None of the above

Solution:

As shown in the solution of the previous question, this is not an asterim.

41. There are three main processes by which elements heavier than iron are formed in stars: rapid neutron process (r-process), rapid proton process (rp-process) and slow neutron process (s-process). The figure below shows some of the possible elements one can form from one of these three processes. The black cells are stable nuclei.

Xe	123 2 h	124	125 17 h	126	127 36 d	128	129	130	131	132	133 5 d	134	135 9 h	136	137 4 min
I	122 4 min	123 13 h	124 4 d	125 59 d	126 13 d	127	128 25 min	129 10^7 y	130 12 h	131 8 d	132 2 h	133 21 h	134 52 min	135 7 h	136 84 s
Te	121 19 d	122	123	124	125	126	127 9 h	128	129 70 min	130	131 25 min	132 76 h	133 13 min	134 42 min	135 19 s
Sb	120 16 min	121	122 3d	123	124 60 d	125 3 y	126 12 d	127 4 d	128 9 h	129 4 h	130 40 min	131 23 min	132 3 min	133 3 min	134 0.8 s

Given the half-life in the figure are their beta decay half-life. Which of the following statement about the process to make the isotopes is **wrong**?

(A) Tellurium-130 cannot be formed through the s-process.

(B) Most Xenon-136 found is likely to be produced in the s-process.

(C) It is unlikely to have most of the Tellurium-135 produced through the rp-process.

(D) All elements in the figure can be made through the r-process during a supernova explosion.

(E) There is no s-process as the speed of proton inside the star is not enough to overcome the Coulombic repulsion of the nuclei and there is not enough proton flux in the core of the star to result in significant fusion of heavier nuclei.

Solution:

Since s-process involves slow moving neutron at small neutron flux (number of neutron incident on the atom per second), it is unlikely they are able to form most of the Xenon-136 as most of the neutron that enter Xenon-134 to form Xenon-135 would have decayed by the time another neutron could have enter the atom and Iodine-136 (which would decay to Xenon-136) is not

produced in large enough quantity in the s-process as well for a similar reason, it is however produced in large quantity in the r-process.

42. A main sequence star is observed and studied. The star's spectrum peaks at a wavelength of around 291 nm. The star has an apparent magnitude determined to be 0.87. Estimate the distance of the star from Earth, assuming that the radii of main sequence stars are directly proportional to their masses. (Hint: The Sun is also a main sequence star).

(A) 1.14 pc

(B) 1.69 pc

(C) **1.96 pc**

(D) 2.25 pc

(E) None of the above

Solution:

From the formulae book, for main sequence stars, $L \propto M^{\frac{7}{2}}$. Combining that with the hint, $R \propto L^{\frac{2}{7}}$. Rewriting Stephan-Boltzmann Law, $\frac{L}{R^2} \propto T^4$. Combining the last two relationships, for main sequence stars, $L \propto T^{\frac{7}{12}}$.

With Wien's displacement law, the temperature of the star is $\frac{2.8978\times10^{-3}}{291\times10^{-9}} = 9958.1$ K. Using the temperature of the Sun (the Sun is also a main sequence star) in the formulae book, $\log_{10}(\frac{L}{L_{Sol}}) = \frac{7}{12}\log_{10}(\frac{9958.1}{5778}) = 0.13790$. Using the formula for absolute magnitude, $M = -2.5 \times 0.13790 + 4.7554 = 4.4107$. Finally, using the formula for distance modulus, $0.87 - 4.4107 = 5\log_{10}(d/10)$. Solving the equation gives $d = 1.96$ pc.

43. Europa is one of the potential candidates for human colonization and terraforming in the future. In order to live on Europa long term, one needs to establish a time keeping system on Europa. One way to do so is to define the so called Io month and Ganymede month where each of the respective Galilean moon return to the same position in Europa's sky. What is the ratio between the time for an Io month to that of a Ganymede month assuming the three Galilean moons are in an exact 1:2:4 orbital resonance and all orbits of the moons are circular.

(A) $\frac{1}{2}$

(B) $(\frac{1}{4})^{\frac{2}{3}}$

(C) $\frac{1}{4}$

(D) $(\frac{1}{4})^{\frac{3}{2}}$

(E) Not enough information to determine an answer

Solution:

This question is asking for the ratio of the Synodic period of Io and Ganymede when viewed from

Europa.

For Io, since it is inferior to Europa, its Synodic period can be calculated using

$\frac{1}{S} = \frac{1}{P} - \frac{1}{E}$

where S is the synodic period of Io, P the sidereal Period of Io and E the sidereal period of Europa.

For Ganymede, since it is superior to Europa, its Synodic period can be calculated using

$\frac{1}{S} = \frac{1}{E} - \frac{1}{P}$

where S is the Synodic period of Ganymede$P$ the sidereal Period of Ganymede and E the sidereal period of Europa. Let us use the Europa month (aka sidereal period of Europa) as a reference unit. With this choice of unit, you can find that the Io month has the duration of 1 Europa month and the Ganymede month is 2 Europa months.

44. The Sun's analemma shows the apparent change in the Sun's position. It is captured on a fixed location on Earth at the same mean solar time throughout the year. In a composite photo, the sun's movement will form a somewhat asymmetrical 'number 8' shape. Below are some proposed contributing factors for the shape of the Sun's analemma:

I Eccentricity of the Earth's orbit around the Sun

II Difference between the Earth's synodical and sidereal day

III The Earth's axis tilt relative to the ecliptic plane

Which of them are valid contributing factors?

(A) I and II only

(B) **I and III only**

(C) II and III only

(D) I, II and III

(E) None of the above

Solution:

Eccentricity affects the analemma by producing the apparent motion along the 'height'of the shape, while the Earth's axis tilt serves as the cause for the perpendicular sinusoidal motion. The combination of these two motions will thus create a 'number 8' shape. Earth's synodical and sidereal day don't play a factor, as the Sun is captured at the same mean solar time.

45. When observing the Sun using a suitable solar filter, we observe that the edge of the Sun is slightly dimmer than the centre, a phenomenon known as limb darkening. What is the likely reason for this phenomenon?

(A) **The average temperature of the gas on the edge is cooler than at the center thus leading to a lower brightness.**

(B) The atmosphere of the Sun mixes with space causing a lower brightness.

(C) The Earth's atmosphere preferentially scatters light coming from the edge of the Sun due to its higher angle of incidence into the atmosphere.

(D) It is due to light diffracting through the setup causing the Sun to appear as a fuzzy disk with a dimmer edge.

(E) It is just an optical illusion and there is no physical mechanism.

Solution:

The answer is the explanation.

46. Isabel is setting up the telescope for a private viewing session with her friends. She wants to show them Saturn (currently at an altitude of 70°) and determines that a magnification of 300 times would be the ideal. Given that she owns a Schmidt-Cassegrain NexStar 6SE telescope with a focal length of 1500 mm, and that she has the following items with her:

Eyepiece: 4 mm, 10 mm, 12 mm, 26 mm and 32 mm

Barlow: 2× and 5×

1 Star Diagonal

What combination of the accessories should she use for the best viewing experience?

(A) 10 mm eyepiece, 2× barlow

(B) 12 mm eyepiece, 2× barlow

(C) 4 mm eyepiece, star diagonal

(D) 12 mm eyepiece, 2× barlow, star diagonal

(E) **26 mm eyepiece, 5× barlow, star diagonal**

Solution:

Saturn is at a high altitude. So, a star diagonal is necessary and we are left with options C/D/E. Option E has a magnification closest to 300×.

47. Tom and Jerry are bored and decide to count pulses from a very distant pulsar in the y direction. The pulsar has a frequency of 400 Hz in Tom's frame. Jerry decides to travel in the x direction at a speed of $\frac{12}{13}c$ for 50 seconds and back at the same speed for 50 seconds. How many pulses does Jerry count in the first and second half of his journey respectively? (Hint: $\frac{1}{f} = T$, where T is period.)

(A) 4000 and 4000

(B) 20000 and 20000

(C) **52000 and 52000**

(D) 100000 and 100000

(E) None of the above

Solution:

This is the Transverse Doppler effect. Let Jerry's frame be S and the frame of the source be S'. S' sees Jerry get hit by a flash every $T' = \frac{1}{f'}$ seconds in its frame. This follows from the fact that the photons received by Jerry are received at points essentially equidistant from the source so we can ignore longitudinal effects in the S' frame. S' also sees Jerry's clock running slow, so Jerry gets hit by a flash every $T = \frac{T'}{\gamma} = \frac{1}{\gamma f'}$ seconds on his clock. The frequency in Jerry's frame is thus $f = \gamma f'$ The number of pulses in both to and from is thus $50 \times f = 52000$.

Note: It is much easier to analyse things in the frame of the source as com- pared to Jerry's frame. In Jerry's frame, the photons received by him have a longitudinal component, which complicates the analysis.

48. A rocky satellite is on its way to being tidally locked to its parent planet. Which of the following would speed up the tidal-locking process?

(A) Decrease the diameter of while maintaining the density of the satellite

(B) Increase the mass of while maintaining the diameter of the satellite

(C) Increase the orbital radius of the satellite

(D) **Increase the mass of the planet**

(E) None of the above

Solution:

Although moment of inertia is smaller due to increased radius, this is offset by the reduction in net torque applied to the satellite. Therefore, Option A is not the answer.

With a heavier satellite, its moment of inertia is larger, and as such will take a longer time for the torque to bring the satellite into tidal locking. Therefore, Option B is not the answer.

Increasing the orbit radius will weaken the gravitational pull of the planet on the satellite, weakening the torque it applies and therefore slowing down the process. Therefore, Option C is not the answer.

With a stronger gravitational pull of the planet, the tidal forces exert a greater net torque on the satellite, resulting in faster tidal locking. Therefore, Option D is the answer.

49. Suppose that to a particular heliocentric 3D-Cartesian coordinate, the Earth is located at (1,0,0) A.U.. You tried to observe Nanyang Star (with the same luminosity as the Sun) from the Earth. Nanyang Star is located at (1,0,1) kpc in the same coordinate system. If you obtained a visual magnitude of +17.2 from the observation, what is the coefficient of absorption, defined as the visual magnitude increase per unit distance due to interstellar matter, in the star's direction? (It is given that Sun's absolute magnitude is +4.83.)

(A) $0.02\ \text{kpc}^{-1}$

(B) $0.17\ \text{kpc}^{-1}$

(C) **$1.13\ \text{kpc}^{-1}$**

(D) $1.55\ \text{kpc}^{-1}$

(E) $2.42\ \text{kpc}^{-1}$

Solution:

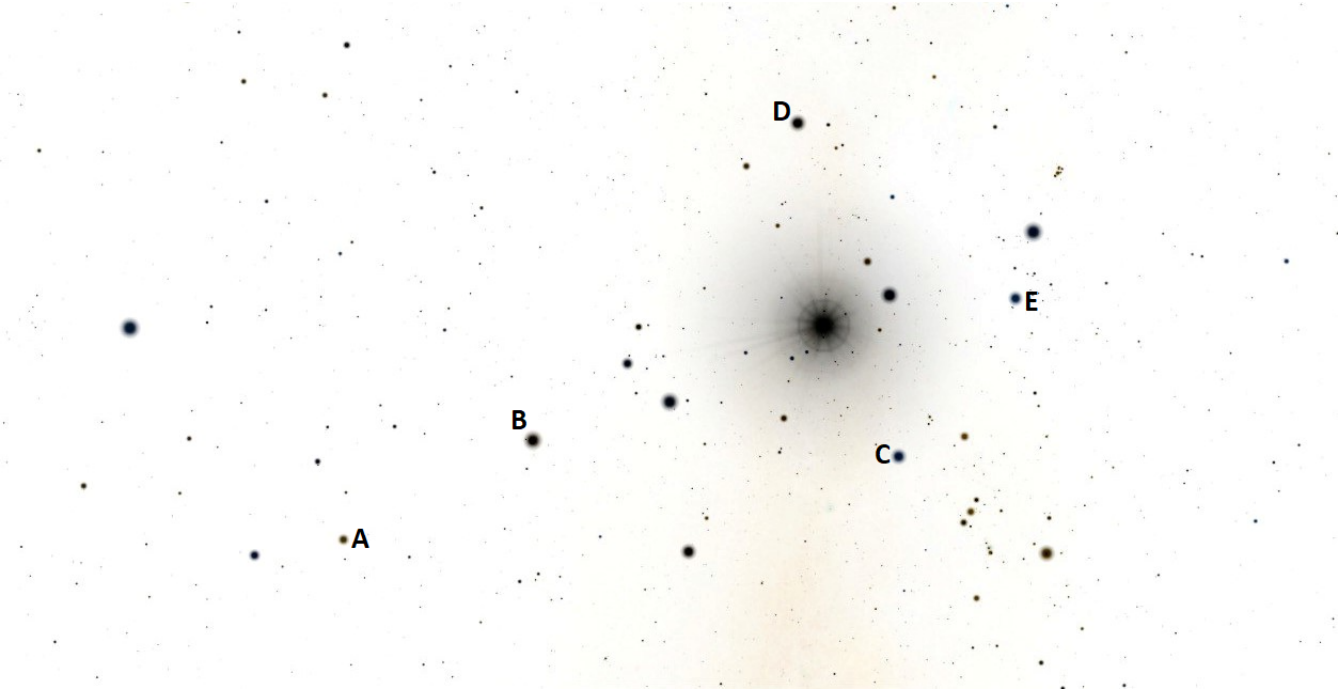
First, we will calculate the distance from the Earth to Nanyang Star by Pythagoras' theorem and using appropriate units. We will obtain 1.4142 kpc.

Then, as we know that the coefficient of absorption is of the unit kpc^{-1}, we can modify the equation for distance modulus:

$m - M = -5 + 5\log(d(\text{in pc})) + \alpha d(\text{in kpc})$, where α is the coefficient of absorption.

Finally, we solve for α, obtaining $1.13\ \text{kpc}^{-1}$.

50. It's the Year 2097 (don't ask why it's so specific), and humans (including you) have settled on our next-door neighbour, Mars. You decide to turn on your special visor that renders the ground invisible. Can you locate our original homeland? (Options are labeled in the diagram.)



(A) A

(B) **B**

(C) C

(D) D

(E) E

Solution:


Venus
Elnath
Mercury
Aldebaran
Sun
Pollux
Jupiter
Earth
Bellatrix
Betelgeuse
Alnilam
Regulus
Procyon
Mars, +90°00'00", +0°00'00"
FOV 60°
18.2 FPS
2097-05-30 09:00:00

 [End of paper]