Astrochallenge Mathematical Formula Booklet

INSTRUCTIONS

- This booklet consists of 3 printed pages, excluding this cover page.
- Do **NOT** make any markings on this booklet.
- Return this booklet to the invigilator at the end of this round of competition together with your answer script.

Differentiation

$$[(f(x) \pm g(x)]' = f'(x) \pm g'(x)$$

$$[k \cdot f(x)]' = k \cdot f'(x)$$

$$[f(x)g(x)]' = g(x) \cdot f'(x) + f(x) \cdot g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}, \qquad g(x) \neq 0$$

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

$$\frac{d}{dx}[e^x] = e^x$$

$$\frac{d}{dx}[a^x] = a^x \ln a, \quad a > 0$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}, \quad x > 0$$

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

Integration

$$\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C, \qquad n \neq -1$$

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

$$\int e^x\, dx = e^x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \cos x\, dx = \sin x + C$$

$$\int u(x) \cdot v'(x)\; dx = u(x) \cdot v(x) - \int v(x) \cdot u'(x)\; dx$$

Binonimal Expansion

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}, \qquad n, k \in \mathbb{Z}; \quad n, k \geqslant 0; \quad 0 \leqslant k \leqslant n$$

$$(a+b)^n = a^n + \binom{n}{1} \cdot a^{n-1}b + \binom{n}{2} \cdot a^{n-2}b^2 + \ldots + \binom{n}{k} \cdot a^{n-k}b^k + \ldots + b^n, \quad n \in \mathbb{N}$$

$$(1+x)^n = 1 + \frac{n}{1!} \cdot x + \frac{n(n-1)}{2!} \cdot x^2 + \ldots + \frac{n(n-1)\ldots(n-k)}{k!} \cdot x^k + \ldots, \qquad n \notin \mathbb{N}, \quad |x| < 1$$

Taylor expansion

$$f(x) = f(0) + f'(0) \cdot x + f''(0) \cdot \frac{x^2}{2!} + \ldots + f^{(n)} \frac{x^n}{n!} + \ldots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \frac{(-1)^n \cdot x^{2n+1}}{(2n+1)!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + \frac{(-1)^n \cdot x^{2n}}{(2n)!} + \ldots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + \frac{(-1)^{n+1} \cdot x^n}{n} + \ldots, \qquad -1 < x \leqslant 1$$

Series

$$a + ar + ar^2 + \ldots + ar^n = \frac{a(1 - r^{n+1})}{1 - r} \qquad r \neq 1$$

$$a + ar + ar^2 + \ldots + ar^n + \ldots = \frac{a}{1 - r}, \qquad |r| < 1$$

$$a + (a + b) + (a + 2b) + \ldots + (a + nb) = \frac{n+1}{2}(2a + nb)$$

Trigonometry

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$$

$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$

$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$$

$$\sin\theta + \sin\phi = 2\sin\left(\frac{\theta + \phi}{2}\right)\cos\left(\frac{\theta - \phi}{2}\right)$$

$$\sin\theta - \sin\phi = 2\cos\left(\frac{\theta + \phi}{2}\right)\sin\left(\frac{\theta - \phi}{2}\right)$$

$$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta + \phi}{2}\right)\cos\left(\frac{\theta - \phi}{2}\right)$$

$$\cos\theta - \cos\phi = -2\sin\left(\frac{\theta + \phi}{2}\right)\sin\left(\frac{\theta - \phi}{2}\right)$$

Double angle formula

$$\begin{aligned}\sin(2\theta) &= 2\sin\theta\cos\theta \\ \cos(2\theta) &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta \\ &= \cos^2\theta - \sin^2\theta \\ \tan(2\theta) &= \frac{2\tan\theta}{1-\tan^2\theta}\end{aligned}$$

R-formula

$$\begin{aligned}a\sin\theta \pm b\cos\theta &= R\sin(\theta \pm \alpha), \\ a\cos\theta \pm b\sin\theta &= R\cos(\theta \mp \alpha),\end{aligned}$$

where $a, b > 0$; $R = \sqrt{a^2 + b^2}$; $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$ is acute

Laws of sines and cosines

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Spherical trigonometry

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

$$\cos c = \cos a\cos b + \sin a\sin b\cos C$$

Dot product

$$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\vec{a}||\vec{b}|\cos\theta$$

$$(\vec{a}+\vec{b})\cdot\vec{c} = \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c}$$

$$|\vec{a}| = \sqrt{\vec{a}\cdot\vec{a}}$$

Cross product

$$\begin{aligned}\vec{a}\times\vec{b} &= |\vec{a}||\vec{b}|(\sin\theta)\hat{\mathbf{n}} \\ |\vec{a}\times\vec{b}| &= |\vec{a}||\vec{b}||\sin\theta| \\ \vec{a}\times\vec{b} &= -\vec{b}\times\vec{a} \\ (\vec{a}\times\vec{b})\cdot\vec{c} &= (\vec{b}\times\vec{c})\cdot\vec{a} \\ &= (\vec{c}\times\vec{a})\cdot\vec{b} \\ (\vec{a}+\vec{b})\times\vec{c} &= \vec{a}\times\vec{c} + \vec{b}\times\vec{c} \\ \hat{\mathbf{i}}\times\hat{\mathbf{j}} &= \hat{\mathbf{k}} \\ \vec{a}\times\vec{b} &= \begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3\end{vmatrix}\end{aligned}$$