

1. Identify which of the following types of Supernovae is fictitious.

- A. Type 1 Supernova
- B. Type 2 Supernova
- C. Pair-instability Supernova
- D. **Meganova**
- E. All of the above are real categories of Supernovae

2. One night at an unknown location, the Moon was observed to be rising over the horizon. An observer reads off her broken watch and found that the time on her watch right then was 22:50. The phase of the Moon is a...

- A. Waxing crescent
- B. Waxing gibbous
- C. Waning crescent
- D. Waning gibbous
- E. **Insufficient information to answer the question**

Solution:

First of all, the question reads, "broken watch". If that wasn't a clear indication of the answer, the next hint is that the local time doesn't necessarily correspond to the local solar time, just as in Singapore in which for economic purposes, we are GMT+8 whereas geographically we should be GMT+7.

3. Several statements are given about the Trojan Asteroid:

- I. Jupiter's Trojan Asteroids are located approximately at the same distance from the Jupiter as compared to its distance to the Sun
- II. There are 3 different major classes of asteroid: metallic, carbonaceous and silicaceous; M-type, C-type and S-type respectively.
- III. Trojan Asteroids can orbit in the L4 and L5 of any planet because a small mass orbiting in L4 or L5 of two astronomical objects are always stable (L stands for Lagrangian point)

Which of the above statement(s) is/are true?

- A. Option I only
- B. Option II only
- C. **Options I and II**
- D. Options I, II and III
- E. None of the above statements is true

Solution:

First and second fact is true. The third one is incorrect. While the only stable Lagrangian point is L4 and L5, they are not always stable. For the orbit in L4 and L5 to be stable, the bigger parent mass should exceed the smaller parent mass by a certain amount. Below a certain ratio, the orbit will eventually be unstable.

4. A planet orbits an unknown star with 2 times the mass of Sun. When it is at periapsis, it receives a flux of two times greater than the flux it receives at apoapsis. The orbit has semi minor axis of the length 1.3 AU. An asteroid with half the mass of the planet hits the planet when it is at the apoapsis and sticks to it. What is the minimum velocity of the asteroid so that the planet will escape the gravitational field of the star in sufficiently long time after the collision?

- A. 58 km/s
- B. 64 km/s
- C. 70 km/s
- D. 76 km/s
- E. 82 km/s

Solution:

Given that it at periapsis, it receives twice as much flux at apoapsis, the aphelion is $\sqrt{2}$ perihelion. Using the formula in the formula booklet for eccentricity would give us 0.17 for eccentricity. Further work, using the semi-minor axis value, would find us the aphelion of planet to be 1.32AU away from the parent star. Or approximately 1.10 AU for the perihelion.

$$b = \sqrt{1 - \epsilon^2} a$$

which hence would leave us with $a = 1.54 \text{ AU}$,

$$G = 6.67 * 10^{-11};$$

$$M = 2 * 2 * 10^{30}$$

$$\epsilon = 0.17$$

$$v_{apo}[a] = \sqrt{GM \left(\frac{2}{(1 + \epsilon)a} - \frac{1}{a} \right)}$$

$$v_{exc,apo}[a] = \sqrt{\frac{2GM}{(1 + \epsilon)a}}$$

$$v_{min}[v_{apo}, v_{exc,apo}] = \text{Solve} \left[0.5v_{min} + v_{apo} == \frac{3}{2}v_{exc,apo}, v_{min} \right]$$

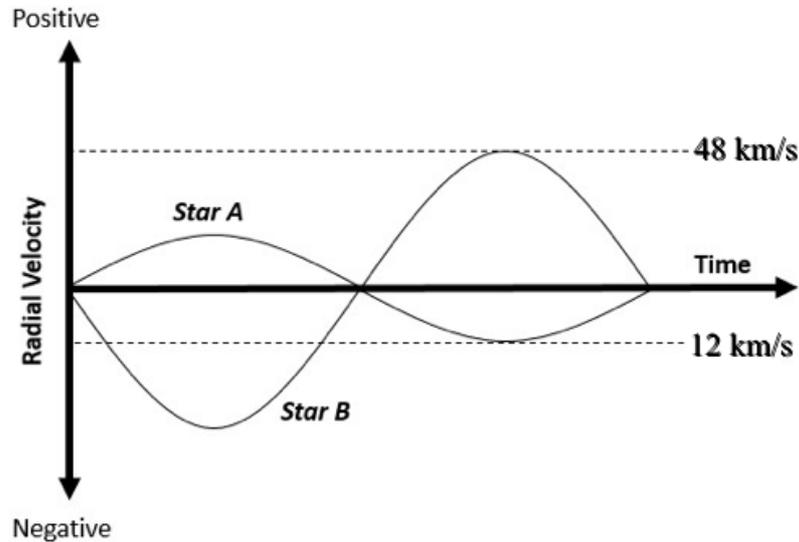
5. Moon whose angular diameter is 30 arc minutes has its image captured by a telescope with focal length equals to 5400 mm. If a CCD (*Charged Couple Device*) camera with dimension 512x512 pixels (12 microns/pixel) is used to do this imaging, what is the percentage of moon image can be covered by CCD's field of view, given that its illumination is 50%? The equation to calculate pixel size is given as follows:

$$focal_length (mm) = \frac{object_size [pixels] \times pixel_size [microns] \times 206.3}{object_size [arcsecs]}$$

- A. 0.69 %

- B. 1.09 %
- C. 1.38 %
- D. 2.17 %
- E. 3.51 %

6. A visual binary star system is known to have orbital period of 2 years with both stars separated for 4 AU.



Based on the graph above, what is mass of each star?

- A. $M_a = 9.6 M_{\text{sun}}, M_b = 6.4 M_{\text{sun}}$
- B. $M_a = 10.7 M_{\text{sun}}, M_b = 5.3 M_{\text{sun}}$
- C. $M_a = 12.0 M_{\text{sun}}, M_b = 4.0 M_{\text{sun}}$
- D. $M_a = 12.8 M_{\text{sun}}, M_b = 3.2 M_{\text{sun}}$
- E. Insufficient information for calculations

Solution:

The ratio of the velocities is 1:4 for Star A to Star B. That by itself would give us D. Question is, rather, why. Let $r = r_A + r_B = 4 \text{ AU}$, where r is the distance between the two stars and r_i is the radius of orbit about centre of mass.

$$\frac{GM_A M_B}{r^2} = M_A r_A \omega^2 = M_B r_B \omega^2$$

$$M_A r_A = M_B r_B$$

After some manipulation, we would get that

$$M_A + M_B = 16M_{\odot}$$

That would eliminate A.

Since, $v_A = r_A \omega$ and $v_B = r_B \omega$,

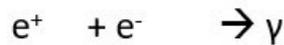
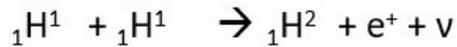
And their period of orbit must be the same and given to be the same, 2 years, we find

$$\frac{r_A}{r_B} = 4$$

And hence,

$$\frac{M_A}{M_B} = 4$$

7. In main sequence evolution phase, a Sun-like star has dominant energy source originating from proton-proton (pp) chain nuclear reaction. Below is the complete reaction for pp-chain reaction:



Legend:

${}_1\text{H}^1$ → Hydrogen

e^+ → Positron

ν → Neutrino

e^- → Electron

γ → Gamma Ray

${}_1\text{H}^2$ → Deuterium

${}_2\text{He}^3$ → Helium-3

${}_2\text{He}^4$ → Helium-4

$$M_{\text{H}} = 1.00794 \text{ a.m.u}^*$$

$$M_{\text{He}} = 4.002602 \text{ a.m.u}^*$$

$$*1 \text{ atom mass unit} = 1.66 \times 10^{-27} \text{ kg}$$

How long will the star shine for each kilogram of hydrogen mass conversion?

- A. 1.70 nanoseconds
- B. 3.38 nanoseconds
- C. 6.76 nanoseconds
- D. 115.70 nanoseconds
- E. 174.52 nanoseconds

Solution:

Find the change in rest mass energy of the whole p-p chain, then using the average luminosity of the sun, you may find the value required. Details not provided.

8. You want to observe a galaxy with apparent magnitude +7.86 and an apparent size of 40.85 arcmin. Your telescope has a focal length of 950mm. Which eyepiece should you use?

- A. 5mm, 80° AFOV eyepiece
- B. 12mm, 70° AFOV eyepiece
- C. 25mm, 80° AFOV eyepiece
- D. 32mm, 70° AFOV eyepiece
- E. 40mm, 80° AFOV eyepiece

Solution:

aFOV and tFOV are related by the magnification of the telescope-eyepiece combination. Find the appropriate eyepiece that would give tFOV larger than 40.85'.

9. What is the minimum aperture of a telescope required to resolve Albireo into its two component stars?

Given:

Angular separation: 0.4 arcsec,

wavelength of blue light: 475 nm, wavelength of red light: 720 nm

- A. 450 mm
- B. 400 mm
- C. 350 mm
- D. 300 mm
- E. 250 mm

Solution:

Use 475nm and use the Rayleigh Criterion using the appropriate units.
Remember to change degree to radians for the approximation.

10. There are many methods currently in use to detect exoplanets as well as their associated limitations. The radial velocity method attempts to detect planets around a host star by measuring “wobbles” around the host star.

Which of the statements below correctly describe the limitations of the radial velocity method?

- I. False signals may be caused by stellar activities
- II. It typically cannot determine the exact mass of any planets found.
- III. High sensitivity is required, limiting the effective range of the method
- IV. The radial velocity method is not able to provide any other information other than the presence of an exoplanet.

- A. Options I and II
- B. Options II, III and IV
- C. Options I, II and III
- D. Options I, II and IV
- E. Options I, II, III and IV

11. During which time period after the Big Bang was the Cosmic Microwave Background Radiation emitted?

- A. Photon epoch
- B. Matter-Dominated era
- C. Dark Age
- D. Recombination
- E. Re-ionisation

12. Nearly the same face of the moon points towards the Earth all the time. The period of time between successive full moons are 29.53 days. Hence, according to an inertial reference frame, the moon as seen from the northern celestial pole is
- Rotating clockwise with a period of 27.32 days
 - Rotating counter-clockwise with a period of 27.32 days
 - Rotating clockwise with a period of 29.53 days
 - Rotating counter-clockwise with a period of 29.53 days
 - Not rotating

Solution:

Rises in the east, sets in the west gives counter-clockwise rotation as seen from us. Then, the 29.53 days is sidereal period, while 27.32 is the synodic period.

13. In the early 2018, the phases of the moon will be as follows:

25 January	First Quarter
31 January	Full Moon
7 February	Last Quarter
16 February	New Moon

Suppose that an astronaut landed on the moon on 11 February, the phase of Earth as seen by him is a

- Waxing crescent
- Waning crescent
- Waxing gibbous
- Waning gibbous
- Cannot be determined

Solution:

Drawing out would help.

14. The ISS orbits at a relatively low orbit around the Earth. According to Newton's law, gravity at orbital altitudes should be nearly identical to that on the ground. However, astronauts on board float in a microgravity (effectively zero-g) environment. How is this so? Why aren't aeroplanes, which are also circling the Earth, not subject to the same effect?
- At high velocities, Einstein's theory of relativity show that the effects of gravity can be cancelled. However, planes do not travel fast enough to cancel the gravitational force

- B. The ISS is essentially in freefall as it orbits the Earth; effectively creating a local microgravity environment. Planes do not experience zero-g environments as they do not travel at orbital velocities
- C. The effects of gravity decrease exponentially with altitude, eventually reaching negligible levels at the orbital altitude of space stations
- D. Gravitational fields require a medium for transfer, and the vacuum of space prevents astronauts from feeling the effects of gravity
- E. At the position where the ISS orbits the Earth, gravity from the Earth is counterbalanced by the moon's gravitational force, creating zero resultant forces

15. On the night of 3-4 November 1971, the Moon, about one and a half days after full phase, passed through the Pleiades cluster. The right ascension of the Sun was 14h 30m at that time. Hence, the right ascension of the Pleiades cluster is approximately

- A. 1h 15m
- B. 3h 45m
- C. 5h 45m
- D. 8h 15m
- E. 15h 45m

Solution:

At full moon, the Moon and Sun are 12h RA apart. And then we know that the Moon rises 50min later every day (in local solar time). That would give us $2\text{h } 30\text{m} + 50 \times 1.5 = 3\text{h } 45\text{m}$.

16. Quasars must be small because they

- A. Have larger redshift
- B. Are very luminous
- C. Often emit radio waves
- D. Possess quasar fuzz
- E. Fluctuate in brightness rapidly

17. Which property of the cosmic microwave background (CMB) radiation suggest most strongly that it has cosmic origin?

- A. it has a blackbody spectrum
- B. its present temperature is small
- C. its intensity is nearly the same from all directions
- D. its peak intensity is in the millimetre part of the spectrum
- E. it has a very large redshift

18. The limiting magnitude of human eye having 7-mm eye pupil aperture is +6.00 mag. If a telescope has limiting magnitude of +16.16 mag because 20% of incident radiation is blocked, what is the real diameter of the telescope? **Hint:** *limiting magnitude defines the faintest star that is still observable by an instrument.*
- A. 602.8 mm
 - B. 674.0 mm
 - C. 769.3 mm
 - D. 842.5 mm
 - E. 941.9 mm

Solution:

Ratio of total light flux if no radiation is blocked = $(100/80) \times 2.512^{(16.16-6)} = 1.4491 \times 10^4$

Actual telescope diameter = $[(1.4491 \times 10^4)]^{0.5} \times 7 = 842.6608 \text{ mm}$

19. An asteroid has an elliptical orbit. The length of its semi-major axis is 2.52 AU. On January 2007, the asteroid was at its perihelion. When will it be at its aphelion?
- A. July 2010
 - B. January 2011
 - C. January 2008
 - D. July 2009
 - E. January 2009
20. Suppose C is a galaxy that is visible from a planet in a circular orbit around the star X. Galaxy C travels at a constant speed away from star X and the distance between galaxy C and X is very large. Given that the redshift of the galaxy C observed when the planet is at point A is 0.01 and the redshift of C observed when the planet is at point B is 0.02, determine the orbital speed of the planet orbiting star X. **Note:** *the redshift referred to here is the z-value, and that points A and B are points where the velocities are along the line of sight.*
- A. $1.48 \times 10^6 \text{ ms}^{-1}$
 - B. $1.51 \times 10^6 \text{ ms}^{-1}$
 - C. $3.00 \times 10^6 \text{ ms}^{-1}$
 - D. $4.46 \times 10^6 \text{ ms}^{-1}$
 - E. $4.55 \times 10^6 \text{ ms}^{-1}$

Solution:

From formula booklet,

$$z = \sqrt{\frac{c+v}{c-v}} - 1$$

Rearranging,

$$(z + 1)^2(c - v) = c + v$$
$$v = \frac{(z + 1)^2 - 1}{(z + 1)^2 + 1}c$$

Notice that at point A, $v_a = v_C - v_P$.

At point B, $v_b = v_C + v_P$.

Hence,

$$v_P = \frac{c}{2} \left[\frac{(z_b + 1)^2 - 1}{(z_a + 1)^2 + 1} - \frac{(z_a + 1)^2 - 1}{(z_a + 1)^2 + 1} \right] = 1.48 \times 10^6 \text{ms}^{-1}$$

21. Suppose instead that there two observers, who are stationary with respect to star X, at point A and point B respectively, what would be the redshift, z'_A and z'_B , observed by them?

- A. $z'_A < z_A, z'_B > z_B$
- B. $z_A < z'_A < z_B, z_A < z'_B < z_B$
- C. $z'_A > z_B, z'_B < z_A$
- D. $z'_A = z_A, z'_B = z_B$
- E. None of the above

Solution:

Since $v_C - v_P < v_C < v_C + v_P$, it is natural to expect that the new redshift would take a value in between the original redshifts.

22. Which of the following statements about the fate of the universe is false?

- A. If the universe has density more than that of the critical density, it will eventually collapse on itself.
- B. According to the Second law of thermodynamics (Entropy is always increasing), the Universe will experience heat death, where the eventual temperature of the universe would be extremely great.
- C. If the geometry of our universe is perfectly flat, our universe would not collapse on itself.
- D. The Big Rip is a theoretical model where expansion of the universe is so great that everything would be ripped apart because forces can no longer interact.
- E. The Big Crunch refers to a model where the universe would eventually collapse on itself.

23. Why can't we usually see green stars?

- A. There are green stars, but light from them have not reached us yet
- B. Green stars only appear when they are nearing the end of their life cycles

- C. A star which has a peak emission wavelength in green light also emits relatively similar amounts of red and blue light, causing it to appear nearly white
- D. The luminosity of green stars is usually too low for us to be able to even detect them
- E. Light from other stars drowns out the green light

24. If the temperature of the Sun was doubled, and its radius kept constant, by what (additive) factor will its absolute bolometric magnitude change?

- A. -16.0
- B. -3.0
- C. -2.0
- D. -0.3
- E. 3.0

25. Four statements about stars are written as follows.

- I. A yellow star (spectral type G) is, with 100% certainty, older than a blue star (Spectral type O & B).
- II. Blue main sequence stars (Spectral type O & B) burn out much faster than yellow stars (spectral type G).
- III. Betelgeuse is definitely older than our sun.
- IV. The more massive a star is, the longer its lifespan on the main sequence.

Which of the above statements is/are true?

- A. Statements II, III and IV
- B. Statement II only
- C. Statements II and III
- D. Statements I and II
- E. All of the statements are true

Solution:

I is wrong, it is possible (though rather unlikely) for a fresh main sequence G star to be younger than a highly evolved OB star.

III is incorrect, Betelgeuse has a higher mass than our Sun and thus has evolved off the main sequence first. The same reason explains why IV is wrong,

26. Which of the following statements regarding Population III stars is correct?

- A. Population III stars are very metal-poor due to their creation in the halo of galaxies, where heavy metals are rarely found
- B. Population III stars are very metal-poor due to their creation soon after the Big Bang, when heavy metals were extremely rare.
- C. Population III stars can be observed readily in the Universe today

- D. Population III stars formed late in the universe after population I and II stars
- E. Population III stars is a fictional concept

Solution:

Population III stars are the theorized, first-generation stars born immediately after the Big Bang. Because the Big Bang mostly produced hydrogen and helium and produced very little of heavy elements, Population III stars are theorized to be extremely low in metals.

27. One fateful winter evening, Orion the Hunter was found to have just crossed the local meridian. Given that Anilam, the middle star of the 3 along the "Belt of Orion" has RA/DEC: 5h 36min/-1°12'1", what is the local sidereal time at that instance?
- A. 23h 56min
 - B. 13h 36min
 - C. 12h 36min
 - D. 5h 36min**
 - E. 0h 00min

Remarks: Local Sidereal Time = RA of object passing through local meridian.

28. On a different day at local midnight, Orion the Hunter was found to have just crossed the local meridian too. Which constellation is the Sun approximately found in? Hint: at local midnight, the hour angle of the sun is defined to be 12h if we were to set 0h to be at noon.
- A. Taurus the Bull
 - B. Sirius the Dog
 - C. Aquila the Eagle
 - D. Virgo the Maiden
 - E. Scorpius the Scorpion**

Remarks: Scorpius and Orion the Hunter are practically opposites. So for one to be at "midnight", Scorpius would be at "noon" position. Knowing the story of Scorpius and Orion would help too.

29. It is the autumnal equinox on September 21st in Ecuador (1.8312° S, 78.1834° W, GMT -5h). At what local time would Ptolemy Cluster, M7, with RA/DEC: 17h 53min/-34° 47' 34", rise from the east?
- A. 22h 53min
 - B. 17h 53min
 - C. 11h 53min**
 - D. 7h 53min

E. 2h 53min

Remarks: At vernal equinox, local time = local sidereal time. At 17h 53min, the object would cross the local meridian. Hence 6 hours before it would have just risen from the east.

30. Where can we see the two brightest stars, Sirius (Right Ascension = $06^{\text{h}} 45^{\text{m}} 08.92^{\text{s}}$, Declination = $-16^{\circ} 42' 58.02''$) and Canopus (Right Ascension = $06^{\text{h}} 23^{\text{m}} 57.11^{\text{s}}$, Declination = $-52^{\circ} 41' 44.38''$), approximately rise at the same time? (assume they are visible in all the cities below)

- (A) Bandar Seri Begawan, Brunei Darussalam ($4^{\circ}53'N$, $114^{\circ}57'E$)
- (B) Washington DC, United States ($38^{\circ}54'N$, $77^{\circ}02'W$)
- (C) Jakarta, Indonesia ($6^{\circ}12'S$, $106^{\circ}48'E$)
- (D) Rio de Janeiro, Brazil ($22^{\circ}55'S$, $43^{\circ}12'W$)
- (E) Kathmandu, Nepal ($27^{\circ}42'N$, $85^{\circ}20'E$)

Solution:

1. The RA of Sirius and Canopus is almost the same.
2. The declination of Canopus is lower than Sirius.

Comparing RAs, we know that Sirius would rise 22 minutes later than Canopus at the Equator. Thus, to ensure both stars rise at exactly the same time, the location must be slightly north of the Equator. The only answer that is close to equator and on the northern hemisphere is option A (Bandar Seri Begawan).

Remarks:

If you have ever been stargazing in Singapore, you know that Sirius' and Canopus' rising time is very close. One can guess that the city's latitude must not be very far from Singapore.

31. Which of the following best describes why heavy-element production in the core of a massive star stops at Fe?

- A. When reaching certain temperature, the Fe atoms in the core will undergo a fission reaction to produce lighter elements.
- B. Fe has the lowest mass per nucleon among all the elements, and fusion of iron into heavier elements will require rather than release energy.
- C. The neutron degeneracy pressure in the core halts the fusion of Fe atom with other elements in the core to produce heavier elements.
- D. Heavier elements can only be produced during supernovae after the massive star collapses.
- E. The star is not massive enough to start the fusion reaction of Fe.

32. Given that the sun has been in existence for 10 billion years (10^{10} years) and that energy produced from the p-p chain ($4p \rightarrow \text{He} + 2e + 2\nu_e$) is approximately 26.73 MeV how much Helium is produced from this process in the sun, given that mass of Helium is approximately $4.00u$?
- A. 1.21×10^{26} kg
 - B. 4.83×10^{26} kg
 - C. 1.89×10^{29} kg
 - D. 4.83×10^{29} kg
 - E. 1.93×10^{30} kg

Solution:

Average Luminosity of Sun = 3.846×10^{26} W

Energy produced from the chain = 26.73×10^6 eV

Number He atoms produced over 10 bn years

$$= \frac{(L * \text{seconds in a year})}{(\text{energy produced for each He atom})} = \frac{(3.846 * 10^{26} * 10^{10} * 365 * 24 * 60 * 60)}{(26.73 * 10^6 / 1.602 * 10^{-19})}$$

Mass of Helium

$$= \frac{(L * \text{seconds in a year})}{(\text{energy produced for each He atom})} * \text{mass of each Helium atom}$$

$$= \frac{(3.846 * 10^{26} * 10^{10} * 365 * 24 * 60 * 60)}{(26.73 * 10^6 * 1.602 * 10^{-19})} * (4.00 * 1.661 * 10^{-27})$$

Evaluating the above gives 1.89×10^{29} kg

Read the following paragraph to answer questions 33 to 36:

The Lagoon Nebula, M8 is relatively bright summer object that is visible to the naked eye under dark skies. It is found that it has a local hour-angle (HA) of 18h 27min on the evening of 15th June 2017 at 2000H (SGT). Use this information to answer question 4 and 5. The local hour-angle is defined hours since crossing the local meridian (00h 00min).

33. Understanding that the light pollution in Singapore is bad, Ivan wishes to image this object while it is higher up in the night sky (HA: 20h 00min). At approximately what time would you recommend that he begin imaging M8 on 24th June 2017 (9 days from 15th June 2017) to get the most exposure duration before it reaches the local meridian. The following times listed are all in the Singapore time (SGT).
- A. 1930H
 - B. 2000H
 - C. 2030H
 - D. 2100H
 - E. 2130H

Solution:

Given that celestial objects and stars alike rises 4 minutes earlier every day it hence also has an increase in the local hour-angle by 4 minutes every day too. 9×4 minutes = 36 minutes. HA of Lagoon Nebula on 24th June at 2000H is in fact 19h 03min. As such, to allow for sufficient clearance Ivan would have to wait approximately an hour later (2100H) for it to have an HA of 20h 00min to begin imaging.

34. If Ivan were to begin imaging M8 at 2200H on 24th June 2017, approximately how much time would he have before the object reaches local meridian? Find the maximum time he would have.

- A. 1h 30min
- B. 2h 00min
- C. 2h 30min
- D. 3h 00min
- E. 3h 30min

Solution:

Having found out that M8 would have HA of 19h 03min at 2000H that evening, it would reach the local meridian at 0100H the following morning. Ivan would hence have approximately 3h to image the Lagoon Nebula.

Use the information below to answer questions 35 and 36:

In preparation for this day, 24th June 2017, Ivan was deciding between the Lagoon Nebula and the Ring Nebula, M57 to image. Some of the considerations include the surface brightness and the size of the object. Some parameters of these objects are given below:

Lagoon Nebula
RA/DEC: 18h 03m/-24° 22'
Magnitude (m): 6.00
Size (A): 1°30' 00" × 40' 00"

Ring Nebula in Lyra (M57)
RA/DEC: 18h 54m/33° 01'
Magnitude (m): 8.80
Size (A): 4'00" × 2' 00"

Surface brightness, $S = m + 2.5 \text{ Log}_{10}$ (Area of object in square arc-minutes)

35. Calculate the surface brightness of both the Lagoon Nebula as well as the Ring Nebula respectively. You may assume the surface area of the objects to be rectangular.

- A. 14.89; 11.06

- B. 14.89; 21.06
- C. 15.29; 11.06
- D. 15.29; 21.06
- E. 25.29; 21.06

Remarks: Apply given formula with appropriate conversion of units.

36. However, Ivan chose to image the Lagoon Nebula instead, suggest which of the following is the best possible reason why he made this choice.

- A. The Lagoon Nebula would fill up his image frame better as compared to the Ring Nebula which is much too small.
- B. He would be able to fit M42, the Orion Nebula within the frame of view should he decide to image the Lagoon Nebula.
- C. The Ring Nebula would be setting and hidden behind the HDB buildings while the Lagoon Nebula would have just cleared the buildings.
- D. The magnitude of an object is a better indication of its suitability to be imaged/viewed as compared to the surface brightness.
- E. He made a wrong choice; the Ring Nebula is the better choice given that it has a larger surface brightness value.

Solution:

The Orion Nebula is on the opposite end of the celestial sphere; if you go out to observe that would be obviously. Further, the ring Nebula would only rise approximately 1 hour later than the Lagoon Nebula (from the RA values), how could it possibly set before the Lagoon Nebula? As for the last option, surface brightness shouldn't be your sole consideration in imaging; otherwise we would be imaging only bright stars like Sirius!

37. Five statements about eclipses are given as follows:

- I. Globally, lunar eclipses (in all its phases) occur approximately as frequently as solar eclipses (in all its phases).
- II. It is highly likely for an eclipse to occur on 8th March 2035, 19 years from 8th March 2016 according to the Metonic Cycle. Recall that there was a solar eclipse on 8th March 2016.
- III. Lunar eclipses can occur during a full moon and new moon.
- IV. Total lunar eclipses are expected to increase in frequency over the years.
- V. Total solar eclipses are expected to increase in frequency over the years.

Which of the above statements are incorrect?

- A. Statements I, II, V
- B. Statements I, III, V

- C. Statements I, IV, V
- D. Statements II, III, IV
- E. Statements III, IV, V

Solution:

I: A rather little-known fact: lunar eclipses do occur approximately as frequently as solar eclipses if we were to include penumbral eclipses. II: Recall the Metonic Cycle. III: Lunar eclipses only occur on a full moon, almost duh. IV and V: While lunar and solar eclipses are expected to decrease in frequency as the moon slowly moves further and further away from us. How slowly? That's for another day.

Read the following paragraph to answer questions 38 and 39:

Bella is a Jupiter size (mass) planet that orbits around the main sequence star, Bella A with a luminosity double that of the Sun ($L = 2L_{\odot}$). It was found to have an orbital semi-major axis equivalent to Jupiter around the Sun.

38. Assuming Bella to be the only planet orbiting around Bella A, calculate the gravitational force of attraction that Bella A exerts on Bella. You may assume circular orbits.

- A. 3.68×10^{22} N
- B. 4.16×10^{23} N
- C. 5.07×10^{23} N
- D. 8.32×10^{23} N
- E. 4.71×10^{24} N

Solution:

$$\frac{L}{L_A} = \left(\frac{M}{M_A}\right)^{3.5},$$

where L and M refers to the Luminosity and Mass of the Sun respectively. While L_A and M_A refers to the Luminosity and Mass of Bella A respectively.

$$M_A = M \times 2^{\frac{1}{3.5}}$$

$$F = \frac{GmM_A}{R^2} = \frac{6.67384 \times 10^{-11} \times 1.899 \times 10^{27} \times 2^{\frac{1}{3.5}} \times 1.989 \times 10^{30}}{(7.785 \times 10^{11})^2}$$

where m and R refers to the mass of the Bella and orbital radius (orbital semi-major axis, a).

39. Would you expect the period of orbit to be larger or smaller for Bella as compared to the orbital period of Jupiter (around the Sun)? Would you expect the period to increase or decrease if the orbit is found to have an eccentricity of 0.8?

- A. Smaller; the same
- B. Smaller; decrease

- C. Smaller; increase
- D. Larger; increase
- E. Larger; decrease

Solution:

Larger force, smaller than period given by the equation for circular motion, $F = mR\omega^2$, where ω is inversely proportional to the period T of orbit.
 Eccentricity essentially does not affect the calculation of the period as the calculations actually uses the orbital semi-major axis, a.

40. A photograph of the Andromeda galaxy (diffuse object) was taken with the following two optical instruments (assume 100% reflectivity and transmissivity where applicable):
- I. Refracting telescope: 100mm aperture, focal ratio f/8
 - II. Newtonian Reflector: 200mm aperture, focal ratio f/4, Diameter of Central Obstruction: 60mm

In order to generate the signal-to-noise ratio (SNR) of a 5-minute exposure on system I, what is the required exposure length on system II, assuming all other factors are constant?

- A. 1.4 minutes
- B. 2.8 minutes
- C. 3.5 minutes
- D. 5.0 minutes
- E. 10.0 minutes

Solution:

Both instruments have the same focal length of 800mm.
 The effective light collecting area of Instrument I is $100^2 = 10,000\text{mm}^2$
 The effective light collecting area of Instrument II is $200^2 - 60^2 = 36400\text{mm}^2$
 Ratio of light collecting power of instrument I to instrument II is 1:3.64
 For a 5-minute exposure on I, $5/3.64 = 1.4$ minutes is required on instrument II.

41. The largest known galaxy in the Universe is IC 1101, which lies at a distance of some 1.04 billion light years from the Earth. Estimate the wavelength of the Hydrogen Alpha line emitted from the Earth as observed from this galaxy.
 (Given: $\lambda_{em} = 656.281\text{nm}$)
- A. 642.1nm
 - B. 662.5nm
 - C. 684.3nm

D. 696.2mm

E. 703.5nm

Solution:

$$1.04\text{bly} = 319 \text{ Mpc}$$

$$\text{Recession velocity} = 67.8 \times 319 = 21,600 \text{ km/s}$$

$$z = v/c = 21,600/300,000 = 0.072$$

$$\lambda_{\text{obs}} = (0.072\lambda_{\text{em}}) + \lambda_{\text{em}} = 703.5 \text{ nm}$$

42. The Kennedy Space Center is located in Florida, USA and has the geographical coordinates of 28.5729° N , 80.6490° W . In 2050, Singapore decided to begin its own space program and created a launch site at Changi Airport (1.3644° N , 103.9915° E).

Calculate the difference in energy required to launch a mass of 10,000 kg into Low Earth Orbit at 300km above sea level into the direction of Earth's rotation.

Take radius of Earth to be 6,380km. Pick the option closest to your answer.

A. $1.5 \times 10^7 \text{ J}$

B. $3.6 \times 10^7 \text{ J}$

C. $6.2 \times 10^7 \text{ J}$

D. $1.2 \times 10^8 \text{ J}$

E. $2.5 \times 10^8 \text{ J}$

Solution:

Calculate difference in initial GPE + KE and final GPE + KE

Final GPE + KE is the same for both cases

Assume Initial GPE to be the same. Difference in initial KE can be calculated as follows:

$$\text{At Florida, distance from surface to Earth's axis, } r_1 = 6,380,000 \cdot \cos 28.57 = 5.603 \times 10^6 \text{ m}$$

$$\text{At Singapore, distance from surface to Earth's axis, } r_2 = 6,380,000 \cdot \cos 1.36 = 6.378 \times 10^6 \text{ m}$$

$$\text{Angular velocity of Earth's rotation} = 2\pi / T = 2\pi / (23.9 \cdot 3600) = 7.293 \times 10^{-5} \text{ s}^{-1}$$

Therefore, velocity due to Earth's rotation is as follows:

$$\text{At Florida, } v_1 = r_1 \cdot \omega = 5.603 \times 10^6 \times 7.293 \times 10^{-5} = 408.6 \text{ m/s}$$

$$\text{At Singapore, } v_2 = r_2 \cdot \omega = 6.378 \times 10^6 \times 7.293 \times 10^{-5} = 465.2 \text{ m/s}$$

$$\text{Difference in KE} = 0.5 \times 10,000 \times (465.2^2 - 408.6^2) = 2.5 \times 10^8 \text{ J}$$

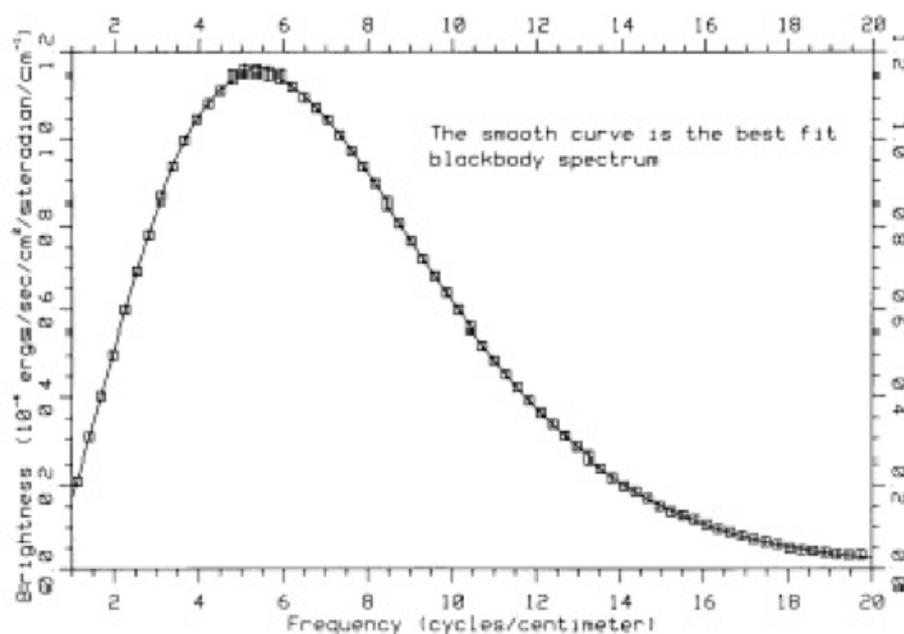
43. Which of the following is not a method of reducing the visibility of chromatic aberration in refracting telescopes?

- A. Increasing the focal length of the telescope
- B. Viewing the object at higher magnification
- C. Using low dispersion glass
- D. Upgrading to an apochromatic set-up
- E. Use of violet fringing filters

Solution:

- A) An increasing focal length creates a larger depth of field, decreasing the degree to which each wavelength of light becomes out of focus with respect to another
- B) Viewing an object at a higher magnification doesn't reduce CA, and actually makes it more visible because the effect now becomes magnified
- C) Using low dispersion glass reduces the effect of optical dispersion, the cause of CA
- D) Adding an additional optical element allows more wavelengths of light to be focused onto the same point
- E) Violet Fringe filters remove blue-violet wavelengths which are usually uncorrected in achromatic refractors

44. The figure below (Mather et al., 1990) shows the radiation spectrum of a certain body plotted together with its best-fit blackbody spectrum (the smooth curve). The y-axis is the brightness (in $10^{-7} \text{ W m}^{-2} \text{ steradian}^{-1} \text{ cm}^{-1}$ converted to SI units) while the x-axis is the frequency (in number of cycles per cm). What does this tell us about the blackbody and the significance of this result?



- A. This body is the Sun. It shows that the Sun can be modeled very well as a blackbody and this gives a very good estimation of the intensity of the Sun on the Earth's surface.
- B. This shows the Stefan-Boltzman radiation law, which produced the best-fit curve, matches very well with experimental results, showing the beauty of physics.
- C. This body is the cosmic microwave background radiation (CMBR) of the universe. The detection and measurement of the CMBR has cemented the Big Bang theory's place as the leading model for cosmology.
- D. This figure was found in the Nobel Prize winning paper for physics in 1964 "for fundamental work in the field of quantum electronics, which has led to the construction of oscillators and amplifiers based on the maser-laser principle".
- E. The above is an attempt to plot the absorption spectrum of a star. The presence of a curve that can fit the data smoothly shows that the absorption lines are few and far in between. More precise measurements are needed to obtain the absorption spectrum.

Solution:

A is wrong, the brightness of the sun is way too high (certainly not 10^{-7} W at certain frequencies). Neither does B make sense, the Stefan-Boltzmann law is a relationship between total luminosity versus temperature (which isn't on the X-axis). E is also rather wrong, all real stars have absorption lines at the temperature ranges we deal with. As for D, it should hopefully be obvious that blackbody radiation has little apparent relationship with the construction of masers/lasers

This is a graph in the paper titled *A preliminary measurement of the cosmic microwave background spectrum by the Cosmic Background Explorer (COBE) satellite* by the mentioned authors. It was a measurement of the spectrum of the CMBR. The temperature of the blackbody can be calculated from the graph using the frequency variant of Wien's displacement law: $f_{\max}/T = 5.88 \times 10^{10} \text{ Hz K}^{-1}$, which is found to be around 2.73K. This corresponds to the temperature of the CMBR measured by the WMAP.

45. The Roche limit states that a body will disintegrate due to tidal forces, yet many earth-orbiting satellites still come within the Roche limit. Which of the following adequately explains this discrepancy?
- A. The Roche limit only applies for large bodies
 - B. The Roche limit only applies to objects whose integrity primarily relies on gravitational equilibrium
 - C. The Roche limit only applies to objects with a fluid core
 - D. The Roche limit applies to bodies with primarily rocky mass
 - E. The Roche limit applies to completely fluid objects

Solution:

This is indeed the key reason why satellites in low-earth orbit can hold together. If we only considered self-gravity, calculations can easily show that these satellites must have densities much greater than that of lead. To keep them together, tension within the satellite resists the force of gravity, preventing the satellite from breaking up.

F.

46. Which of these is a consequence of the postulates of Special Relativity?

- A. Observers would observe a different speed of light in different inertial frames
- B. The observer in the stationary frame would notice that time is moving slower in his/her frame
- C. The observer in the moving frame would notice that he/she can move faster than the speed of light
- D. Observers in different inertial frames may observe a different order of events
- E. None of the above

Solution:

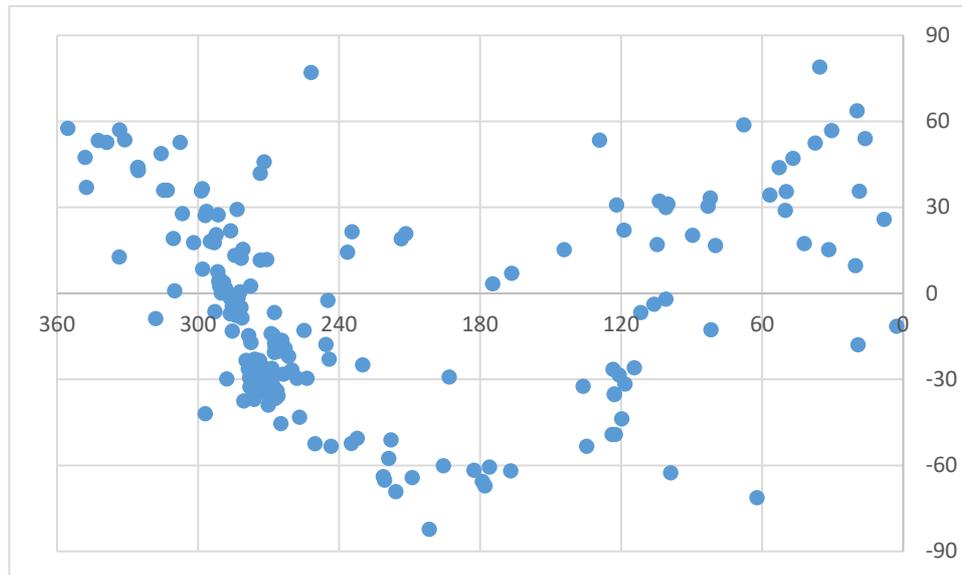
The first option is wrong, because one of the postulate states that speed of light is the same in all inertial frames (covariance). The third option is also wrong because nothing can move faster than the speed of light by SR. The second option tackles time dilation, but time should always appear FASTER in your own frame.

47. An astronomer is observing 5 different galaxies to determine its angular velocities which will then be plotted. On which of the following type of galaxies can we use the Tully-Fisher relation?

- A. Elliptical galaxies
- B. Lenticular galaxies
- C. Spiral galaxies
- D. Irregular galaxies
- E. All of the above may be plotted

48. The following diagram shows all observed instances of a specific type of event (up to an apparent magnitude of 10.0), plotted onto the celestial sphere. The horizontal and vertical axes refer to RA and DE respectively, in units of degrees.

NB: In this map, 0 degrees in right ascension would correspond to 0h, and so on.
Note that the map is in reverse order

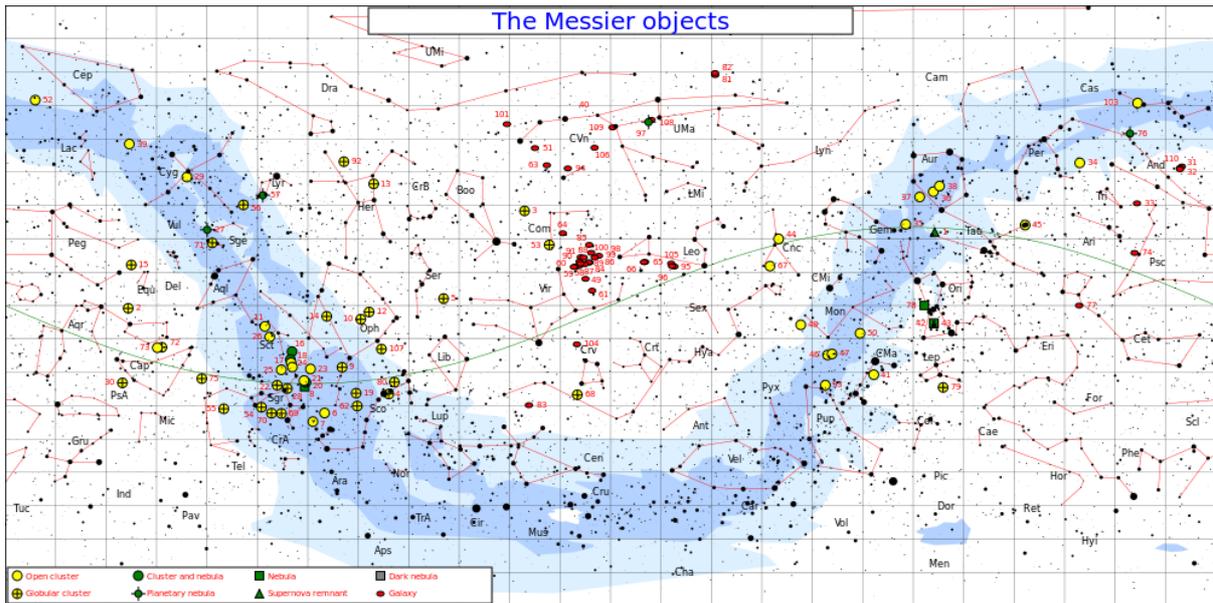


Given the spatial distribution of events, the diagram most likely captures:

- A. Observed supernovae at peak brightness
- B. Position of bright comets at perihelion (at a specific observing location)
- C. Position of bright novae at peak brightness
- D. Position of Uranus at opposition (at a specific observing location)
- E. Apparent radiant of bright meteors over the course of a month

Solution:

Firstly, note that these events are clustered along the plane of the Milky Way (Easily verified using the attached star map). This quickly rules out B, D and E: we do not expect these occurrences to be remotely linked to the Milky Way. This leaves A. Recall that supernovae are rare occurrences: we'll expect to see only one occur in our galaxy every century or so (before accounting for their visibility). Thus, even if we go back to the earliest recorded supernova (AD 185), there are way too many events in this map to depict observed supernovae.



49. After looking at this star map of the Messier Catalogue, a Japanese astronomer comments that spring is his favourite season. Which of the following rationales could plausibly justify this choice?

- During spring, the galactic center culminates near midnight, allowing spectacular views of the Milky Way bulge all night
- With right ascensions of $10^{\text{h}}-13^{\text{h}}$, treasures like Omega Centauri, the Carina Nebula and the Coalsack Nebula are visible nearly all night in Japan during this period.
- With right ascensions near 0^{h} , spring is a good opportunity to observe objects in the Perseus-Cassiopeia region of the Milky Way. Plus, the famous Andromeda Galaxy is visible all night.
- The rewarding sight of spying Markarian's Chain and the challenge of finding the Sombrero Galaxy amidst the crowded field of the Virgo Cluster.
- Sweet spring memories with a special someone, watching the rising Pleiades fade into twilight while waiting for sunrise.

Hint: The Right Ascension axis is plotted in reverse order (decreasing from left to right). 0^{h} is on the right edge of this map.

Solution:

A: the Milky Way center does not culminate at midnight in spring, it only does so in late June (early summer)

B: This would be accurate...if not for the fact that these objects are so far South. They're thus extremely difficult (if not impossible) to observe from these latitudes

C/E: These objects are best seen in Autumn. The Pleiades are best seen in early

winter, thus it is only possible to view the rising Pleiades fade into the sunrise in late summer/autumn (definitely not spring).

50. Procyon (RA 7h 39m, DE +05° 13') is known to culminate at local midnight on January 14th for an observer in Moscow (55°45'N, 37°37'E). On which date will Procyon culminate at local midnight for an observer located in Singapore (1° 17' N, 103° 51' E)?

- A. January 1st
- B. January 14th
- C. January 27th
- D. March 20th
- E. November 8th

Solution:

Local midnight in astronomy has a very specific connotation: it refers to the point when the Sun is crossing the lower meridian, NOT 12 midnight on the clock. Thus, given that we are working on local solar time (as implied in the question), culmination time does not depend on location (assuming it is visible, of course)